

## ADJACENT DEGREE POLYNOMIAL OF GRAPHS

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**Abstract**

We introduce a special type of polynomial associated to every  $(p, q)$ -graph  $G$ . The adjacent degree polynomial of a graph  $G$  depends the sum of the degrees of the vertices in the neighborhood of all vertices in  $G$ . This paper mainly focuses on adjacent degree polynomial of graphs and shall attempt to compute the adjacent degree index of some special type of graphs. We also establish some fundamental properties of adjacent degree index of certain graphs.

**Keywords**

Adjacent degree polynomial of a graph, adjacent degree index of a graph.

**1. Introduction**

By a  $(p, q)$ -graph  $G = (V(G), E(G))$  we mean a finite undirected simple graph with  $q > 1$  and having no isolated vertex. Let  $v \in V(G)$  and the neighborhood of  $v$  is the set  $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$ . The degree of  $v \in V(G)$  is equal to the cardinality of  $N_G(v)$  and is denoted by  $d_G(v)$ .  $\delta$  is the minimum degree of a graph. or various graph theoretic notations and terminology we follow F. Harrary [1] and for basic number theoretic results we refer [4].

**2. Adjacent degree polynomial of graphs**

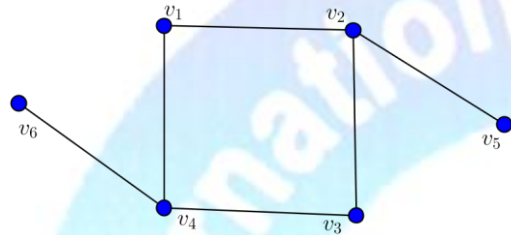
**Definition 2.1** Let  $G$  be a  $(p, q)$ -graph with vertex set  $V(G) = \{v_1, v_2, \dots, v_p\}$ .

Let  $\sum_{v_i \in V(G)} |N(v_i)| = k_i$  for  $1 \leq i \leq p$ .

Then the polynomial defined by  $\mu_G(x) = \sum_{i=1}^p x^{k_i}$  is called the adjacent degree polynomial associated with  $G$  and  $\mu_G(q)(\text{mod } p)$  is called the adjacent degree index of  $G$  and is denoted by  $In_{adj}(G)$ .

The following are some simple observations which follow immediately from the definition of a adjacent degree polynomial.

Example :



$$\mu_G(x) = 2x^3 + 2x^5 + 2x^6.$$

**Observation 2.2** Sum of the coefficient of  $\mu_G(x)$  is the number of vertices of  $G$ . **Observation 2.3** For an Euler graph  $G$ ,  $\mu_G(x)$  is an even function.

**Observation 2.4** For a  $k$ -regular graph  $G$ ,  $\mu_G(x) = px^{k^2}$  and thus for a complete graph  $K_n$ ,  $\mu_{K_n}(x) = nx^{(n-1)^2}$ .

**Observation 2.5** For Star graphs  $K_{1n}$ ,  $\mu_{K_{1n}}(x) = (n+1)x^n$

**Observation 2.6** For Paths  $P_n$ ,  $\mu_{P_2}(x) = 2x, \mu_{P_3}(x) = 3x^2$

For  $n \geq 4$ ,  $\mu_{P_n}(x) = 2x^2 + 2x^3 + (n-4)x^4$

**Observation 2.7** For Cycles  $C_n$ ,  $\mu_{C_n}(x) = nx^4$

**Observation 2.8** For Complete Bipartite graphs  $K_{mn}$ ,  $\mu_{K_{mn}}(x) = (n+m)x^{mn}$

**Observation 2.9** For Wheels  $W_{1n}$ ,  $\mu_{W_{1n}}(x) = nx^{6+n} + x^{3n}$

**Theorem 2.10** : Let  $G$  is a  $k$ -regular graph of order  $n$  then adjacent degree polynomial of  $G^c$  is  $\mu_{G^c}(x) = nx^{(n-1-k)^2}$ .

**Proof** : Let  $G$  be a  $k$ -regular graph of order  $n$  then  $\mu_G(x) = nx^{k^2}$ .  $G^c$  is  $(n-1-k)$ -regular graph of order  $n$ . So  $\mu_{G^c}(x) = nx^{(n-1-k)^2}$ .

