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MULTI-ITEMS INVENTORY MODEL OF DETERIORATING PRODUCTS WITH LIFE TIME

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ABSTRACT:

In this paper, the work of Mahapatra & Maiti has been extended by considering the more realistic assumption that the deterioration of the items starts after some time i.e. the concept of life time has been considered. The production starts from zero stock level and goes on upto the stock level Q_i at time μ when the deterioration starts. At this level the production is stopped and then stock level reduces due to demand and deterioration to stock level zero at time T_{1i}. The model has been developed in the case of shortages and without shortages.

Key Words: ***Deteriorating Products, Life Time, Multi Objective, Linearly Dependent etc.***

1 INTRODUCTION:

Normally the demand of any item depends upon several factors, such as quality of the item, selling price, time and stock level at the show room etc. In practice it is seen that the higher selling price of an item has negative impact on the demand of that item while the less price has positive effect. **Abad** (2000) discussed the effect of selling price on non-deteriorating items while **Cohen** (1977) and **Mukherjee** (1987) discussed the effect of selling price on deteriorating items. Large stock of goods displayed in a super market also has great impact on the customers to buy that product. **Levin et al.** (1972), **Baker & Urban** (1989) discussed the inventory models with stock dependent demand.

The quality of an item plays big role to stimulate the demand of that item. But in developing countries where majority of people live under poverty line price of the item is the deciding factor for the demand of that item.

It is normally assumed that the life time of the items is either infinite or the deterioration starts at the beginning of the inventory. But in reality it is not true. Due to poor preservation conditions some portion of items like food grains, vegetables fruits and drugs etc. are damaged or decayed due to dryness, spoilage and vaporization etc. and are of no use. **Mandal & Phaujdar** (1989) and **Mandal & Maiti** (1999) discussed the inventory models for deteriorated items assuming deterioration rate to be constant or linearly dependent on time or stock level. Also in most of the cases deterioration of items does start in the beginning of the inventory. It starts after some time called life time. Some authors studied the inventory model with life time.

Mahapatra & Maiti (2005) developed multi objective and single objective inventory models of stochastically deteriorating items in which demand was considered as a function of inventory level and selling price of the commodity. Recently Madhu etal. [2010] and Hari Kishan etal. [2012] developed an inventory model of deteriorating product with life time and variable deterioration rate with declining demand.

In this paper, the work of **Mahapatra & Maiti** has been extended by considering the more realistic assumption that the deterioration of the items starts after some time i.e. the concept of life time has been considered. The production starts from zero stock level and goes on upto the stock level Q_i at time μ when the deterioration starts. At this level the production is stopped and then stock level reduces due to demand and deterioration to stock level zero at time T_{1i}. The model has been developed in the case of shortages and without shortages.

2 Assumptions and Notations:

The assumptions used in this chapter are as follows:

(i) There are n items in the inventory system.

(ii) Lead time is considered negligible.

(iii) The planning horizon is infinite.

(iv) There is no repair or replacement of the deteriorated items.

(v) The production rate is a function of quality level.

(vi) The demand rate is considered as function of selling price and instantaneous stock level. It is given by

$$d_i = d_{0i} + d_{1i} q_i(t), \text{ for } q_i > 0$$

$$d_i = d_{0i}, \text{ for } q_i \le 0.$$

(vii) Deterioration is a function of quality level as well as time given by

$$\theta_i(t, qu_i) = \alpha_1 qu_i^{-\delta_i} \beta_i t^{\beta_i - 1}$$

where α_i and β_i are positive and are scale and shape parameters respectively.

(viii) Deterioration of items starts after time μ. At this stage the production of items is stopped.

The notations used in this chapter are as follows:

(i) n: number of products in the inventory system

(ii) P: total average profit

(iii) S: available storage space

For ith product:

(iv) $q_i(t)$: inventory level at time t.

(v) d_i: demand rate

(vi) K_i: production rate.

