Examination of ΔI = 2 Energy Staggering In the Superdeformed Bands of ^{194}Hg

A. M. Khalaf[(1)], T.M. Awwad [(2)] and M.F. Elgabry [(3)]

(1) Physics Department, Faculty of Science, Al-AZhar University, Cairo – Egypt
(2) Physics Department, Faculty of Girls, Ain Shams University, Cairo – Egypt
(3) Engineering Mathematics and Physics Department, Faculty of Engineering, Al-AZhar University,Cairo – Egypt

Abstract

The presence of the regular ΔI = 2 energy staggering in any of the three superdeformed (SD) bands in ^{194}Hg has been investigated. The gamma-ray transition energies have been determined by proposing a formula based on collective rotational model containing four inertial parameters plus perturbed term linearly dependent on spin. Staggering indices containing four and five consecutive transition energies points have been extracted which represent the third and fourth order derivatives of gamma ray energies at given spin. The experimental dynamical moment of inertia $J^{(2)}$ values have been fitted with Harris formula to assign the band head spins of the SD bands. By using these assigned spin values, the kinematic moments of inertia $J^{(1)}$ of the SD bands have been calculated. $J^{(1)}$ and $J^{(2)}$ are plotted as a function of rotational frequency ħω, it is found that $J^{(1)}$ is appreciably smaller than $J^{(2)}$, both show a smooth rise as ħω increase, this rise was attributed to the successive gradual alignment of a pair of nucleons occupying specific high N intruder orbitals in the presence of the pair correlations. The excellent agreement between the calculated transition energies and the observed ones give good support to the proposed model in describing the yrast SD and the excited SD bands in ^{194}Hg.

Keywords

Harris Model, Superdeformed Rotational Bands, Staggering in Superdeformed Rotational Bands.

1. Introduction

Recently, the discovery of superdeformation is one of the most exciting events in nuclear physics. A great number of superdeformed (SD) bands were established in different mass regions A ~ 60, 80, 130, 150 and 190 [1, 2]. In particular rich experimental information was obtained for nuclei in A ~ 190 mass region, more than 85 SD bands are now known in this region. A characteristical feature in this region is that, the SD bands exhibits the same smooth increasing trend in dynamical moment of inertia $J^{(2)}$ with increasing rotational frequency ħω. This rise was suggested mainly due to the gradual alignment of pair of quasiparticle in high N intruder orbitals and from the gradual disappearance of pairing correlations with the collective rotation. Mean filed calculations predict stable minima in the total energy surface of many nuclei in this mass region A ~ 190 at large deformation, This is due to the presence of large shell gaps at proton number Z = 80 and Z = 82 and neutron number N = 112 for values of the quadrupole deformation parameter β_2 ~ 0.5.

In general for the SD bands, gamma-ray transition energies between levels differing by two units of angular momenta are the only experimentally determined quantity. The problem of experimentally spin assignment for SD bands is difficult and still an unsolved problem up to now; this is due to the difficulty of establishing the de-excitation of SD into known yrast states. Several approaches to assign the spins of SD bands ware proposed [3-7], by fitting the experimentally dynamical moment of inertia $J^{(2)}$ values with Harris formula [8-11], or by fitting the gamma-ray transition energies E_γ with collective rotational models like Bohr-Mottelson formula [12], variable moment of inertia model [13, 14] and empirical formulae [15].

The ΔI = 2 energy staggering effects is corresponding to an up-ward (down-ward) shift of states with spin sequence I, I+4, I+8, …, with respect to down-ward (up-ward) states with spin sequence I+2, I+6, I+10, …, that is states differing by four units of angular momentum show an energy shift of about some hundred eV to a few KeV relative to a smooth rotational sequence. Theoretical efforts were established to explain the origin of ΔI = 2 staggering phenomenon in SD rotational bands. Among them the possibility of presence of an inherent fourfold C_4 symmetry of a prolate deformed nucleus with respect to the long axis [16-18]. Also, it was suggested that such perturbations in the level energies arise from Y_{44} deformations of nuclear shapes [17] and the staggering can occur as a result of tunneling between the four equivalent minima in angular momentum space generated by a potential related to the Y_{44} deformation [16]. The ΔI = 2 staggering can also arise without introducing Y_{44} deformation in the Hamiltonian like band crossing [19], projected shell model [20] on the other hand some models were proposed for gamma-ray transition energies and extracting some staggering parameters to explain the experimental data [21-26].

