PRIORITY QUEUE MODEL ALONG INTERMEDIATE QUEUE UNDER FUZZY ENVIRONMENT WITH APPLICATION

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ABSTRACT:

This paper deals a mathematical model of intermediate queue before two semi parallel but heterogeneous servers under fuzzy environment. Queue discipline before the servers are considered on the basis of high priority while the service of intermediate queue is considered to be low priority. Steady state analysis of the model has been derived with the help of generating function technique, laws of calculus and statistical techniques.

In some of these kinds of systems, it happens that the arrival and service parameters both are uncertain. Hence, we have considered the fuzzy arrival and fuzzy service rate which result in deriving the state probabilities and queue length of system in fuzzy expression. α- cut has been applied to shorten the length of the interval. A numerical illustration has also been given in order to clarify the concept. The model finds its application to public transportation problem.

***Key words**: Steady state, Triangular fuzzy number, marginal queue length, Poisson distribution, Exponential distribution, pre empty priority.*

1. INTRODUCTION:

A vast amount of research has been conducted by different researchers and academic community in the field of serial and bi-serial queue network during past several decades. Jackson (1957) presented the solution of the steady state serial queue system in product form. Maggu (1970) introduced the concept of bi- series in queuing theory. Maggu and Sharma (1984) discussed the problem of intermediate queue before heterogeneous servers in the stochastic environment. Later on T.P.Singh (1986) and his scholars did a remarkable work in the field of queuing theory with many new ideas and concepts. T.P. Singh, Deepak Gupta et.al. (2005, 2010, 2012) extended the work of earlier researchers in the field of queuing theory with different parameters and augmentations. Most of the researchers assume that customers arrival, inter arrival time and service rates are required to follow some probability distribution with fixed parameters, however, in many real world situations the parametric distribution are characterized subjectively i.e. linguistic variables as slow, moderate, fast etc . Fuzzy queue models are potentially more useful and realistic than commonly stochastic models. The researchers could able to fuzzify the queue system under different situations.

Li & Lee (1989) investigated the analytical results for two special queue models M/F/1/∞ and FM/FM/1/∞ where F denotes fuzzy time and FM denotes fuzzified exponential distribution. Negi & Lee (1992) proposed new method using α-cut. T.P.Singh, Kusum et.al. (2012) fuzzified the machine repairing queue model. T.P. Singh et. al. (2012, 2013) investigated the fuzzy queue model for interdependent communication and voice packetized system. Recently Meenu,T.P.Singh and Deepak Gupta (2015) analysed fuzzy queue model with threshold effect using α-cut concept. The results found very encouraging.

The present study first discusses a stochastic behaviour of an intermediate queue arising before two semi parallel but heterogeneous servers on the basis of a pre emptive priority service rule, while services before S_1 & S_2 follow FCFS rule. The customers getting service before S_1 & S_2 are high priority while customers getting service before intermediate service are low priority. Then we convert the system in fuzzified form by applying α- cut in order to shorten the length of the interval, making the model more relevant and realistic for a real world situation. The model finds its applications to public transportation for a long route bus stoppage where there is a common station from one place to another.

2. MODEL WITH PRACTICAL SITUATION:

We consider a routing situation at a bus stop with two service counters S_1 and S_2 distributing tickets. Also consider two types of passengers travelling to different destinations A & B through a common station C on the routes via A & B. The queue of passengers demanding service for C is intermediate queue and the service is being served on the basis of low priority service discipline by one of the servers S_1 or S_2 otherwise servers S_1 & S_2 prefer to serve long route passengers to the destinations A & B with high priority. The Servers S_1 & S_2 only serves C Station passengers if number of passengers in queue before S_1 or S_2 requiring service for routes A & B is zero (see fig.1)

