Restrictions of interval Assignment problem using Hungarian method

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ABSTRACT

In this paper to discuss the Restrictions on assignment problem, sometime technical, space, legal or other restrictions do not permit the assignment of a particular facility to a particular job. Such problem can be solved by assigning a very heavy cost (infinite cost) to the corresponding cell. Such a job will then be automatically exclude from further consideration (making assignments). In the entries of the cost matrix is not always crisp. In many application this parameters are uncertain and this uncertain parameters are represented by interval. In this contribution we propose interval Hungarian method and consider interval analysis concept for solving interval linear assignment problems.

Keywords : Interval Analysis, Assignment Problems, Hungarian assignment method , Matrix zeros assignment method (MZAM) ,optimization

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1. INTRODUCTION

An assignment problem deals with the question how to assign n objects to m other objects in an injective fashion in the best possible way. An assignment problem is completely specified by its two components the assignments, which represent the underlying combinatorial structure, and the objective function to be optimized, which models "the best possible way".The assignment problem refers to another special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one to one basis so as to minimize total costs of performing the tasks at hand or maximize total profit of allocation.

2. Arithmetic Conditions:

The interval form of the parameters may be written as where is the left value [$\underline{x}$] and is the right value [$\overline{x}$] of the interval respectively. We define the centre is m = $\frac{\overline{x}+x}{2}$ and w = $\overline{x} - \underline{x}$ is the width of the interval [$\overline{x}, \underline{x}$]

Let [$\overline{x}, \underline{x}$] and [$\overline{y}, \underline{y}$] be two elements then the following arithmetic are well known

(i) $[\overline{x}, \underline{x}] + [\overline{y}, \underline{y}] = [\overline{x} + \overline{y}, \underline{x} + \underline{y}]$

(ii) $[\overline{x}, \underline{x}] \times [\overline{y}, \underline{y}]$ =[min{$\overline{x}\underline{y}$ $\underline{x}\overline{y}$, $\overline{xy}$, $\underline{xy}$} ,max{$\overline{x}\underline{y}$ $\underline{x}\overline{y}$, $\overline{xy}$, $\underline{xy}$}]

(iii) $[\overline{x}, \underline{x}] \div [\overline{y}, \underline{y}]$ =[min{$\overline{x} \div \underline{y}$ $\underline{x} \div \overline{y}$, $\overline{x} \div \overline{y}$, $\underline{x} \div \underline{y}$} , max{$\overline{x} \div \underline{y}$ $\underline{x} \div \overline{y}$, $\overline{x} \div \overline{y}$, $\underline{x} \div \underline{y}$}] provide if $[\overline{y}, \underline{y}] \neq [0, 0]$,

3. Hungarian method:

Find the opportunity cost table by: (a). subtracting the smallest number in each row of the original cost table or matrix from every number in that row and. (b). Then subtracting the smallest number in each column of the table obtained in part (a) from every number in that column.

Examine the rows successively until a row with exactly one unmarked zero is found. Enclose this zero in a box ( ) as an assignment will be made there and cross (X) all other zeros appearing in the corresponding column as they will not be considered for future assignment. Proceed in this way until all the rows have been examined.

Examine the columns successively until a column with exactly one unmarked zero is found. Make an assignment to this single zero by putting square ( ) around it and cross out (X) all other zeros appearing in the corresponding row as they will not be used to make any other assignment in that row, Proceed in this manner until all columns have been examined.

Until one of the following situations arises: (i). All the zeros in rows /columns are either marked ( ) or corssed (X) and there is exactly one assignment in each row and in each column. In such a case optimal assignment policy for the given problem is obtained.

(ii). there may be some row (or column) without assignment, i.e., the total number of marked zeros is less than the order of the matrix. In such a case, proceed to next

Draw the minimum number of vertical and horizontal lines necessary to cover all the zeros in the reduced cost table obtained by adopting the following procedure:

i. Mark (√) all rows that do not have assignments.

ii. Mark (√) all columns (not already marked) which have zeros in the marked rows

iii. Mark (√) all rows (not already marked) that have assignments in marked columns

iv. Repeat steps (ii) and (iii) until no more rows or columns can be marked.

Draw straight lines through each unmarked row and each marked column. If the number of lines drawn (or total assignments) is equal to the number of rows (or columns) then the current solution is the optimal solution,

Example :-

(Assignment Problem- Restrictions on Assignment)

Four new machines M1, M2, M3 and M4 are to be installed in a machine shop. There are five vacant places A,B, C, D and E available. Because of limited space, machine M1 cannot be place at C, machine M2 cannot be place at C, machine M3 cannot be place at A and M4 cannot be placed at E. Cij the assignment cost of machine i to place j in rupees is shown below.

Table

	A	B	C	D	E
M1	4	6	∞	5	6
M2	7	4	∞	5	4
M3	∞	6	9	6	2
M4	9	3	7	2	∞

Find the optimal assignment schedule

Solution:

Step I. Prepare a Square Matrix.

As the given matrix is non-square, we add a dummy machine and associate zero cost with the corresponding cells. As machine M1 cannot be place at C, machine M2 cannot be placed at C and M3 cannot be placed at A and M4 cannot be placed at E, we assign infinite cost (M1, C) in cell (M2, C), (M3 A) and (M4, E) resulting in the following matrix.