(vii) qu_i: quality level of the product.

(viii) μ: the life time.

(ix) h_i: the holding cost per unit per unit time.

(x) s_i: the shortage cost per unit per unit time.

(xi) Q_i: the maximum inventory level at time $t = \mu$.

(xii) S_i: the maximum shortage level at time $t = T_{2i}$.

(xiii) D_{ui}: the total deteriorated units

(xiv) D_i: the deteriorated cost during one cycle.

(xv) dc_i: the deteriorated cost per unit.

(xvi) p_i: the unit production cost

(xvii) P_{ri}: the total production cost

(xviii) s_{ei}: the unit selling price

(xix) P_i : the total average profit

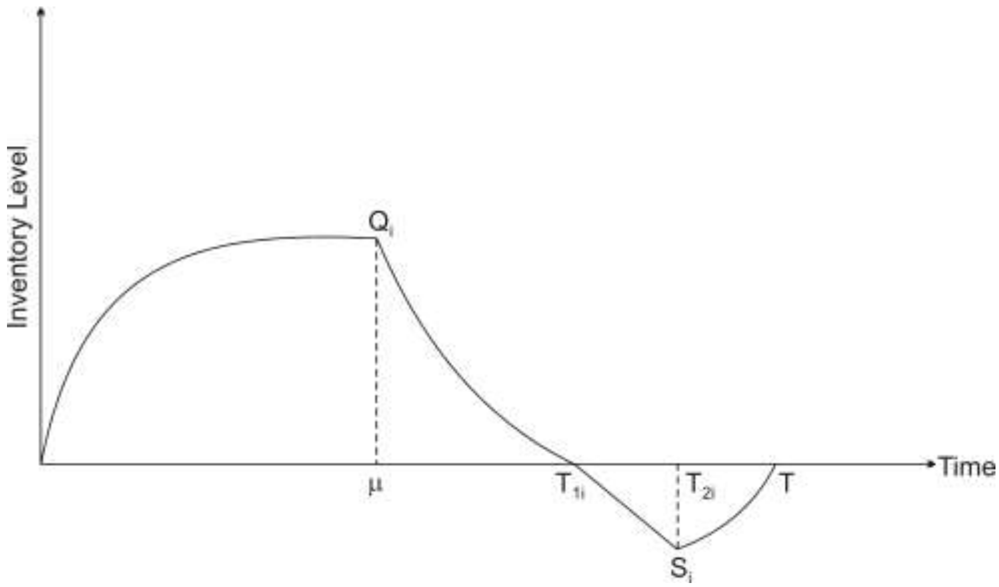
(xx) S_{ei} : the setup cost

(xxi) sp_i : space required for one unit of item

3 Mathematical Formulation:

(i) With Shortage:

Let the production of ith product starts at time t=0 and reaches stock level Q_i at time μ when the deterioration of the items starts. At this stage the production of the product is stopped. The stock level reduces due to demand as well as due to deterioration upto time T_{1i} when stock level reduces to zero. After this shortage starts and goes upto level S_i. Then production of the product restarts and shortage is backlogged upto time T_{2i}. This inventory model has been shown in figure 1. Our objective is to obtain the optimal values of different variables to maximize the profit.



If $q_i(t)$ be the inventory level of the ith product at time t then the governing differential equations are given by

$$\frac{dq_i}{dt} = K_i - (d_{0i} + d_i q_i), \qquad 0 \le t \le \mu \qquad \ldots(1)$$

$$\frac{dq_i}{dt} = -(d_{0i} + d_i q_i) - \theta_i(t, qu_i)q_i, \qquad \mu \le t \le T_{1i} \qquad \ldots(2)$$

$$\frac{dq_i}{dt} = \frac{-d_{0i}}{1 + b_i(T_{1i} - t)}, \qquad T_{1i} \le t \le T_{2i} \qquad \ldots(3)$$

$$\frac{dq_i}{dt} = K_i - d_{0i}, \qquad T_{2i} \le t \le T \qquad \ldots(4)$$

The boundary conditions are $q_i(t) = 0$ at $t = 0, T_{1i}$ and T.

