Nonlinear Electron-Acoustic Waves In Relativistic Plasma In The Presence Of Nonthermal Electrons

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ABSTRACT

The problem of relativistic electron acoustic waves is discussed using the reductive perturbation theory in Collisionless, Unmagnetized plasma consisting of relativistic cold electrons, hot electrons obeying non maxellian distributions and stationary ions. The Korteweg De Vries (KdV) equation has been derived. finally the effects of relativistic electrons on the amplitude and width of small amplitude electron acoustic waves are investigated.

Keywords: *Electron acoustic wave; Relativistic electrons; KdV equation.*

1. INTRODUCTION

The investigations of the electron acoustic waves (EAW) have attracted much attention in the studying of nonlinear waves in plasmas because of their potential importance in interpreting the electrostatic component of the broadband electrostatic noise observed in the cups of the terrestrial magnetosphere [1,2] and in the geomagnetic tail [3]. The electron acoustic wave plays an important role in space environment [4-7]. EA wave is characterized by two electron populations, referred to as cool and hot electrons. The cool electrons provide the inertia necessary to maintain the electrostatic oscillations, while the restoring force comes from hot electron pressure. The propagation of EASWs in a plasma system has been studied by several investigators in Unmagnetized two electon plasmas [9-11] as well as in magnetized plasma [12-16]. Space plasma observations indicate clearly the presence of electron populations which are far away from their thermodynamic equilibrium [17–25]. Numerous observations of space plasmas [26–27] clearly prove the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. Ghosh et al. demonstrated that both negative as well as positive nonplanar ion acoustic solitons are formed in the plasmas with superthermal electrons and positrons.

Superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas or because of wave–particle interactions. Plasmas with an excess of superthermal (non-Maxwellian) electrons are generally characterized by a long tail in the high energy region. Thus the hot electrons follow a vortex - like distribution in the most cases [8-12]. Most of these investigations on linear and non-linear phenomenon are confined to non relativistic plasmas. But when electron velocity approaches the velocity of light, relativistic effects may significantly modify the soliton behaviour. Relativistic plasmas occur in a variety of situations e.g. in laser-plasma interaction, space plasma phenomenon, plasma sheet boundary layer of earth's magnetosphere. As plasma with relativistic velocities are frequently observed in astrophysical and space environments, so there is a need to study electron-acoustic waves in three component plasma with weakly relativistic electrons.

In this paper we will drive the phase velocity from the basic set of equation for plasma by reductive perturbation theory and study the effect of relativistic electrons on Unmagnetized Collisionless plasma consisting of cold relativistic electron fluid, non Maxwellian hot electrons and stationary ions. This paper is organised as follow, in section 2, we present the basic set of fluid equations in three component plasma model and KdV equation is derived using reductive perturbation method. Results are discussed in section 3 by plotting graph. Finally some conclusions are given in section 4.

2. BASIC EQUATIONS

We consider Unmagnetized Collisionless plasma consisting of cold relativistic electron fluid, non Maxwellian hot electrons and stationary ions. The non linear dynamics of the electron acoustic solitary waves is governed by the continuity and motion equations for relativistic cold electron and Poisson's equation.

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial (\gamma_c u_c)}{\partial t} + u_c \frac{\partial (\gamma_c u_c)}{\partial x} + \frac{3\alpha(1+\alpha)^2 n_c}{\theta} \frac{\partial n_c}{\partial x} - \alpha \frac{\partial \phi}{\partial x} = 0 \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\alpha} n_c + n_h - \left(1 + \frac{1}{\alpha}\right) \quad (3)$$

where

$$n_h = (1 - \beta\phi + \beta\phi^2)e^{\phi}, \gamma_c = 1 + \frac{u_c^2}{2c^2}$$

$\theta = \frac{T_h}{T_c}$ and $\alpha = \frac{n_{h0}}{n_{c0}}$, n_c and n_h are the normalized number densities of cold and hot electrons, respectively. ϕ is the normalized electrostatic potential, u_c is the normalized velocities of cold electron in the x direction. x and t are also normalized. The normalization is as follows. The densities of cold and hot electrons are normalized by n_{co} and n_{ho}. The space coordinates x time t, velocity and electrostatic potential ϕ are normalized by the hot electron Debye length $(K_B T_h / 4\pi\, n_{ho} e^2)^{1/2}$, inverse of cold electron plasma frequency $\omega_{pc}^{-1} = \sqrt{(m/4\pi\, n_{co} e^2)}$, $c_e = \sqrt{(K_B T_h/\alpha m)}$, and $\frac{K_B T_h}{e}$, respectively. Here m is the electron mass, e is the magnitude of the electron charge and K_B is the Boltzmann constant. According to RPM, the independent variables are scaled as