The present paper, is an extension to the previous paper [23], the ΔI = 2 energy staggering in the three SD bands of doubly even ^{194}Hg nucleus are examined in a framework of energy formula contains seven parameters based on the collective rotational model and a perturbed term. In section 2 we applied Harris expansion for dynamical moment of inertia to assign the band head spins for the SD bands. We outline our proposed theoretical formula for SD transition energies which can explore the ΔI = 2 energy staggering in section 3. In section 4 we present the numerical calculations and the obtained results for the three SD bands in Mercury ^{194}Hg and comparison with experimental data is discussed. Finally a brief conclusion is given in section 5.

2. Spin Assignment of SD Bands

The rotational energy is given by Harris formula [27] of the cranking model, where one has even powers of angular velocity expansion as follows:

$$E(\omega) = \tfrac{1}{2}\sum_{n=0}^{3} J_n\left(\tfrac{2n+1}{n+1}\right)\omega^{2n+2} = \tfrac{1}{2}J_0\omega^2 + \tfrac{3}{4}J_1\omega^4 + \tfrac{5}{6}J_2\omega^6 + \tfrac{7}{8}J_3\omega^8 \qquad (1)$$

The standard way to analyze SD bands is to consider the dynamical moment of inertia $J^{(2)}$ because it does not require any knowledge of the spin value which is not determined experimentally. $J^{(2)}$ is defined through the relation:

$$\frac{J^{(2)}}{\hbar^2} = \frac{1}{\omega}\frac{dE}{d\omega} = \sum_{n=0}^{3} J_n(2n+1)\omega^{2n} = J_0 + 3J_1\omega^2 + 5J_2\omega^4 + 7J_3\omega^6 \qquad (2)$$

The expansion parameters J_0, J_1, J_2 and J_3 which results from fitting $J^{(2)}$ with experimental values are used to determined the intermediate spin $\hat{I}=\sqrt{I(I+1)}$ by integrating $J^{(2)}$ with respect to ω:

$$\hbar\hat{I} = \int J^{(2)} d\omega = \sum_{n=0}^{3} J_n\omega^{2n+1} = J_0\omega + J_1\omega^3 + J_2\omega^5 + J_3\omega^7 \qquad (3)$$

We define a kinematic moment of inertia $J^{(1)}$ by:

$$J^{(1)} = \hbar\frac{\hat{I}}{\omega} = \sum_{n=0}^{3} J_n\omega^{2n} = J_0 + J_1\omega^2 + J_2\omega^4 + J_3\omega^6 \qquad (4)$$

For rigid rotation $J^{(1)}$, $J^{(2)}$ will be equal.

The values of ω^2 associated with the quantum state I can be obtained from the relation:

$$\hbar\hat{I}^2 = \omega^2[J^{(1)}]^2 \qquad (5)$$

In general, the above Harris approximation converges faster than the Bohr-Mottelson expansion [28] in powers of $\hat{I}^2$:

$$E(I) = A\hat{I}^2 + B\hat{I}^4 + C\hat{I}^6 + D\hat{I}^8 \quad (6)$$

where A, B, C and D are the constant rotational parameters. Substituting from equations (4, 5) into equation (6), yield:

$$\begin{aligned} E(I) &= [AJ_0^2]\omega^2 + [2AJ_0J_1 + BJ_0^4]\omega^4 \\ &+ [A(2J_0J_2 + J_1^2) + 4BJ_0^3J_1 + CJ_0^6]\omega^6 \\ &+ [A(2J_0J_3 + 2J_1J_2) + B(4J_0^3J_2 + 6J_0^2J_1^2) + 6CJ_0^5J_1 + DJ_0^8]\omega^8 \end{aligned} \quad (7)$$