Solving the differential equation (1) and using the boundary condition, we get

$$q_i = \frac{(K_i - d_{0i})}{d_i}\left(1 - e^{-d_i t}\right), \qquad \ldots(5)$$

Solving the differential equation (2) and using the boundary condition, we get

$$q_i = -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \int_{\mu}^{t} e^{d_i t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}}\, dt$$

$$= -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \int_{\mu}^{t} \left(1 + d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i} + \frac{1}{2} d_{1i}^2 t^2 + \frac{1}{2}\left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i}\right) dt$$

$$= -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \left[t + \frac{d_{1i}t^2}{2} + \frac{\theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i + 1}}{(\beta_1 + 1)} \right.$$

$$\left. + \frac{1}{6} d_{1i}^2 t^3 + \frac{1}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 1} \right]_{\mu}^{t}$$

$$= -d_{0i} \left[t - \frac{d_{1i}t^2}{2} - \frac{\beta_1 \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i + 1}}{(\beta_1 + 1)} \right.$$

$$\left. + \frac{1}{6} d_{1i}^2 t^3 + \frac{1}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 1} - \frac{d_{1i}}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 2} \right]$$

$$+ d_{0i} a_{0i} \left(1 - d_{1i}t - \theta_i \alpha_i q u_i^{-\delta_1} t^{\beta_1} + \frac{1}{2} d_{1i}^2 t^2 \right)$$

where $a_{0i} = e^{-\left(d_{1i}T_{1i} + \theta_i \alpha_i q u_i^{-\delta_i} T_{1i}^{\beta_1}\right)} \left[T_{1i} + \frac{d_{1i}T_{1i}^2}{2} + \frac{\theta_i \alpha_i q u_i^{-\delta_i} T_{1i}^{\beta_1 + 1}}{(\beta_1 + 1)} \right.$

$$\left. + \frac{1}{6} d_{1i}^2 T_{1i}^3 + \frac{1}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 T_{1i}^{2\beta_1 + 1} \right]. \qquad \ldots(6)$$

Since $q_i(t)$ is continuous at $t = \mu$ and $q_i(\mu) = Q_i$, we have from (5) and (6)

$$Q_i = \frac{(K_i - d_{0i})}{d_i}\left(1 - e^{-d_i \mu}\right)$$

$$= -d_{0i} \left[\mu - \frac{d_{1i}\mu^2}{2} - \frac{\beta_1 \theta_i \alpha_i q u_i^{-\delta_i} \mu^{\beta_i + 1}}{(\beta_1 + 1)} \right.$$

$$\left. + \frac{1}{6} d_{1i}^2 \mu^3 + \frac{1}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 \mu^{2\beta_i + 1} - \frac{d_{1i}}{2(2\beta_1 + 1)} \left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 \mu^{2\beta_i + 2} \right]$$

$$+ d_{0i} a_{0i} \left(1 - d_{1i}\mu - \theta_i \alpha_i q u_i^{-\delta_1} \mu^{\beta_1} + \frac{1}{2} d_{1i}^2 \mu^2 \right). \qquad \ldots(7)$$

Equation (7) gives the relation between t_{1i} and μ.

Solving the differential equation (3) and using the boundary condition, we get

$$q_i = d_{0i} \log\left(1 + b_i (T_{1i} - t)\right). \qquad \ldots(8)$$

Solving the differential equation (4) and using the boundary condition, we get

$$q_i = (K_i - d_{0i})(T - t). \qquad \ldots(9)$$

Since $q_i(t)$ is continuous at $t = t_{2i}$ and $q_i(t_{2i}) = S_i$, we have from (8) and (9)

$$S_i = d_{0i} \log\left(1 + b_i\left(T_{1i} - T_{2i}\right)\right)$$
$$= \left(K_i - d_{0i}\right)\left(T - T_{2i}\right). \qquad \ldots(10)$$

Equation (10) gives relations among T_{1i}, T_{2i} and T.