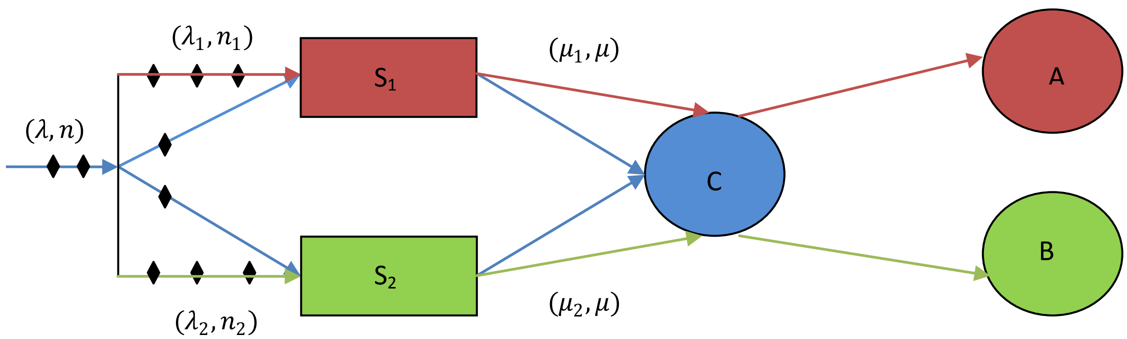


fig.1

3. ASSUMPTIONS AND NOTATIONS:

3.1 Notations: The following notations are addressed for the purpose of mathematical analysis of queue system:

$\lambda_{1,}\lambda_2$**:** Poisson arrival rates before servers S_1, S_2 demanding service for long routes A & B respectively.

λ :Poisson arrival rate to intermediate route C

$\mu_{1,}\mu_2$**:** Exponential service parameters at S_1& S_2

μ: Service parameter for the destination C regarding intermediate queue

$\tilde{\lambda}_1, \tilde{\lambda}_2$: Fuzzy arrival parameters

$\tilde{\mu}_1, \tilde{\mu}_2$: Fuzzy service parameters at S_1 & S_2

$\tilde{\mu}$: Fuzzy service parameter for intermediate queue

Q_{H_1}:High priority queue before S_1 when servers are busy

Q_{H_2}: High priority queue before S_2 when servers are busy

$Q_{int\,.L}$: Low priority queue before server S_1 or S_2

3.2 ASSUMPTIONS:

- Intermediate queue service follows low priority discipline.
- Service within each queue is done on the basis of FCFS rule.
- Before S_1, there is a queue Q_{H_1} of high priority customers for long route A.
- Before S_2, there is a queue Q_{H_2} of high priority customers for long route B.
- There is a common station C via routes A & B.

4. STEADY STATE ANALYSIS: Let P_{n_1,n,n_2}(t) denotes the probability that any time t there are n_1, n, n_2 number of customers in $Q_{H_1}, Q_{int.L}, Q_{H_2}$ respectively where n_1, n, $n_2 \geq 0$. Connecting the various state probabilities at any time t and $(t+\Delta t)$ also letting $t \rightarrow \infty$ & $\Delta t \rightarrow 0$ the steady state equations governing the model can be depicted as:

$$(\lambda_1+\lambda+\lambda_2+\mu_1+\mu_2)P_{n_1,n,n_2} = \lambda_1 P_{n_1-1,n,n_2} + \lambda P_{n_1,n-1,n_2} + \lambda_2 P_{n_1,n,n_2-1} + \mu_1 P_{n_1+1,n,n_2} + \mu_2 P_{n_1,n,n_2+1}$$

for n_1, n, $n_2 > 0$ (1)

$$(\lambda_1+\lambda+\lambda_2+\mu+\mu_2)P_{0,n,n_2} = \lambda P_{0,n-1,n_2} + \lambda_2 P_{0,n,n_2-1} + \mu_1 P_{1,n,n_2} + \mu_2 P_{0,n,n_2+1} + \mu P_{0,n+1,n_2}$$

for n_1,=0 n, $n_2 > 0$ (2)

$$(\lambda_1+\lambda+\lambda_2+\mu_1+\mu_2)P_{n_1,0,n_2} = \lambda_1 P_{n_1-1,0,n_2} + \lambda_2 P_{n_1,0,n_2-1} + \mu_1 P_{n_1+1,0,n_2} + \mu_2 P_{n_1,0,n_2+1}$$

for n=0,n_1, $n_2 > 0$ (3)

$$(\lambda_1+\lambda+\lambda_2+\mu_1+\mu_2)P_{n_1,n,0} = \lambda_1 P_{n_1-1,n,0} + \lambda P_{n_1,n-1,0} + \mu_1 P_{n_1+1,n,0} + \mu_2 P_{n_1,n,1} + \mu P_{n_1,n+1,0}$$