$$\xi = \epsilon^{1/2}(x - v_g t), \quad (4)$$

$$\tau = \epsilon^{3/2} t, \quad (5)$$

Where ϵ is small (0< ϵ<1) expansion parameter charcterizing the strength of the non-linearity and v_g is the phase velocity of the electron-acoustic wave.The dependent variables are expanded around the equilibrium values in the power of ϵ in the following forms :

$$n_c = 1 + \epsilon n_1 + \epsilon^2 n_2 + \epsilon^3 n_3 + \ldots\ldots\ldots$$

$$u_c = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 u_3 + \ldots\ldots\ldots$$

$$\phi_c = \epsilon\phi_1 + \epsilon^2\phi_2 + \epsilon^3\phi_3 + \ldots\ldots\ldots \quad (6)$$

Using stretching coordinates Eqs.(4-5) and Eq.(6) in Eqs.(1-3) and collecting the terms of ϵ . We obtain the following equations in lowest power of :

$$-v_g\frac{\partial n_1}{\partial \xi} + \frac{\partial u_1}{\partial \xi} + u_0\frac{\partial n_1}{\partial \xi} = 0 \quad (7)$$

$$-v_g\gamma_1\frac{\partial u_1}{\partial \xi} + u_0\gamma_1\frac{\partial u_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta}\frac{\partial n_1}{\partial \xi} - \alpha\frac{\partial \phi_1}{\partial \xi} = 0$$
(8)

$$(1-\beta)\phi_1 + \frac{1}{\alpha}n_1 = 0 \quad (9)$$

From (7) to (9), we can express the first order quantities in terms of ϕ_1 as

$$n_1 = -\alpha(1-\beta)\phi_1 \quad (10)$$

$$u_1 = -\alpha(1-\beta)(v_g - u_0)\phi_1 \quad (11)$$

where

$\gamma_1 = 1 + \frac{3}{2}U^2$, U = $\frac{u_0}{c}$

Algebraic manipulations of these equations lead to the following dispersion relation

$$V = v_g - u_0 = \sqrt{\frac{1}{\gamma_1}\left[\frac{3\alpha(1+\alpha)^2}{\theta} + \frac{1}{1-\beta}\right]} \quad (12)$$

The phase velocity given by Eq.(12) depends on relativistic factor U(u_0/c), cold to hot electron density ratio(α), hot to cold electron temperature ratio(θ) and nonthermal electrons distribution parameter β.

To next order of ε, we get

$$\frac{\partial n_1}{\partial \tau} - (v_g - u_0)\frac{\partial n_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \frac{\partial(n_1u_1)}{\partial \xi} = 0$$
(13)

$$\gamma_1\frac{\partial u_1}{\partial \tau} - (v_g - u_0)\gamma_1\frac{\partial u_2}{\partial \xi} + [\gamma_1 - 2\gamma_2\left(\frac{v_g - u_0}{u_0}\right)u_1\frac{\partial u_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta}n_1\frac{\partial n_1}{\partial \xi} + \frac{3\alpha(1+\alpha)^2}{\theta}\frac{\partial n_2}{\partial \xi} - \alpha\frac{\partial \phi_2}{\partial \xi} = 0$$
(14)

$$\frac{\partial^2\phi_1}{\partial \xi^2} = (1-\beta)\phi_2 + \frac{1}{\alpha}n_2 + \frac{1}{2}\phi_1^2$$
(15)

where

$\gamma_1 = 1 + \frac{3}{2}U^2$, $\gamma_2 = \frac{3}{2}U^2$, $U = \frac{u_0}{c}$

After some algebraic manipulations, second order quantities are eliminated and ϕ_1 is found to satisfy the following KdV equation

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \qquad (16)$$

where A is the coefficient of non - linear term and B is the coefficient of dispersion term.