Comparing equation (7) with equation (1), yield the relations:

$$A = \tfrac{1}{2}J_0^{-1}\hbar^2 \quad (8)$$

$$B = -\tfrac{1}{4}J_1J_0^{-4}\hbar^4 \quad (9)$$

$$C = (\tfrac{1}{2}J_1^2J_0^{-1} - \tfrac{1}{6}J_2)J_0^{-6}\hbar^6 \quad (10)$$

$$D = (-\tfrac{3}{2}J_1^3J_0^{-2} + J_1J_2J_0^{-1} - \tfrac{1}{8}J_3)J_0^{-8}\hbar^8 \quad (11)$$

The experimentally determined quantities are the gamma-ray transition energies between levels differing by two units of angular momenta, and then we can write:

$$\begin{aligned} E_\gamma(I) &= E(I) - E(I-2) \\ &= 2(2I-1)[A + 2(I^2 - I + 1)B \\ &\quad + (3I^4 - 6I^3 + 13I^2 - 10I + 4)C \\ &\quad + 4(I^6 - 3I^5 + 10I^4 - 15I^3 + 15I^2 - 8I + 2)D] \\ &= C_0 + C_1I + C_2I^2 + C_3I^3 + C_4I^4 + C_5I^5 + C_6I^6 + C_7I^7 \end{aligned} \quad (12)$$

where:

$$C_0 = -2(A + 2B + 4C + 8D) \quad (13)$$

$$C_1 = 4(A + 3B + 9C + 24D) \quad (14)$$

$$C_2 = -2(6B + 33C + 124D) \quad (15)$$

$$C_3 = 8(B + 8C + 45D) \quad (16)$$

$$C_4 = -10(3C + 32D) \quad (17)$$

$$C_5 = 4(3C + 46D) \quad (18)$$

$$C_6 = -56D \quad (19)$$

$$C_7 = 16D \quad (20)$$

3. Analysis of Δ = 2 Staggering Effect in Transition Energies

Axially symmetric prolate deformed nuclei posses a symmetry that leaves the system invariant following rotations through π about an axis perpendicular to the symmetry axis. Evidence for higher symmetry terms come with the observation of ΔI = 2 staggering (ΔI = 4 bifurcation) in the SD bands [29]. Here, the gamma-ray transition energies from every second SD level are shifted by approximately few hundred eV relative to other level. This perturbation that occur every four units of angular momentum has been interpreted [16, 17] as evidence for a C_{4v} invariant (π/2) term in the Hamiltonian. This new term is a small perturbation on the main quadrupole shape. A ΔI = 2 energy staggering can be arise from a rotational Hamiltonian of the form:

$$H = C_3I^3 + C_2I^2 + C_1I + C_0 + \delta E_\gamma(I) \quad (21)$$

where:

$$\delta E_\gamma(I) = \begin{cases} \alpha I + \beta \text{ for } & 4,8,12,.... \\ & 5,9,13,.... \\ \gamma \quad \text{for } & 6,10,14,.... \\ & 7,11,15,.... \end{cases} \quad (22)$$

To analyze the energy staggering in the SD bands, several tests were used [21-26]. In our analysis, we use the finite differences approximation $S^n(I) = \Delta^n E_\gamma/\Delta I^n$ of the transition energies $E_\gamma(I)$:

$$S^{(n)}(I) = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} E_\gamma(I)(X + 2k) \quad (23)$$

where X = I, I-2, I-2 and I-4 for first, second, third and fourth derivative and the binomial coefficient is given by:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (24)$$

For each band the deviation of the gamma-ray transition energies from a smooth reference has been determined.

Therefore:

$$S^{(1)}(I) = E_\gamma(I+2) - E_\gamma(I)$$
$$= (3I^2 + 6I + 4)C_3 + 2(I+1)C_2 + C_1 + \begin{cases} (I+2)\alpha + \beta - \gamma \\ I\alpha - \beta + \gamma \end{cases} \quad (25)$$

$$S^{(2)}(I) = E_\gamma(I-2) - 2E_\gamma(I) + E_\gamma(I+2)$$
$$= 6IC_3 + 2C_2 \pm \tfrac{1}{2}(I\alpha + \beta - \gamma) \quad (26)$$

$$S^{(3)}(I) = E_\gamma(I-2) - 3E_\gamma(I) + 3E_\gamma(I+2) - E_\gamma(I+4)$$
$$= -24(I+2)C_3 \pm [(I+1)\alpha + \beta - \gamma] \quad (27)$$

$$S^{(4)}(I) = E_\gamma(I-4) - 4E_\gamma(I-2) + 6E_\gamma(I) - 4E_\gamma(I+2) + E_\gamma(I+4)$$
$$= \pm(I\alpha + \beta - \gamma) \quad (28)$$