The deteriorated units during $\left(\mu, T_{1i}\right)$ are given by

$$\int_{\mu}^{T_{1i}} \theta_i\left(t, qu_i\right) q_i(t)\, dt. \qquad \ldots(11)$$

The holding cost is given by

$$H_i = h_i\left[\int_0^{\mu} q_i(t)\,dt + \int_{\mu}^{T_{1i}} q_i(t)\,dt\right]$$

$$= h_i\left[\int_0^{\mu} \frac{\left(K_i - d_{0i}\right)}{d_i}\left(1 - e^{-d_i t}\right)dt\right.$$

$$\int_{\mu}^{T_{1i}} - d_{0i}\left[t - \frac{d_{1i}t^2}{2} - \frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i} t^{\beta_i+1}}{\left(\beta_1+1\right)}\right.$$

$$\left. + \frac{1}{6}d_{1i}^2 t^3 + \frac{1}{2\left(2\beta_1+1\right)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2 t^{2\beta_i+1} - \frac{d_{1i}}{2\left(2\beta_1+1\right)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2 t^{2\beta_i+2}\right]dt$$

$$\left. + \int_{\mu}^{T_{1i}} d_{0i}a_{0i}\left(1 - d_{1i}t - \theta_i\alpha_i qu_i^{-\delta_1} t^{\beta_1} + \frac{1}{2}d_{1i}^2 t^2\right)dt\right]$$

$$= h_i\left[\frac{\left(K_i - d_{0i}\right)}{d_i}\left(\mu + \frac{\left(e^{-d_i\mu} - 1\right)}{d_1}\right)\right.$$

$$- d_{0i}\left[\frac{\left(T_{1i}^2 - \mu^2\right)}{2} - \frac{d_{1i}\left(T_{1i}^3 - \mu^3\right)}{6} - \frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}\left(T_{1i}^{\beta_1+2} - \mu^{\beta_1+2}\right)}{\left(\beta_1+1\right)\left(\beta_1+2\right)}\right.$$

$$\left. + \frac{1}{24}d_{1i}^2\left(T_{1i}^4 - \mu^4\right) + \frac{1}{2\left(2\beta_1+1\right)\left(\beta_1+1\right)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2\left(\beta_1+1\right)} - \mu^{2\left(\beta_1+1\right)}\right)\right]$$

$$\left. - \frac{d_{1i}}{2\left(2\beta_1+1\right)\left(2\beta_1+3\right)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2\beta_i+3} - \mu^{2\beta_i+3}\right)\right]$$

$$\left. + d_{0i}a_{0i}\left(\left(T_{1i} - \mu\right) - \frac{d_{1i}\left(T_{1i}^2 - \mu^2\right)}{2} - \theta_i\alpha_i qu_i^{-\delta_1}\frac{\left(T_{1i}^{\beta_1+1} - \mu^{\beta_1+1}\right)}{\left(\beta_1+1\right)} + \frac{1}{6}d_{1i}^2\left(T_{1i}^3 - \mu^3\right)\right)\right]. \qquad \ldots(12)$$

The deterioration cost is given by

$$D_i = dc_i\left[\int_{\mu}^{T_{1i}} q_i(t)\,dt\right]$$

$$= dc_i\left[- d_{0i}\left[\frac{\left(T_{1i}^2 - \mu^2\right)}{2} - \frac{d_{1i}\left(T_{1i}^3 - \mu^3\right)}{6} - \frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}\left(T_{1i}^{\beta_1+2} - \mu^{\beta_1+2}\right)}{\left(\beta_1+1\right)\left(\beta_1+2\right)}\right.\right.$$