for n_2=0 ,n, $n_1 > 0$ (4)

$$(\lambda_1+\lambda+\lambda_2+\mu+\mu)P_{0,n,0} = \lambda P_{0,n-1,0} + \lambda_2 P_{n_1,n,n_2-1} + \mu_1 P_{1,n,0} + \mu_2 P_{0,n,1} + 2\mu P_{0,n+1,0}$$

for n_1, n_2 =0, n> 0 (5)

$$(\lambda_1+\lambda+\lambda_2+\mu_2)P_{0,0,n_2} = \lambda_2 P_{0,0,n_2-1} + \mu_1 P_{1,0,n_2} + \mu_2 P_{0,0,n_2+1} + \mu P_{0,1,n_2}$$

for n_1& n=0, $n_2 > 0$ (6)

$$(\lambda_1+\lambda+\lambda_2+\mu_1)P_{n_1,0,0} = \lambda_1 P_{n_1-1,0,0} + \mu_1 P_{n_1+1,0,0} + \mu_2 P_{n_1,0,1} + \mu P_{n_1,1,0}$$

for n_2,=0 n, $n_1 > 0$ (7)

$$(\lambda_1+\lambda+\lambda_2)P_{0,0,0} = \mu_1 P_{1,0,0} + \mu_2 P_{0,0,1} + 2\mu P_{0,1,0}$$

for n_1, n, n_2 = 0 (8)

4.1 SOLUTION OF QUEUE EQUATIONS: To solve the equations (1-8) we use the generating function technique defined as:

$$F(X,Y,Z)=\sum_{n_1=0}^{\infty}\sum_{n=0}^{\infty}\sum_{n_2=0}^{\infty}P_{n_1,n,n_2}X^{n_1}\,Y^{n}\,Z^{n_2} \tag{9}$$

where $|X|\le 1$, $|Y|\le 1$ and $|Z|\le 1$

Also define the following partial generating functions:

$$\left.\begin{aligned} F_{n,n_2}(X)&=\sum_{n_1=0}^{\infty}P_{n_1,n,n_2}X^{n_1}\\ G_{n_1,n_2}(Y)&=\sum_{n=0}^{\infty}P_{n_1,n,n_2}Y^{n}\\ \mathrm{H}_{n_1,n}(Z)&=\sum_{n_2=0}^{\infty}P_{n_1,n,n_2}Z^{n_2}\end{aligned}\right\} \tag{10.1}$$

$$\left.\begin{aligned} I_{n_2}(X,Y)&=\sum_{n=0}^{\infty}F_{n,n_2}(X)Y^{n}=\sum_{n=0}^{\infty}G_{n_1,n_2}(Y)X^{n_1}\\ A_{n_1}(Y,Z)&=\sum_{n=0}^{\infty}G_{n_1,n_2}(Y)Z^{n_2}=\sum_{n=0}^{\infty}H_{n_1,n}(Z)Y^{n}\end{aligned}\right\} \tag{10.2}$$

Multiplying equation (1),(3),(4),(7) by X^{n_1},summing over n_1 from 0 to ∞ and using equations (2),(6),(5),(8),(10.1)(10.2) then resultant equations multiplying by Y^{n}, summing over n from 0 to ∞ and then multiplying by Z^{n_2} summing over n_2 from 0 to ∞ ,on the basis of solution given by T.P. Singh (2005), we get

$$F(X,Y,Z)=\frac{I_0(X,Y)\left[\mu_2\left(1-\frac{1}{Z}\right)-\mu\left(1-\frac{1}{Y}\right)\right]+A_0(Y,Z)\left[\mu_1\left(1-\frac{1}{X}\right)-\mu\left(1-\frac{1}{Y}\right)\right]+\mu\left(1-\frac{1}{Y}\right)H_{0,0}(Z)+\mu\left(1-\frac{1}{Y}\right)F_{0,0}(X)}{\lambda_1(1-X)+\lambda(1-Y)+\lambda_2(1-Z)+\mu_1\left(1-\frac{1}{X}\right)+\mu_2(1-\frac{1}{Z})} \tag{11}$$

The value of $F(X,Y,Z)$ determined by (11) at X=Y=Z=1reduces to indeterminant form 0/0.