Table

	A	B	C	D	E
M1	4	6	∞	5	6
M2	7	4	∞	5	4
M3	∞	6	9	6	2
M4	9	3	7	2	∞
Dummymachine	0	0	0	0	0

Step II. Reduce the matrix:

Table

	A	B	C	D	E
M1	0	2	∞	1	2
M2	3	0	∞	1	0
M3	∞	4	7	4	0
M4	7	1	5	0	∞
Dummymachine	0	0	0	0	0

Step III. Check if Optimal Assignment can be made in the Current Feasible Solution or not.

Table

	A	B	C	D	E
M1	**0**	2	∞	1	2
M2	3	**0**	∞	1	0
M3	∞	4	7	4	**0**
M4	7	1	5	**0**	∞
Dummymachine	0	0	**0**	0	0

As there is no now and no column without assignment optimal assignment can be made in the initial feasible solution. The optimal assignment of various machines is as follows:

Machine M1 to place A,
Machine M2 to place B,
Machine M3 to place, E
Machine M4 to place D,
And place C will remain vacant.

Total assignment cost = Rs. (4+4+2+2)= Rs 12

4. Interval Assignment problem

In this section we are proposed interval Hungarian method to solve the interval linear assignment problems. Find out the mid values of each interval in the cost matrix. Subtract the interval which has smallest mid value in each row from all the entries of its row. Subtract the interval which has smallest mid value from those columns which have no intervals contain zero from all the entries of its column. Draw lines through appropriate rows and columns so that all the intervals contain zero of the cost matrix are covered and the minimum number of such lines is used. Test for optimality (i) If the minimum number of covering lines is equal to the order of the cost matrix, then optimality is reached.

(ii) If the minimum number of covering lines is less than the order of the matrix, Determine the smallest mid value of the intervals which are not covered by any lines .Subtract this entry from all un-crossed element elements and add it to the crossing having an interval contain zero.

Example:-

(Assignment Problem- Restrictions on Assignment)

Four new machines M1, M2, M3 and M4 are to be installed in a machine shop. There are five vacant places A,B,C, D and E available. Because of limited space, machine M2 cannot be place at C and M3 cannot be placed at A. Cij the assignment cost of machine i to place j in rupees is shown below.

Table

	A	B	C	D	E
M1	4	6	∞	5	6
M2	7	4	∞	5	4
M3	∞	6	9	6	2
M4	9	3	7	2	∞

Find the optimal assignment schedule

Solution :

Step I. Prepare a Square and **interval cost of Matrix.**

As the given matrix is non-square, we add a dummy machine and associate zero cost with the corresponding cells. As machine M1 cannot be place at C, machine M2 cannot be placed at C and M3 cannot be placed at A & M4 cannot be placed at E, we assign infinite cost (M1, C) in cell (M2, C), (M3 A) and (M4, E) resulting in the following matrix.

Table

Interval cost of Matrix.

	A	B	C	D	E
M1	[3,5]	[5,7]	[∞,∞]	[4,6]	[5,7]
M2	[6,8]	[3,5]	[∞,∞]	[4,6]	[3,5]
M3	[∞,∞]	[5,7]	[8,10]	[5,7]	[1,3]
M4	[8,10]	[2,4]	[6,8]	[1,3]	[∞,∞]
Dummymachine	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]

Step II. Reduce the matrix:

Table

	A	B	C	D	E
M1	[0,0]	[2,2]	[∞,∞]	[1,1]	[2,2]
M2	[3,3]	[0,0]	[∞,∞]	[1,1]	[0,0]
M3	[∞,∞]	[4,4]	[7,7]	[4,4]	[0,0]
M4	[7,7]	[1,1]	[5,5]	[0,0]	[∞,∞]
Dummymachine	[0,0]	[0,0]	[0,0]	[0,0]	[0,0]

Step III. Check if Optimal Assignment can be made in the Current Feasible Solution or not.

Table

	A	B	C	D	E
M1	**[0,0]**	[2,2]	[∞,∞]	[1,1]	[2,2]
M2	[3,3]	**[0,0]**	[∞,∞]	[1,1]	[0,0]
M3	[∞,∞]	[4,4]	[7,7]	[4,4]	**[0,0]**
M4	[7,7]	[1,1]	[5,5]	**[0,0]**	[∞,∞]
Dummymachine	[0,0]	[0,0]	**[0,0]**	[0,0]	[0,0]

As there is no now and no column without assignment optimal assignment can be made in the initial feasible solution. The optimal assignment of various machines is as follows:

Machine M1 to place A,
Machine M2 to place B,
Machine M3 to place, E
Machine M4 to place D,
And place C will remain vacant.

Total assignment cost = Rs.[8,16] and optimal assignment is Rs 12

CONCLUSION:

A new and simple method was introduced for solving assignment problems. The MZA-method can be used for all kinds of assignment problems, whether maximize or minimize objective. The new method is based on creating interval zeros in the assignment matrix and finds an assignment in terms of the zeros. As considerable number of methods has been so for presented for assignment problem in which the Hungarian Method is more convenient method among them. Also the comparisons between both the methods have been shown in the paper. Therefore this paper attempts to propose a method for solving assignment problem which is different from the preceding methods.

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