$$\left. + \frac{1}{24}d_{1i}^2\left(T_{1i}^4 - \mu^4\right) + \frac{1}{2\left(2\beta_1+1\right)\left(\beta_1+1\right)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2\left(\beta_1+1\right)} - \mu^{2\left(\beta_1+1\right)}\right)\right]$$

$$-\frac{d_{1i}}{2(2\beta_1+1)(2\beta_1+3)}\left(\theta_i\alpha_i q u_i^{-\delta_i}\right)^2\left(T_{1i}^{2\beta_i+3}-\mu^{2\beta_i+3}\right)\Bigg]$$

$$+d_{0i}a_{0i}\left(\left(T_{1i}-\mu\right)-\frac{d_{1i}\left(T_{1i}^2-\mu^2\right)}{2}-\theta_i\alpha_i q u_i^{-\delta_1}\frac{\left(T_{1i}^{\beta_1+1}-\mu^{\beta_1+1}\right)}{\left(\beta_1+1\right)}+\frac{1}{6}d_{1i}^2\left(T_{1i}^3-\mu^3\right)\right)\Bigg]. \quad \ldots(13)$$

The shortage cost is given by

$$S_i=s_i\left[\int_{T_{1i}}^{T_{2i}}q_i(t)dt+\int_{T_{2i}}^{T}q_i(t)dt\right]$$

$$=s_i\left[\int_{T_{1i}}^{T_{2i}}d_{0i}\log\left(1+b_i\left(T_{1i}-t\right)\right)dt+\int_{T_{2i}}^{T}(K_i-d_{0i})(T-t)dt\right]$$

$$=s_i\left[d_{0i}\left[T_{2i}\log\left(1+b_i\left(T_{1i}-T_{2i}\right)\right)+\frac{1}{b_i}\left\{\left(T_{1i}-T_{2i}\right)+\left(1+b_iT_{1i}\right)\log\left(1+b_i\left(T_{1i}-T_{2i}\right)\right)\right\}\right]\right.$$

$$\left.+\left(K_i-d_{0i}\right)\left(\frac{T^2}{2}-TT_{2i}+\frac{T_{2i}^2}{2}\right)\right]. \quad \ldots(14)$$

$i=1, 2, \ldots, n$. …(16)

Case I: Non-integrated Management: Total production cost is given by

$$P_{ri}=p_i\left[\int_0^{\mu}K_i dt+\int_{T_{2i}}^{T}K_i dt\right]$$

$$=p_i\left[K_i\left(\mu+T-T_{2i}\right)\right]. \quad \ldots(15)$$

Total average profit is given by

$$P_i=\frac{1}{T}\left[s_{ei}\left\{K_i\left(\mu+T-T_{2i}\right)-D_{ui}\right\}\right.$$

$$\left.-\left\{p_iK_i\left(\mu+T-T_{2i}\right)+H_i+S_i+S_{ei}\right\}\right],$$

The model can be presented as the following multi objective problem

maximize $\left(P_1, P_2, \ldots P_n\right)$

subject to $\sum_{i=1}^{n} sp_iQ_i \le S$. …(17)

Case II: Integrated Management:

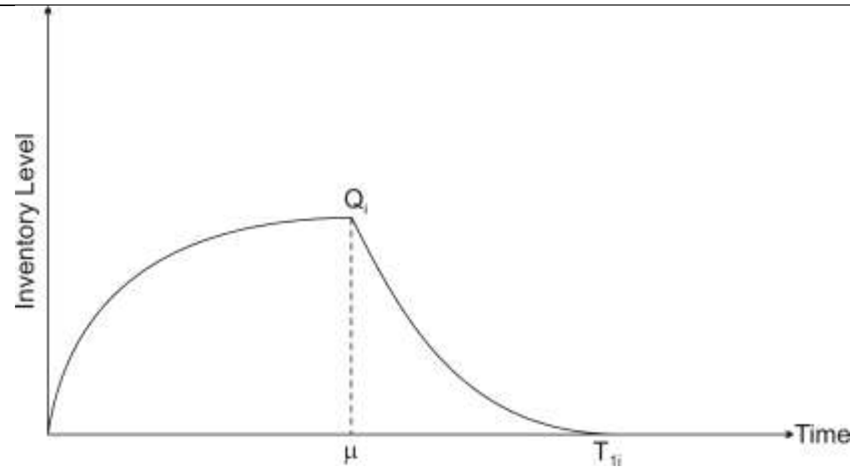
The model can be presented as the following single objective problem