The expansion $S^{(4)}(I)$ was previously used by Cederwall [30], and is identical to the expansion $\Delta^4 E_\gamma(I)$ in our previous paper [23]. We chose to use $S^{(3)}(I)$ and $S^{(4)}(I)$ in order to be able to follow higher order changes in the moment of inertia of the SD bands.

4. Numerical Calculations and Discussion

In the ^{194}Hg nucleus the SD1 consists of 21 transitions spanning the energy range of 212 KeV to 903 KeV, SD2 consists of 20 transitions spanning the energy range of 201 KeV to 867 KeV, SD3 consists of 20 transitions spanning the energy range of 222 KeV to 883 KeV. To assign band head spins, we have applied Harris formula for dynamical moment of inertia $J^{(2)}$ to fit the corresponding experimental values, we choose to minimize the mean square deviation χ^2:

$$\chi^2(J^{(2)}) = \frac{1}{N}\sum_j \left(\frac{J^{(2)}_{exp}(j) - J^{(2)}_{cal}(j)}{\Delta J^{(2)}_{exp}(j)}\right)^2$$

where $\Delta J^{(2)}_{exp}(j)$ are the experimental error in $J^{(2)}$ and N is the number of the experimental data points entering the fitting procedure. The calculated lowest band head spin for the three bands SD1, SD2 and SD3 are 8, 8 and 9 respectively. Band 2 and 3 are considered to be signature partners based on a strongly coupled configuration. In the framework of our proposed formula for gamma-ray transition energies E_γ, the seven parameters C_0, C_1, C_2, C_3, α, β and γ occurring in equation (21) are obtained by using a simulated computer search program to fit the calculated E_γ with the experimental E_γ^{exp}, to perform the fitting we choose also the minimization function χ^2:

$$\chi^2(E_\gamma) = \frac{1}{N}\sum_j \left(\frac{E_\gamma^{exp}(j) - E_\gamma^{cal}(j)}{\Delta E_\gamma^{exp}(j)}\right)^2$$

Table 1. The calculated optimized best model parameters C_3, C_2, C_1, C_0, α, β and γ (all in KeV) and suggested band head spin I_o for SDRB's in ^{194}Hg nuclei which exhibit staggering in energies.

parameters	^{194}Hg (SD1)	^{194}Hg (SD2)	^{194}Hg (SD3)
C_3	-17.44×10^{-4}	-1.36×10^{-3}	-1.28×10^{-3}
C_2	26.16×10^{-4}	2.064×10^{-3}	1.92×10^{-3}
C_1	22.0037	21.1259	21.0060
C_0	-11.0023	-10. 5633	-10.5033
α	1.1851×10^{-3}	-2.32×10^{-3}	-3.4×10^{-3}
β	-2.3702×10^{-3}	4.64×10^{-3}	6.8×10^{-3}
γ	3.5553×10^{-3}	-6.96×10^{-3}	-0.0102
I_o	8	8	9
$E_\gamma (I_o+2 \rightarrow I_o)$	211.7	200.79	222.0

Table 2. The calculated transition energies $E_\gamma(I)$ for SDRB's in ^{194}Hg and comparison with experimental data. The model parameters and the band head spins are listed in Table 1.