Now letting Y=Z=1 and taking as X→1in (11), we get

$$A_0(1,1)\mu_1=-\lambda_1+\mu_1 \tag{12}$$

(12) gives $A_0(1,1)=1-\frac{\lambda_1}{\mu_1}$ (13)

Similarly letting X=Z=1 and taking limit as Y→1 in (11) gives

$$\mu I_0(1,1) + A_0(1,1) - \mu F_{0,0}(1) - \mu H_{0,0}(1) = \lambda \quad (14)$$

And letting X=Y=1 and taking limit as Z→1 in (11) gives

$$I_0(1,1) = 1 - \frac{\lambda_2}{\mu_2} \quad (15)$$

with the aid of (13),(14),(15) gives

$$F_{0,0}(1) + H_{0,0}(1) = \left(1 - \frac{\lambda_1}{\mu_1}\right) + \left(1 - \frac{\lambda_2}{\mu_2}\right) - \frac{\lambda}{\mu} \quad (16)$$

If we let $F(X,Y,Z) = \frac{f(X,Y,Z)}{g(X,Y,Z)}$ (17)

Where

$$f(X,Y,Z) = I_0(X,Y)\left[\mu_2\left(1-\frac{1}{Z}\right) - \mu\left(1-\frac{1}{Y}\right)\right] + A_0(Y,Z)\left[\mu_1\left(1-\frac{1}{X}\right) - \mu\left(1-\frac{1}{Y}\right)\right] + \mu\left(1-\frac{1}{Y}\right)H_{0,0}(Z) + \mu\left(1-\frac{1}{Y}\right)F_{0,0}(X)$$

(18)

And $g(X,Y,Z) = \lambda_1(1-X) + \lambda(1-Y) + \lambda_2(1-Z) + \mu_1\left(1-\frac{1}{X}\right) + \mu_2(1-\frac{1}{Z})$
(19)

Now, clear that

$$f(X,Y,Z)|_{X=Y=Z=1} = 0$$

$$g(X,Y,Z)|_{X=Y=Z=1} = 0$$

Calculation of the values of the partial derivative at X=y=Z=1, gives

$$\frac{\partial f}{\partial X}|_{X=Y=Z=1} = \mu_1 A_0(1,1) = \mu_1 - \lambda_1$$

$$\frac{\partial^2 f}{\partial X^2}|_{X=Y=Z=1} = -2\mu_1 A_0(1,1) = -2(\mu_1 - \lambda_1)$$

$$\frac{\partial f}{\partial Y}|_{X=Y=Z=1} = -\lambda$$

$$\frac{\partial^2 f}{\partial Y^2}|_{X=Y=Z=1} = 2\lambda - 2\mu[A'_0(1,1) - I'_0(1,1)]$$

$$\frac{\partial f}{\partial Z}|_{X=Y=Z=1} = \mu_2 I_0(1,1) = \mu_2 - \lambda_2$$

$$\frac{\partial^2 f}{\partial Z^2}|_{X=Y=Z=1} = -2\mu_2 I_0(1,1) = -2(\mu_2 - \lambda_2)$$

$$\frac{\partial g}{\partial X}|_{X=Y=Z=1} = -\lambda_1 + \mu_1$$

$$\frac{\partial^2 g}{\partial X^2}|_{X=Y=Z=1} = -2\mu_1$$

$$\frac{\partial g}{\partial Y}|_{X=Y=Z=1} = -\lambda$$

$$\frac{\partial^2 g}{\partial Y^2}|_{X=Y=Z=1} = 0$$

$$\frac{\partial g}{\partial Z}|_{X=Y=Z=1} = \mu_2 - \lambda_2$$

$$\frac{\partial^2 g}{\partial Z^2}|_{X=Y=Z=1} = -2\mu_2$$

4.2 MEAN QUEUE LENGTH

The marginal mean queue length for the queue Q_{H_1} denoted by $L_{Q_{H_1}}$ is given by the statistical formula