maximize $\sum_{i=1}^{n} P_i$

subject to $\sum_{i=1}^{n} sp_iQ_i \le S$. …(18)

(ii) Without Shortage:

When shortages are not allowed the inventory model has been shown in figure 2. We have to obtain the optimal values of the variables to maximize the profit.



For ith product the governing differential equations are given by

$$\frac{dq_i}{dt} = K_i - (d_{0i} + d_i q_i), \qquad 0 \le t \le \mu \qquad \ldots(19)$$

$$\frac{dq_i}{dt} = -(d_{0i} + d_i q_i) - \theta_i(t, q_i)q_i, \qquad \mu \le t \le T_{1i} \qquad \ldots(20)$$

The boundary conditions are $q_i(t) = 0$ at $t = 0, T_{1i}$.

Solving the differential equation (19) and using the boundary condition, we get

$$q_i = \frac{(K_i - d_{0i})}{d_i}\left(1 - e^{-d_i t}\right). \qquad \ldots(21)$$

Solving the differential equation (20) and using the boundary condition, we get

$$q_i = -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \int_{\mu}^{t} e^{d_i t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}} dt$$

$$= -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \int_{\mu}^{t} \left(1 + d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i} + \frac{1}{2} d_{1i}^2 t^2 + \frac{1}{2}\left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i}\right) dt$$

$$= -d_{0i} e^{-\left(d_{1i}t + \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i}\right)} \left[t + \frac{d_{1i} t^2}{2} + \frac{\theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i + 1}}{(\beta_1 + 1)} \right.$$

$$\left. + \frac{1}{6} d_{1i}^2 t^3 + \frac{1}{2(2\beta_1 + 1)}\left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 1} \right]_{\mu}^{t}$$

$$= -d_{0i} \left[t - \frac{d_{1i} t^2}{2} - \frac{\beta_1 \theta_i \alpha_i q u_i^{-\delta_i} t^{\beta_i + 1}}{(\beta_1 + 1)} \right.$$

$$\left. + \frac{1}{6} d_{1i}^2 t^3 + \frac{1}{2(2\beta_1 + 1)}\left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 1} - \frac{d_{1i}}{2(2\beta_1 + 1)}\left(\theta_i \alpha_i q u_i^{-\delta_i}\right)^2 t^{2\beta_i + 2} \right]$$

$$+ d_{0i} a_{0i} \left(1 - d_{1i} t - \theta_i \alpha_i q u_i^{-\delta_1} t^{\beta_1} + \frac{1}{2} d_{1i}^2 t^2\right)$$

where $a_{0i} = e^{-\left(d_{1i}T_{1i} + \theta_i \alpha_i q u_i^{-\delta_i} T_{1i}^{\beta_1}\right)} \left[T_{1i} + \frac{d_{1i} T_{1i}^2}{2} + \frac{\theta_i \alpha_i q u_i^{-\delta_i} T_{1i}^{\beta_1 + 1}}{(\beta_1 + 1)} \right.$

$$+\frac{1}{6}d_{1i}^2T_{1i}^3+\frac{1}{2(2\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2T_{1i}^{2\beta_1+1}\Bigg] \quad \ldots(22)$$