^{194}Hg (SD1)			^{194}Hg (SD2)			^{194}Hg (SD3)		
I(ħ)	E_γ (I) (KeV)		I(ħ)	E_γ (I) (KeV)		I(ħ)	E_γ (I) (KeV)	
	EXP	CAL		EXP	CAL		EXP	CAL
10	211.70	207.5555	10	20079	199.521	11	222.00	219.0812
12	253.93	250.4163	12	242.25	240.8486	13	262.27	260.0459
14	295.99	292.7790	14	283.45	281.8286	15	302.68	300.6826
16	337.18	334.5976	16	323.45	322.3220	17	342.50	340.8053
18	377.39	375.7413	18	363.12	362.3555	19	381.68	380.5019
20	416.60	416.1830	20	402.05	401.7519	21	420.08	419.5341
22	454.76	455.7725	22	440.31	440.5752	23	457.79	458.0444
24	491.86	494.5021	24	477.68	478.6110	25	494.77	495.7401
26	527.88	532.2027	26	514.23	515.9604	27	531.01	532.8179
28	562.92	568.8854	28	549.93	552.3720	29	566.26	568.9309
30	596.87	604.3619	30	584.82	587.9839	31	600.92	604.3302
32	629.93	638.6625	32	618.96	622.5075	33	634.60	638.6144
34	662.07	671.5802	34	652.03	656.1183	35	667.84	672.0890
36	693.40	703.1647	36	684.57	688.4904	37	700.11	704.2982
38	723.91	733.1874	38	716.20	719.8362	39	731.70	735.6020
40	753.92	761.7191	40	746.89	749.7931	41	762.77	765.4900
42	783.67	788.5139	42	777.73	778.8610	43	793.51	794.3768
44	813.12	813.6585	44	807.76	805.8882	45	823.65	821.6977
46	842.55	836.8894	46	837.48	831.9133	47	853.85	847.9215
48	872.41	858.3123	48	867.08	856.2486	49	883.60	872.4289
50	903.10	877.6442						

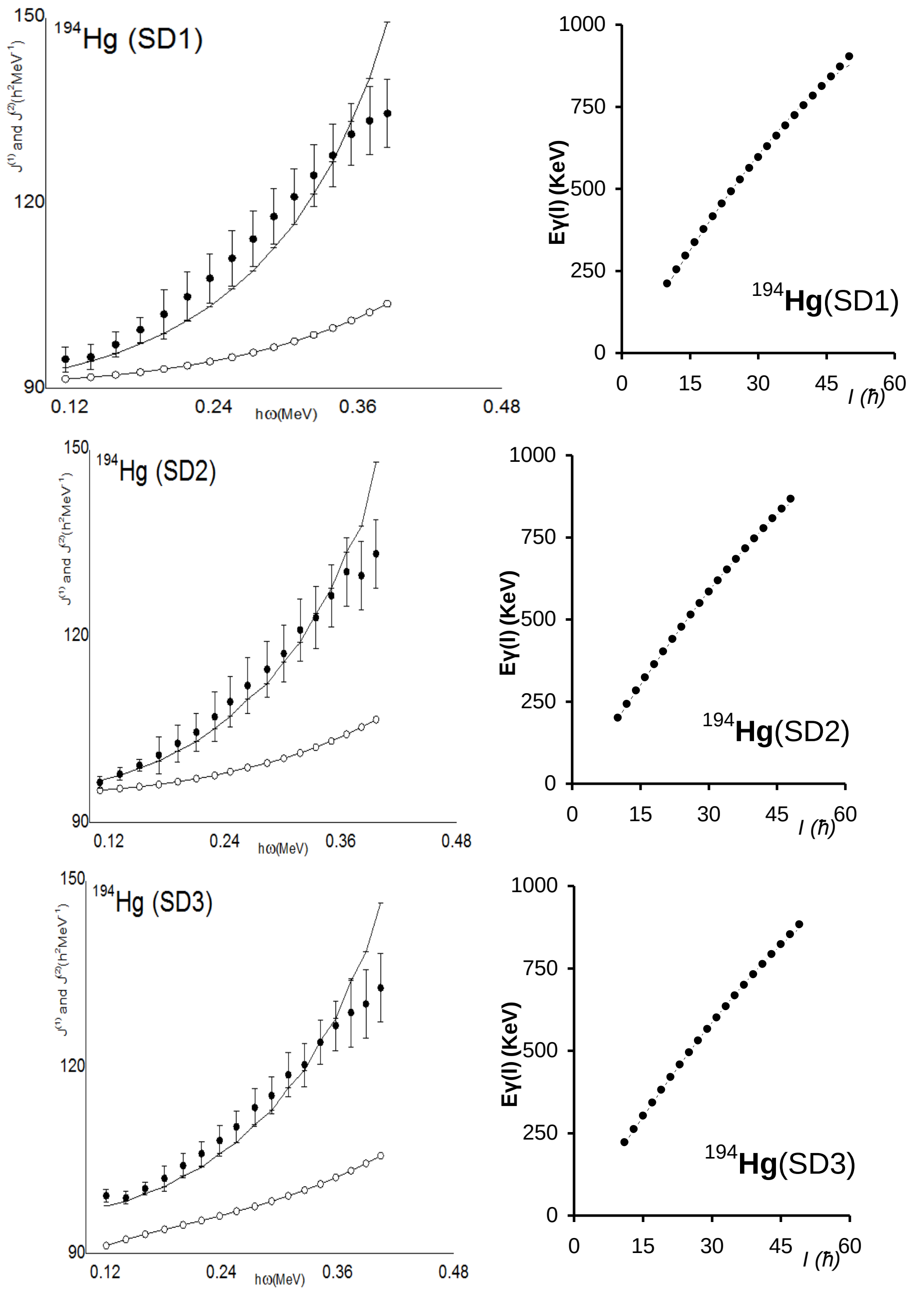
194Hg (SD1)
194Hg (SD2)
194Hg (SD3)
hω(MeV)
Eγ(I) (KeV)
I (ħ)
194Hg(SD1)
194Hg(SD2)
194Hg(SD3)