The non - linearity coefficient A and dispersive coefficient B are defined as follow :

$$A = \frac{(v_g - u_0)}{2(1-\beta)} - \frac{3}{2}(v_g - u_0)(1-\beta)\alpha + \frac{\gamma_2}{u_0}\frac{\alpha(1-\beta)(v_g-u_0)^2}{\gamma_1} - \frac{3\alpha(1+\alpha)^2}{2\theta\gamma_1(v_g-u_0)(1-\beta)} - \frac{3\alpha^2(1+\alpha)^2}{2\theta\gamma_1(v_g-u_0)} \qquad (17)$$

and

$$B = \frac{1}{2\gamma_1(v_g-u_0)(1-\beta)^2} \qquad (18)$$

On introducing the new variable $\eta = \xi - v\tau$, where v is a constant velocity, the solution of Eq.(16) is given as

$$\phi_1(\eta) = \phi_m\, sech^2\left(\frac{\eta}{L}\right) \qquad (19)$$

where peak amplitude ϕ_m and width L of soliton are respectively given by

$$\phi_m = \frac{3v}{A} \qquad (20)$$

And

$$L = \sqrt{\frac{4B}{v}} \qquad (21)$$

3. RESULTS AND DISCUSSION

Using the reductive perturbation technique, we have derived the phase velocity for small amplitude solitons. The phase velocity depends on relativistic factor U (u_0/c), cold to hot electron density ratio(α) , hot to cold electron temperature ratio(θ) and nonthermal electrons distribution parameter β. To see how these parameters effect the formation of waves, we perform the numerical computation and display the results in the form of graphs as shown in figures 1,2 , 3and 4.

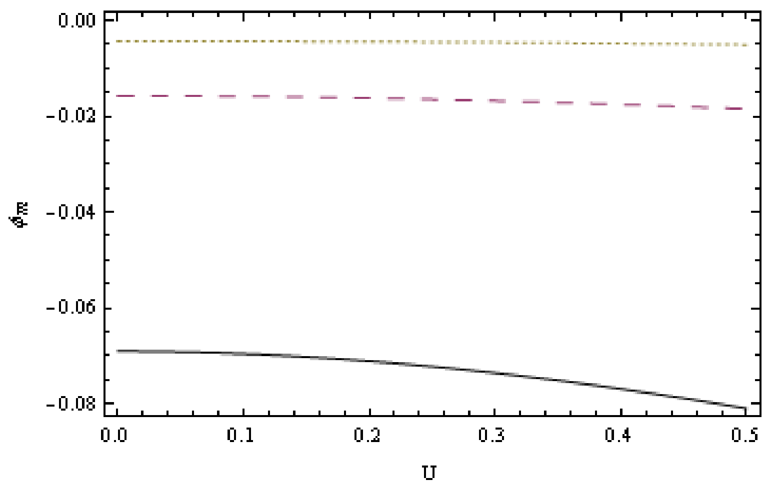

Fig. 1: The amplitude of solitary wave ϕ_m with respect to the relativistic factor U for β=0.1, θ=20, v= 0.01 and α= 0.5, (solid curve), 1.0 (dashed curve), 2.0 (dotted curve).

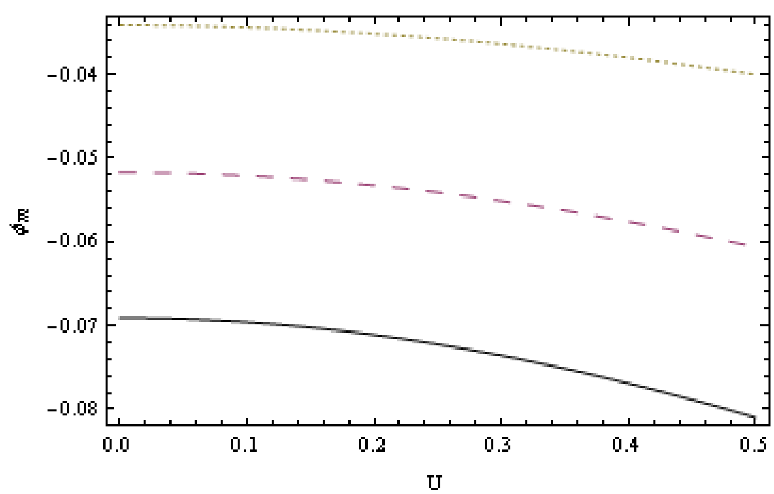

Fig. 2: The amplitude of solitary wave φ_m with respect to the relativistic factor U for α=0.5, θ=20, v= 0.01 and β= 0.1, (solid curve), 0.3 (dashed curve), 0.5(dotted curve).