Since $q_i(t)$ is continuous at $t=\mu$ and $q_i(\mu)=Q_i$, we have from (21) and (22)

$$Q_i=\frac{(K_i-d_{0i})}{d_i}\left(1-e^{-d_i\mu}\right)$$

$$=-d_{0i}\Bigg[\mu-\frac{d_{1i}\mu^2}{2}-\frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}\mu^{\beta_i+1}}{(\beta_1+1)}$$

$$+\frac{1}{6}d_{1i}^2\mu^3+\frac{1}{2(2\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\mu^{2\beta_i+1}-\frac{d_{1i}}{2(2\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\mu^{2\beta_i+2}\Bigg]$$

$$+d_{0i}a_{0i}\left(1-d_{1i}\mu-\theta_i\alpha_i qu_i^{-\delta_1}\mu^{\beta_1}+\frac{1}{2}d_{1i}^2\mu^2\right). \quad \ldots(23)$$

Equation (23) gives the relation between t_{1i} and μ.

The holding cost is given by

$$H_i=h_i\left[\int_0^{\mu}q_i(t)dt+\int_{\mu}^{T_{1i}}q_i(t)dt\right]$$

$$=h_i\Bigg[\int_0^{\mu}\frac{(K_i-d_{0i})}{d_i}\left(1-e^{-d_it}\right)dt$$

$$\int_{\mu}^{T_{1i}}-d_{0i}\Bigg[t-\frac{d_{1i}t^2}{2}-\frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}t^{\beta_i+1}}{(\beta_1+1)}$$

$$+\frac{1}{6}d_{1i}^2t^3+\frac{1}{2(2\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2t^{2\beta_i+1}-\frac{d_{1i}}{2(2\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2t^{2\beta_i+2}\Bigg]dt$$

$$+\int_{\mu}^{T_{1i}}d_{0i}a_{0i}\left(1-d_{1i}t-\theta_i\alpha_i qu_i^{-\delta_1}t^{\beta_1}+\frac{1}{2}d_{1i}^2t^2\right)dt\Bigg]$$

$$=h_i\Bigg[\frac{(K_i-d_{0i})}{d_i}\left(\mu+\frac{\left(e^{-d_i\mu}-1\right)}{d_1}\right)$$

$$-d_{0i}\Bigg[\frac{\left(T_{1i}^2-\mu^2\right)}{2}-\frac{d_{1i}\left(T_{1i}^3-\mu^3\right)}{6}-\frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}\left(T_{1i}^{\beta_1+2}-\mu^{\beta_1+2}\right)}{(\beta_1+1)(\beta_1+2)}$$

$$+\frac{1}{24}d_{1i}^2\left(T_{1i}^4-\mu^4\right)+\frac{1}{2(2\beta_1+1)(\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2(\beta_1+1)}-\mu^{2(\beta_1+1)}\right)\Bigg]$$

$$-\frac{d_{1i}}{2(2\beta_1+1)(2\beta_1+3)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2\beta_i+3}-\mu^{2\beta_i+3}\right)\Bigg]$$

$$+d_{0i}a_{0i}\left((T_{1i}-\mu)-\frac{d_{1i}\left(T_{1i}^2-\mu^2\right)}{2}-\theta_i\alpha_i qu_i^{-\delta_1}\frac{\left(T_{1i}^{\beta_1+1}-\mu^{\beta_1+1}\right)}{(\beta_1+1)}+\frac{1}{6}d_{1i}^2\left(T_{1i}^3-\mu^3\right)\right). \quad \ldots(24)$$

The deterioration cost is given by

$$D_i = dc_i\left[\int_{\mu}^{T_{1i}} q_i(t)dt\right]$$

$$= dc_i\left[-d_{0i}\left[\frac{\left(T_{1i}^2-\mu^2\right)}{2}-\frac{d_{1i}\left(T_{1i}^3-\mu^3\right)}{6}-\frac{\beta_1\theta_i\alpha_i qu_i^{-\delta_i}\left(T_{1i}^{\beta_1+2}-\mu^{\beta_1+2}\right)}{(\beta_1+1)(\beta_1+2)}\right.\right.$$