Figure 1.

(Left): calculated gamma-ray transition energies E_γ (solid curves) versus spin I for the three SD bands of ^{194}Hg compared with the experimental values (closed circles) [1, 2].

(Right): The calculated results of the kinematic $J^{(1)}$ (open circles) and dynamic $J^{(2)}$ (solid curves) moments of inertia plotted as a function of rotational frequency ħω for the three SD bands of ^{194}Hg and comparison with the experimental data for $J^{(2)}$ (closed circles with error bars).

Table (1) shows band head spin I_o, and the best fitted values of the model four parameters C_0, C_1, C_2 and C_3, and the three terms α, β and γ of the perturbed term. Table (2) summarizes the calculated transition energies compared to experimental data for the three SD bands in ^{194}Hg. For the purpose of comparison, Figure (1) display the results of calculated E_γ compared to experimental ones. The calculated results of kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia as a function of rotational frequency ħω of the three SD bands of ^{194}Hg and comparison with experimental ones are showing in Figure (1). From the figure we see that the smooth increase of $J^{(2)}$ with ħω is reproduced well.
For the ΔI = 2 energy staggering, the resulting staggering parameters $S^{(3)}$(I) and $S^{(4)}$(I) defined as the third and fourth derivative of the gamma-ray transition energies against the spin for the three bands are presented in Tables (3, 4) and in Figure (2). It indicates that the experimentally observed staggering is well reproduced our approach. Investigated the tables and figures, we see that the E2 transition gamma energies, the dynamical moment of inertia $J^{(2)}$ and the ΔI = 2 energy staggering parameters $S^{(3)}$(I) and $S^{(4)}$(I) of the three SD bands of ^{194}Hg can be quantitatively described excellently by four initial parameters formula based on collective rotational model plus perturbed term linearly dependent on spin. The calculated results agree with experimental data very well.

Table 3. The calculated ΔI = 1 staggering parameter $S^{(3)}$ (in KeV) obtained by the four point formula for SDRB's ^{194}Hg (SD1, SD2, SD3) and comparison with experiment.