$$L_{Q_{H_1}} = \frac{\partial F(X,Y,Z)}{\partial X}|_{X=Y=Z=1}$$

$$= \frac{\frac{\partial g}{\partial X}|_{X=Y=Z=1} \cdot \frac{\partial^2 f}{\partial X^2}|_{X=Y=Z=1} - \frac{\partial f}{\partial X}|_{X=Y=Z=1} \cdot \frac{\partial^2 g}{\partial X^2}|_{X=Y=Z=1}}{2[\frac{\partial g}{\partial X}|_{X=Y=Z=1}]^2}$$

$$= \frac{\lambda_1}{\mu_1 - \lambda_1} \quad (20)$$

Similarly, the marginal mean queue length for the queue $Q_{int.L}$ denoted by $L_{Q_{int.L}}$ is given by the statistical formula

$$L_{Q_{int.L}} = \frac{\partial F(X,Y,Z)}{\partial Y}|_{X=Y=Z=1}$$

$$= \frac{\frac{\partial g}{\partial Y}|_{X=Y=Z=1} \cdot \frac{\partial^2 f}{\partial Y^2}|_{X=Y=Z=1} - \frac{\partial f}{\partial Y}|_{X=Y=Z=1} \cdot \frac{\partial^2 g}{\partial Y^2}|_{X=Y=Z=1}}{2[\frac{\partial g}{\partial Y}|_{X=Y=Z=1}]^2}$$

$$= \frac{\lambda}{\mu\left(1 - \frac{\lambda_1}{\mu_1}\right) + \mu\left(1 - \frac{\lambda_2}{\mu_2}\right) - \lambda} \quad (21)$$

The marginal mean queue length for the queue Q_{H_2} denoted by $L_{Q_{H_2}}$ is given by the statistical formula

$$L_{Q_{H_2}} = \frac{\partial F(X,Y,Z)}{\partial Z}|_{X=Y=Z=1}$$

$$= \frac{\frac{\partial g}{\partial Z}|_{X=Y=Z=1} \cdot \frac{\partial^2 f}{\partial Z^2}|_{X=Y=Z=1} - \frac{\partial f}{\partial Z}|_{X=Y=Z=1} \cdot \frac{\partial^2 g}{\partial Z^2}|_{X=Y=Z=1}}{2[\frac{\partial g}{\partial Z}|_{X=Y=Z=1}]^2}$$

$$= \frac{\lambda_2}{\mu_2 - \lambda_2} \quad (22)$$

Now the mean queue length of the queuing system,(including service, if any)denoted by L, is given by the formula

$$L = \sum_{n_1=0}^{\infty} \sum_{n=0}^{\infty} \sum_{n_2=0}^{\infty} (n_1 + n + n_2) P_{n_{1,n,n_2}}$$

$$= \frac{\partial F}{\partial X}|_{X=Y=Z=1} + \frac{\partial F}{\partial Y}|_{X=Y=Z=1} + \frac{\partial F}{\partial Z}|_{X=Y=Z=1}$$

$$= L_{Q_{H_1}} + L_{Q_{int.L}} + L_{Q_{H_2}}$$

$$= \frac{\lambda_1}{\mu_1 - \lambda_1} + \frac{\lambda}{\mu\left(1-\frac{\lambda_1}{\mu_1}\right)+\mu\left(1-\frac{\lambda_2}{\mu_2}\right)-\lambda} + \frac{\lambda_2}{\mu_2 - \lambda_2} \quad (23)$$

$$L = \frac{\rho_1}{(1-\rho_1)} + \frac{\rho}{(1-\rho_1)+(1-\rho_2)-\rho} + \frac{\rho_2}{(1-\rho_2)}$$

Where, $\frac{\lambda_1}{\mu_1} = \rho_1, \frac{\lambda}{\mu} = \rho, \frac{\lambda_2}{\mu_2} = \rho_2$

But it has been observed that in many real world situations the parametric distribution may only be characterized subjectively i.e. arrival and service pattern of the passengers are more suitably described in linguistic variables such as slow, moderate, fast arrival, slow service etc. Fuzzy arithmetic plays an important role. Fuzzy queue models are potentially more useful and realistic than commonly stochastic models. We first discuss the triangular fuzzy system & α-cut concepts to explain the model.