$$\left.+\frac{1}{24}d_{1i}^2\left(T_{1i}^4-\mu^4\right)+\frac{1}{2(2\beta_1+1)(\beta_1+1)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2(\beta_1+1)}-\mu^{2(\beta_1+1)}\right)\right]$$

$$\left.-\frac{d_{1i}}{2(2\beta_1+1)(2\beta_1+3)}\left(\theta_i\alpha_i qu_i^{-\delta_i}\right)^2\left(T_{1i}^{2\beta_i+3}-\mu^{2\beta_i+3}\right)\right]$$

$$\left.+d_{0i}a_{0i}\left(\left(T_{1i}-\mu\right)-\frac{d_{1i}\left(T_{1i}^2-\mu^2\right)}{2}-\theta_i\alpha_i qu_i^{-\delta_1}\frac{\left(T_{1i}^{\beta_1+1}-\mu^{\beta_1+1}\right)}{(\beta_1+1)}+\frac{1}{6}d_{1i}^2\left(T_{1i}^3-\mu^3\right)\right)\right]. \qquad \ldots(25)$$

Total production cost is given by

$$P_{ri} = p_i\left[\int_0^{\mu} K_i dt\right]$$

$$= p_i\left[K_i\mu\right]. \qquad \ldots(26)$$

Total average profit is given by

$$P_i = \frac{1}{T}\left[s_{ei}\left\{K_i\mu - D_{ui}\right\}\right.$$

$$\left.-\left\{p_iK_i\mu + H_i + S_{ei}\right\}\right], \qquad i=1, 2, \ldots, n. \qquad \ldots(27)$$

Case I: Non-integrated Management:

The model can be presented as the following multiobjective problem

maximize $\left(P_1, P_2, \ldots P_n\right)$

$$\text{subject to } \sum_{i=1}^{n} sp_iQ_i \le S. \qquad \ldots(28)$$

Case II: Integrated Management:

The model can be presented as the following single objective problem

$$\text{maximize } \sum_{i=1}^{n} P_i$$

$$\text{subject to } \sum_{i=1}^{n} sp_iQ_i \le S. \qquad \ldots(29)$$

The multiobjective and single objective problems given by (17), (28) and (18), (29) respectively can be solved by goal programming (GP) technique and generalized reduced gradient (GRG) method respectively.

4 MATHEMATICAL ANALYSIS:

The non-integrated production inventory problems can be solved by goal programming method as given below:

Step 1: The following single objective problem is solved by using any gradient based nonlinear programming algorithm.

maximize P_1,

subject to $\sum_{i=1}^{n} sp_i Q_i \leq S$.

Let the optimal value of P_1 be P_1^1. The same procedure is repeated for other objectives P_i, i=2, 3, …, n. Let the corresponding optimal values are P_i^1, i=2, 3, …, n respectively.

Thus the ideal objective vector is given by $\left(P_1^1, P_2^1, ..., P_n^1\right)$.

Step 2: The following goal programming problem is formulated by using the ideal objective vector obtained in step 1.

$$\text{minimum} \left[\sum \left(\left(d_i^+\right)^p + \left(d_i^-\right)^p\right)\right]^{1/p},$$

subject to $P_i + d_i^- - d_i^+ = P_i^1$, ($i$=1, 2, …, n)

$$\sum_{i=1}^{n} sp_i Q_i \leq S,$$

$d_i^- d_i^+ = 0$, $d_i^- \geq 0$, $d_i^+ \geq 0$, (i=1, 2, …, n).

Step 3: Finally, the above single objective problem described in step 2 is solved by GRG method and a compromise solution is obtained. Let the compromise solution be $\left(P_1^*, P_2^*, ..., P_n^*\right)$.

6 CONCLUDING REMARKS:

In this paper, a multi-objective multi items inventory model of perishable product with life time has been developed in the cases of shortages and without shortage. The model has been discussed in non-integrated as well as integrated cases.

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