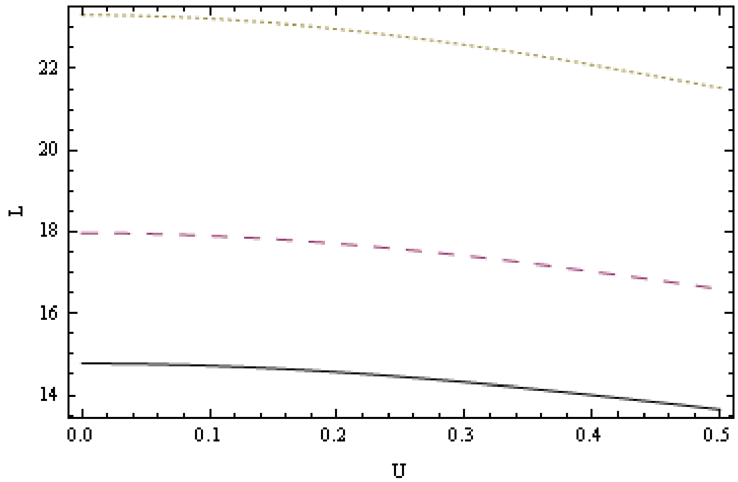

Fig. 3: The width of solitary wave L with respect to the relativistic factor U for β=0.1, θ=20, v= 0.01 and α= 0.5, (solid curve), 1.0 (dashed curve), 2.0 (dotted curve).

In order to study the variation of the amplitude and the width of EASW with respect to the relativistic factor U for different parameters, we select a set of parameters are α= 0.5,β=0.1, θ=20 .Figure 1 shows how the amplitude ϕ_m changes respect to the relativistic factor U for different values of density ratio (α), superthermal parameter (β). Absolute value of the soliton amplitude ϕ_m decreases when ''U'' increases. Increasing in the soliton amplitude is more sensible for the larger values of . This figure also shows that the soliton amplitude is an increasing function of α. Thus the soliton amplitude finds its maximum value in smaller values of U. Figure 2 presents ϕ_m as a function of U with different values of β. This figure clearly shows that the absolute value of the soliton peak increases with an increasing β. This means that the effect of superthermality is noticeable with smaller values of ''β''. Therefore one can conclude that the presence of superthermal electrons decreases the amplitude of the soliton.

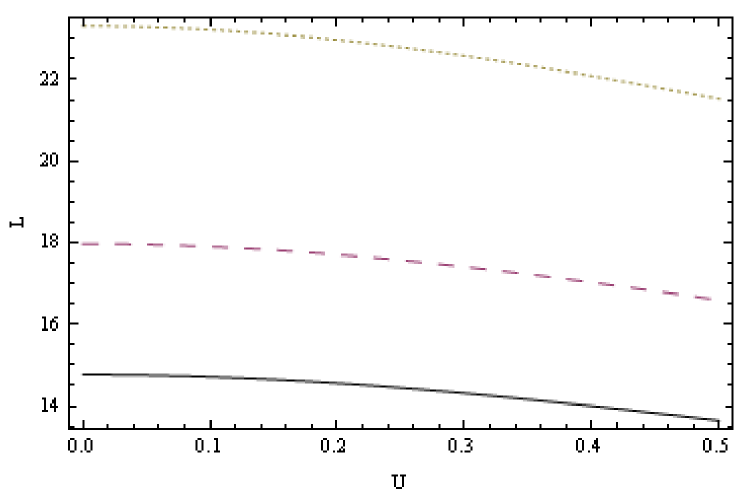

Fig. 4: The width of solitary wave L with respect to the relativistic factor U for α=0.5, θ=20, v= 0.01 and β= 0.1, (solid curve), 0.3 (dashed curve), 0.5(dotted curve)

Figures 3 and 4 show the variation of width of solitary waves with respect to the to the relativistic factor U. It is shown that the width L decreases as the relativistic factor increases. The width of soliton increases with increasing the value of α and β. The maximum value of the soliton width increases as ''β'' increases. It is mentioned that superthermal distribution reduces to Maxwellian distribution for large values of ''β''.

4. CONCLUSION

In this paper, we study the Electron-Acoustic Solitary Waves In Weakly Relativistic three Component Plasma with cold electron fluid and vortex-like hot electron by the linearize method and the reductive perturbation technique. By using the linearize method, we obtain the phase velocity. The KdV equation has been derived by the reductive perturbation technique. With the aid of some numerical simulation results, it is found that the amplitude of EASW decreases as the relativistic factor U increases, but increases as the parameter α and β increases. On the other hand, the width of EASW decreases as the relativistic factor U increases. It can be concluded that the parameter α and β effects on the characters of EASW. It indicates that the free hot electron temperature and the trapped hot electron temperature have very important effects on the amplitude and the width of EASW in an unmagnetized, collisionless, two-temperature electron plasma with vortex-like hot electrons.

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