4.3 FUZZY SET:

Fuzzy logic extends Boolean logic to handle the expression of vague concepts. To express impression quantitatively a set membership function maps elements to real values between 0 & 1. The value indicates the degree to which an element belongs to a set. The degree is not describing probabilities that the item is in the set, but instead describes to what extent the item is in the set.

In the universe of discourse X, a fuzzy subset $\tilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}$ (X) Which maps each element x into X to a real number in the interval [0,1]. $\mu_{\tilde{A}}$ (X) denotes the grade or degree of membership and it is usually denoted by

$\mu_{\tilde{A}}$ (X) : X→[0,1].

If a fuzzy set A is defined on X, for any α ∈ [0,1], the α-cuts $^{\alpha}A$ is represented by the following crisp set:

Strong α-cuts: $^{\alpha+}A = \{x \epsilon X ; \mu_{\tilde{A}}(X) > \alpha; \alpha \epsilon [0,1]\}$

Weak α-cuts: $^{\alpha}A = \{x \epsilon X ; \mu_{\tilde{A}}(X) \geq \alpha; \alpha \epsilon [0,1]\}$

Hence, the fuzzy set A can be treated as crisp set $^{\alpha}A$ in which all the members have their membership values greater than or at least equal to α. It is one of the most important concepts in fuzzy set theory.

4.4 TRIANGULAR FUZZY NUMBER: A fuzzy number is simply an ordinary number whose precise value is somewhat uncertain. Taking [a_1, a_2] supporting interval and the point [a_m,1] as the peak, we define a triangular fuzzy number $\widetilde{A}$ with membership function $\mu_{\tilde{A}}(X)$ defined on R by confidence interval

$$\tilde{A} = \begin{cases} \frac{x-a_1}{a_m-a_1}, a_1 \leq x \leq a_m \\ \frac{a_2-x}{a_2-a_m}, a_m \leq x \leq a_2 \\ 0, \quad otherwise \end{cases}$$

In general practice, the points $a_m \epsilon$ (a_1, a_2) is located at the mid of the supporting interval i.e

$$a_m = \frac{a_1+a_2}{2}$$

Putting these values, we get

$$\tilde{A} = \begin{cases} \frac{2(x-a_1)}{a_2-a_1}, a_1 \leq x \leq \frac{a_1+a_2}{2} \\ \frac{2(a_2-x)}{a_2-a_1}, \frac{a_1+a_2}{2} \leq x \leq a_2 \\ 0, \quad otherwise \end{cases}$$

i.e three values a_1, a_m, and a_2 construct a triangular number denoted by $\tilde{A}$ = (a_1,a_m,a_2).

4.5 FUZZY ARITHMETIC OPERATION:

Let $\tilde{A}$= (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) be two triangular fuzzy numbers, then the arithmetic operation on $\tilde{A}$ and $\tilde{B}$are given as follows:

- Addition : $\tilde{A} + \tilde{B}$ = [a_1+b_1, a_2+b_2, a_3+b_3]
- Subtraction : $\tilde{A} - \tilde{B}$ = [a_1-b_3, a_2-b_2, a_3-b_1]
- Multiplication: $\tilde{A} * \tilde{B}$= [$a_1b_2+a_2b_1-a_2b_2$, a_2b_2, $a_2b_3+a_3b_2-a_2b_2$]
- Division : $\frac{\tilde{A}}{\tilde{B}}$ = [$2a_1/b_1+b_3$, a_2/b_2, $2a_3/b_1+b_3$]
- Square Root : [$\sqrt{a_1, a_2, a_3}$] =[$\sqrt{a_1}$, $\sqrt{a_2}$, $\sqrt{a_3}$]

Provided $\tilde{A}\ and\ \tilde{B}$ are all non-zero positive numbers.