### 3. Adjacent degree index of graphs

**Theorem 3.1** : For star graphs  $K_{1n}, In_{adj}(K_{1n}) = 0$

**Proof** : Star graphs  $K_{1n}$  has  $n+1$  vertices and  $n$  edges.

$$p = n+1, q = n \text{ and } \mu_{K_{1n}}(x) = (n+1)x^n$$

$$In_{adj}(K_{1n}) = \mu_{K_{1n}}(q) \pmod{p} = (n+1)n^n \pmod{n+1} = 0$$

**Theorem 3.2** : For Paths  $P_n, In_{adj}(P_n) = n - 4$ , for  $n \geq 4$

**Proof** For Paths  $P_n$  has  $n$  vertices and  $n-1$  edges.

$$p = n, q = n-1 \text{ and } \mu_{P_n}(x) = 2x^2 + 2x^3 + (n-4)x^4$$

$$\begin{aligned} In_{adj}(P_n) &= \mu_{P_n}(q) \pmod{p} = 2(n-1)^2 + 2(n-1)^3 + (n-4)(n-1)^4 \pmod{n} \\ &= 2 - 2 + n - 4 \pmod{n} = (n-4) \pmod{n}. \end{aligned}$$

**Theorem 3.3** : For  $k$ -regular graph  $G, In_{adj}(G) = 0$

**Proof** : If  $G$  has  $n$  vertices then  $G$  has  $\frac{nk}{2}$  edges.

$$p = n, q = \frac{nk}{2} \text{ and } \mu_G(x) = px^{k^2}$$

$$In_{adj}(G) = \mu_G(q) \pmod{p} = n \left(\frac{nk}{2}\right)^n \pmod{n} = 0$$

**Theorem 3.4** : For Cycles,  $In_{adj}(C_n) = 0$

**Proof** : For Cycles  $C_n$  has  $n$  vertices and  $n$  edges.

$$p = n, q = n \text{ and } \mu_{C_n}(x) = nx^4$$

$$In_{adj}(C_n) = \mu_{C_n}(q) \pmod{p} = n(n)^4 \pmod{n} = 0$$

**Theorem 3.5 :** For Complete Bipartite graphs  $K_{mn}$ ,  $In_{adj}(K_{mn}) = 0$

**Proof :** For Complete Bipartite graphs  $K_{mn}$  has  $m+n$  vertices and  $mn$  edges.

$$p = m+n, q = mn \text{ and } \mu_{K_{mn}}(x) = (n+m)x^{mn}$$

$$In_{adj}(K_{mn}) = \mu_{K_{mn}}(q) \pmod{p} = (n+m)(mn)^{mn} \pmod{m+n} = 0$$

**Theorem 3.6 :** For Wheels  $W_{1n}$ ,  $In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n} \pmod{n+1}$

**Proof :** For Wheels  $W_{1n}$  has  $n+1$  vertices and  $2n$  edges.

$$p = n+1, q = 2n \text{ and } \mu_{W_{1n}}(x) = nx^{6+n} + x^{3n}$$

$$In_{adj}(W_{1n}) = \mu_{W_{1n}}(q) \pmod{p} = n(2n)^{6+n} + (2n)^{3n} \pmod{n+1} = -(-2)^{6+n} + (-2)^{3n} \pmod{n+1}$$

**Corollary 3.7 :** For Wheels  $W_{1n}$ ,  $In_{adj}(W_{1n}) = -2^6 + 1 \pmod{n+1}$ , If  $n+1$  is prime.

**Proof :** For Wheels  $W_{1n}$   $In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n} \pmod{n+1}$

If  $n+1$  is prime,  $(-2)^n = 1 \pmod{n+1}$  then

$$In_{adj}(W_{1n}) = -(-2)^{6+n} + (-2)^{3n} \pmod{n+1}$$

$$= -[(-2)^6(-2)^n] + (-2)^{3n} \pmod{n+1} = -[-2^6] + 1 \pmod{n+1} = -2^6 + 1 \pmod{n+1}$$

**Theorem 3.6:** Let  $G$  be  $(p, q)$  graph with  $p = q$  then  $In_{adj}(G)$  is the constant term of  $\mu_G(x)$

**Proof :** Let  $\mu_G(x) = a_0 + a_1x + \dots + a_kx^k$  For Wheels

$$In_{adj}(G) = \mu_G(q) \pmod{p} = a_0 + a_1q + \dots + a_kq^k$$

$$= a_0 + a_1p + \dots + a_kp^k \pmod{p} = a_0$$

**Corollary 3.6:** Let  $G$  be  $(p, q)$  graph with  $p = q$ ,  $\delta > 0$  then  $In_{adj}(G) = 0$  .

**Proof :** Since there is no isolated vertex  $|N_G(v)| \neq 0$  for any vertex  $v$  in  $G$ .

Then the constant term of  $\mu_G(x) = 0$ . By above theorem  $In_{adj}(G) = 0$ .

**Theorem 3.6:** Let  $G$  be  $(p, q)$  graph then  $\mu_G(x) = 2mx \Leftrightarrow G \cong mK_2, m > 0$

**Proof :** Suppose  $\mu_G(x) = 2mx$  then there is only one adjacent vertex and which have degree one so which is  $mK_2$ . Conversely suppose  $G \cong mK_2$  using the definition clearly we get  $\mu_G(x) = 2mx$ .

**Theorem 3.6:** For any integer  $n$  with  $In_{adj}(G) = n$  there always exist a graph  $G$

**Proof :** Let  $n$  any integer, consider the Path  $P_{n+4}, In_{adj}(P_{n+4}) = n$ .

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