^{194}Hg (SD1)			^{194}Hg (SD2)			^{194}Hg (SD3)		
I(ħ)	$S^{(3)}$ (I) (KeV)		I(ħ)	$S^{(3)}$ (I) (KeV)		I(ħ)	$S^{(3)}$ (I) (KeV)	
	EXP	CAL		EXP	CAL		EXP	CAL
12	0.7	-0.054	12	0.01	0.1401	15	0.05	-0.0879
14	0.11	0.0308	14	0.06	-0.0267	17	0.14	0.2383
16	0.02	-0.0729	16	0.10	0.1772	19	-0.09	-0.1425
18	0.05	0.0502	18	-0.07	-0.064	21	0.04	0.2927
20	0.01	-0.0923	20	0.22	0.2144	23	0.01	-0.1967
22	0.02	0.0691	22	-0.07	-0.1011	25	0.25	0.3469
24	-0.10	-0.1111	24	0.03	0.2514	27	-0.40	-0.2511
26	0.11	0.0883	26	-0.04	-0.1381	29	0.39	0.4014
28	-0.20	-0.1303	28	-0.06	0.2886	31	-0.54	-0.3055
30	0.03	0.107	30	0.32	-0.1755	33	0.53	0.4558
32	-0.11	-0.1497	32	-0.54	0.2259	35	-0.29	-0.36
34	0.01	0.1286	34	0.38	-0.2124	37	-0.16	0.5104
36	-0.32	-0.1708	36	0.03	0.3625	39	-0.19	-0.4146
38	-0.24	0.1457	38	-1.09	-0.4999	41	0.27	0.5647
40	0.04	-0.1867	40	0.96	1.1517	43	-0.66	-0.4688
42	-0.28	0.1635	42	-0.50	-1.0386	45	0.51	0.6193
44	-0.45	-0.2057	44	-0.19	0.6877			
46	-0.40	0.187						

Table 4. The calculated ΔI = 2 staggering parameter $S^{(4)}$ (in KeV) obtained by the five point formula for SDRB's ^{194}Hg (SD1, SD2, SD3) and comparison with experiment.

^{194}Hg (SD1)			^{194}Hg (SD2)			^{194}Hg (SD3)		
I(ħ)	$S^{(4)}$ (I) (KeV)		I(ħ)	$S^{(4)}$ (I) (KeV)		I(ħ)	$S^{(4)}$ (I) (KeV)	
	EXP	CAL		EXP	CAL		EXP	CAL
16	0.09	0.1042	14	-0.05	0.16704	15	0.68	0.2720
18	-0.03	-0.1232	16	-0.04	-0.20416	17	-0.09	-0.3264
20	0.04	0.1422	18	0.17	0.24128	19	0.23	0.3808
22	-0.01	-0.1611	20	-0.29	-0.2784	21	-0.13	-0.4352
24	0.12	0.1801	22	0.29	0.31552	23	0.03	0.4896
26	-0.21	-0.1990	24	-0.10	-0.35264	25	-0.24	-0.5440
28	0.31	0.2180	26	0.07	0.38976	27	0.65	0.5984
30	-0.23	-0.2370	28	0.02	-0.42688	29	-0.79	-0.6528
32	0.14	0.2559	30	-0.38	0.46400	31	0.93	0.7072
34	-0.12	-0.2749	32	0.86	-0.50112	33	-1.07	-0.7616
36	0.33	0.2939	34	-0.92	0.53824	35	0.82	0.8160
38	-0.08	-0.3128	36	0.35	-0.57536	37	-0.13	-0.8704
40	-0.28	0.3318	38	1012	0.61248	39	0.03	0.9248
42	0.32	-0.3507	40	-2.05	-0.64960	41	-0.46	-0.9792
44	0.17	0.3697	42	1.46	0.68672	43	0.93	1.0336
46	-0.05	-0.3887	44	-0.31	-0.72384	45	-1.17	-1.0880