4.6 DEFUZZIFICATION OF TFN:

If $\widetilde{A} = (a_1, a_2, a_3)$ is a TFN then its associated crisp number is given by Yager's formula i.e.

$$A = \frac{a_1+2a_2+a_3}{4}$$

Now we suppose that $\lambda_1, \lambda, \lambda_2, \mu_1, \mu, \mu_2, \rho_1, \rho, \& \rho_2$ are approximately known and can be represented by fuzzy numbers $\tilde{\lambda}_1, \tilde{\lambda}, \tilde{\lambda}_2, \tilde{\mu}_1, \tilde{\mu}, \tilde{\mu}_2 \tilde{\rho}_1, \tilde{\rho}, \tilde{\rho}_2$ can be represented respectively by using α- cut

$\tilde{\lambda}_1(\alpha) = [\tilde{\lambda}_{1,L}(\alpha), \tilde{\lambda}_{1,U}(\alpha)]$, $\tilde{\mu}_1(\alpha) = [\tilde{\mu}_{1,L}(\alpha), \tilde{\mu}_{1,U}(\alpha)]$, $\tilde{\rho}_1(\alpha) = [\tilde{\rho}_{1,L}(\alpha), \tilde{\rho}_{1,U}(\alpha)]$

$\tilde{\lambda}(\alpha) = [\tilde{\lambda}_L(\alpha), \tilde{\lambda}_U(\alpha)]$, $\tilde{\mu}(\alpha) = [\tilde{\mu}_L(\alpha), \tilde{\mu}_U(\alpha)]$, $\tilde{\rho}(\alpha) = [\tilde{\rho}_L(\alpha), \tilde{\rho}_U(\alpha)]$

$\tilde{\lambda}_2(\alpha) = [\tilde{\lambda}_{2,L}(\alpha), \tilde{\lambda}_{2,U}(\alpha)]$, $\tilde{\mu}_2(\alpha) = [\tilde{\mu}_{2,L}(\alpha), \tilde{\mu}_{2,U}(\alpha)]$, $\tilde{\rho}_2(\alpha) = [\tilde{\rho}_{2,L}(\alpha), \tilde{\rho}_{2,U}(\alpha)]$

Considering queue length in system as

$$\tilde{L}(\alpha) = [\tilde{L}_L(\alpha), \tilde{L}_U(\alpha)]$$

$$\tilde{L} = \left[\left\{\frac{\tilde{\rho}_{1,L}}{(1-\tilde{\rho}_{1,L})} + \frac{\tilde{\rho}_L}{(1-\tilde{\rho}_{1,L}) + (1-\tilde{\rho}_{2,L}) - \tilde{\rho}_L} + \frac{\tilde{\rho}_{2,L}}{(1-\tilde{\rho}_{2,L})}\right\}, \left\{\frac{\tilde{\rho}_{1,U}}{(1-\tilde{\rho}_{1,U})} + \frac{\tilde{\rho}_U}{(1-\tilde{\rho}_{1,U}) + (1-\tilde{\rho}_{2,U}) - \tilde{\rho}_U} + \frac{\tilde{\rho}_{2,U}}{(1-\tilde{\rho}_{2,U})}\right\}\right]$$

By using Little's formulae, we may find other effective measures in fuzzy parameters.

5. NUMERICAL ILLUSTRATION:

Consider fuzzy variables in triangular form as $\tilde{\lambda}_1 = (5,6,7)$, $\tilde{\mu}_{1=}(10,11,12)$, $\tilde{\lambda} = (9,10,11)$, $\tilde{\mu} = (22,23,24)$, $\tilde{\lambda}_2 = (8,9,10)$, $\tilde{\mu}_{2=}(15,16,17)$

Using α-cut, we get

$\tilde{\lambda}_1(\alpha) = [5+\alpha, 7-\alpha]$, $\tilde{\mu}_1(\alpha) = [10+\alpha, 12-\alpha]$, $\tilde{\rho}_1(\alpha) = \left[\frac{5+\alpha}{12-\alpha}, \frac{7-\alpha}{10+\alpha}\right]$

$\tilde{\lambda}$ =(9+α, 11-α), $\tilde{\mu} = (22+\alpha, 24-\alpha)$, $\tilde{\rho}_1 = \left[\frac{9+\alpha}{24-\alpha}, \frac{11-\alpha}{22+\alpha}\right]$

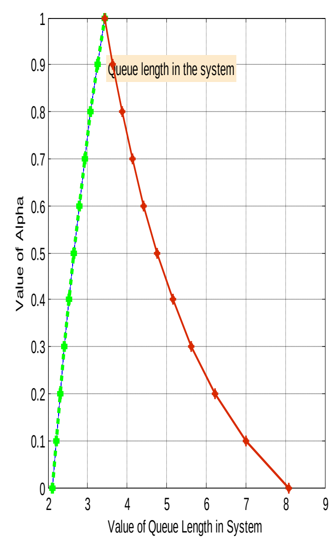
$$\tilde{\lambda}_2 = (8,9,10), \tilde{\mu}_2 = (15,16,17), \tilde{\rho}_2 = \left[\frac{8+\alpha}{17-\alpha}, \frac{10-\alpha}{15+\alpha}\right]$$

For different values of α belongs to (0, 1) the various values of queue length for lower & upper bounds are represented in Table-1 as:

Table-1: Mean Queue Length of the system

S.NO.	Value of α	L(lower)	L(upper)
1	0	2.11	8.08
2	0.1	2.21	6.99
3	0.2	2.31	6.22
4	0.3	2.41	5.62
5	0.4	2.53	5.15
6	0.5	2.65	4.76
7	0.6	2.78	4.42
8	0.7	2.92	4.13
9	0.8	3.08	3.87
10	0.9	3.25	3.64
11	1	3.44	3.44

Graphical Representation



CONCLUSION:

From the above Table-1 and fig.2 we observe that when arrival rates $\tilde{\lambda}_1, \tilde{\lambda}, \tilde{\lambda}_2 and\ service\ rates\ \tilde{\mu}_1, \tilde{\mu}, \tilde{\mu}_2$ are in fuzzy numbers, the performance measures in the system are also expressed by fuzzy system that shows a complete conservation of fuzziness of I/O information. From the graph, it is clear that as the values of α increase, the lower bound of queue length of the system increases and the upper bound correspondingly decreases. We shows that uncertainty decreases and when α=1, the value of queue length becomes equal to crisp value. At this stage, system uncertainty is very less, almost tends to zero which shows the validity of the system.

References

1. (1957), Jackson J.R.," Network of waiting lines", Operation Research,Vol.-5(4),pp(518-521).
2. (1970), P.L.Maggu., "Phase type service queues with two servers in Bi- series, Journal of Operation Research Society of Japan, vol-13(1),pp(1-6).
3. (1978), Zadeh's, L.A.,"Fuzzy sets a basis for a theory of probability", Fuzzy sets and system 1, pp (3-28).
4. (1984), P.L.Maggu & B.K. Sharma," On the service of an intermediate queue by two heterogeneous servers under pre- emptive priority service rule",JISSOR, vol-5,no.1-4.
pp(23-31).
5. (1986), T. P. Singh, "On some network of queuing and scheduling system ", Ph. D. Thesis, Garwhal University, Shrinagar, Garwhal.
6. (1989), Li, R.J. and Lee, E. S., "Analysis of fuzzy queues" Computers and Mathematics with Applications 17,pp(1143-1147).
7. (1992), Negi, D.S. and Lee, E.S.," Analysis and simulation of fuzzy queues", Fuzzy sets & systems 46, pp (321-330).
8. (2005), T. P. Singh, Vinod Kumar & Rajinder, "On transient behaviour of a queue network with parallel bi-serial queues", JMASS, Vol.-2.
9. (2011), T. P. Singh, Kusum & Deepak Gupta, "Trapezoidal fuzzy queue network with blocking", Aryabhatta Journal of Mathematics & Informatics, vol-3, No.-2,pp(185-192).
10. (2012), T. P. Singh, Kusum & Deepak Gupta, "Fuzzy queue model to interdependent communication system with bulk arrival", Aryabhatta Journal of Mathematics & Informatics, vol-4, No.-1,pp(41-48).
11. (2013), Arti & T.P. Singh, "Analytical study of fuzzy queue model to interdependent communication system having voice packetized transmission", Aryabhatta Journal of Mathematics & Informatics, vol-5, No.-1, pp (175-182).
12. (2015) Meenu, T.P. Singh & Deepak Gupta," Threshold effect on a fuzzy queue model with batch arrival ", Aryabhatta Journal of Mathematics & Informatics, vol-7, No.-1, pp (109-118).