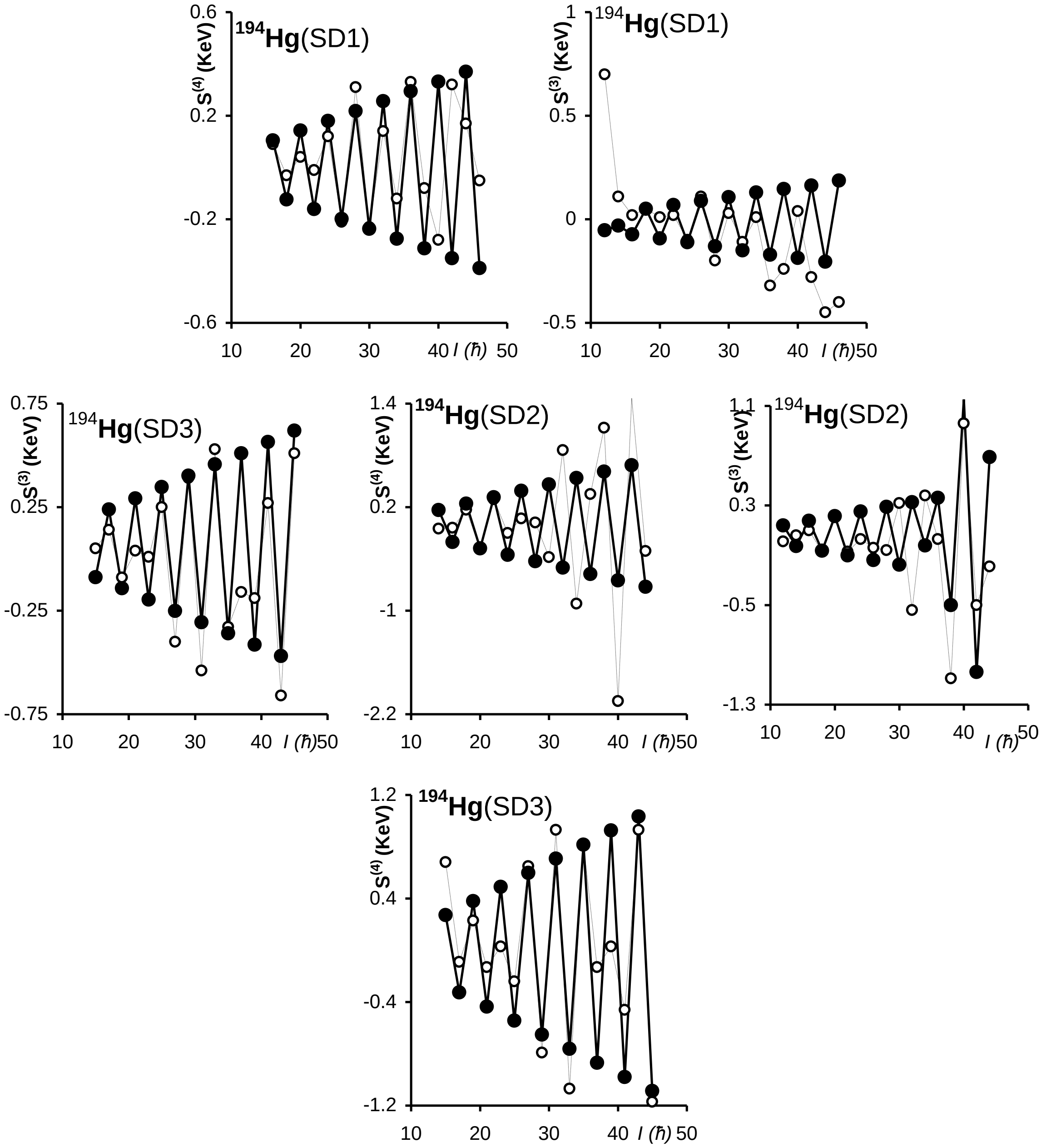

Figure 2.

(Left): The calculated ΔI = 2 Staggering parameter $S^{(3)}$ (closed circles with solid line) obtained by the four point formula plotted as a function of spin I and comparison with experiment (Open circles with dotted lines) for the three SD bands of ^{194}Hg.

(Right): The same as (left) but for Staggering parameter $S^{(4)}$ obtained by the five point formula.

5. Conclusion

Firstly the band head spins for our SD bands are determined by performing Harris expansion for dynamical moment of inertia. For gamma-ray transition energies E_γ, we adopted a rotational Hamiltonian based on collective model, and additional perturbed term to explore the staggering in energies. The appearance of ΔI = 2 staggering effects in E_γ for the three SD bands of ^{194}Hg have been examined by considering a smooth references representing the finite difference approximation to the third and fourth order derivative of E_γ at a given spin. Band 1 show no significant deviation from the references over the entire rotational frequency ħω range, bands 2 is obviously deviated from the references for ħω = 0.3 MeV. A short regular staggering pattern is vissible in band 3 for 0.25 MeV < ħω < 0.325 MeV.

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