SOME NEW INTEGRALS INVOLVING GENERALIZED H-FUNCTION OF TWO VARIABLES

By

Smt. Sreshta Dhiman
Govt. Science College, Rewa (M. P.)

ABSTRACT

The aim of this research paper is to establish some new integrals involving the generalized H–function of two variables.

1. INTRODUCTION:

The generalized H–function of two variables is given by Shrivastava, H. S. P. [2] and defined as follows:

$$H^{m_1, n_1:m_2, n_2;m_3,}_{p_1, q_1:p_2, q_2;p_3, q_3}\left[\begin{matrix}x\\y\end{matrix}\,\middle|\,\begin{matrix}(a_j; \alpha_j, A_j)_{1, p1}:(c_j, \gamma_j)_{1, p2}:(e_j, E_j)_{1, p3}\\(b_j; \beta_j, B_j)_{1, q1}:(d_j, \delta_j)_{1, q2}:(f_j, F_j)_{1, q3}\end{matrix}\right]$$

$$= \frac{-1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi, \eta)\,\theta_2(\xi)\theta_3(\eta)x^{\xi}\,y^{\eta}\,d\xi\,d\eta, \qquad (1)$$

where

$$\phi_1(\xi, \eta) = \frac{\prod_{j=1}^{n_1}\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)\prod_{j=1}^{m_1}\Gamma(b_j - \beta_j\xi - B_j\eta)}{\prod_{j=n_1+1}^{p_1}\Gamma(a_j - \alpha_j\xi - A_j\eta)\prod_{j=1}^{q_1}\Gamma(1 - b_j + \beta_j\xi + B_j\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(d_j - \delta_j\xi)\prod_{j=1}^{n_2}\Gamma(1 - c_j + \gamma_j\xi)}{\prod_{j=m_2+1}^{q_2}\Gamma(1 - d_j + \delta_j\xi)\prod_{j=n_2+1}^{p_2}\Gamma(c_j - \gamma_j\xi)},$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_3}\Gamma(f_j - F_j\eta)\prod_{j=1}^{n_3}\Gamma(1 - e_j + E_j\eta)}{\prod_{j=m_3+1}^{q_3}\Gamma(1 - f_j + F_j\eta)\prod_{j=n_3+1}^{p_3}\Gamma(e_j - E_j\eta)},$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, (i = 1, 2, 3; j = 2, 3). Also, all the A's, α's, B's, β's, γ's, δ's, E's, and F's are assumed to the positive quantities for standardization purpose.

The contour L_1 is in the ξ-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \delta_j\xi)$ (j = 1, ..., m_2) lie to the right, and the poles of $\Gamma(1 - c_j + \gamma_j\xi)$ (j = 1, ..., n_2), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The countor L_2 is in the η-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - F_j\eta)$ (j = 1, ..., m_3) lie to the right, and the poles of $\Gamma(1 - e_j + E_j\eta)$ (j = 1, ..., n_3), $\Gamma(1 - a_j + \alpha_j\xi + A_j\eta)$ (j = 1, ..., n_1) to the left of the contour.

The generalized H-function of two variables given by (1) is convergent if

$$U = \sum_{j=1}^{n_1} \alpha_j + \sum_{j=1}^{m_1} \beta_j + \sum_{j=1}^{n_2} \gamma_j + \sum_{j=1}^{m_2} \delta_j$$

$$- \sum_{j=n_1+1}^{p_1} \alpha_j - \sum_{j=m_1+1}^{q_1} \beta_j - \sum_{j=n_2+1}^{p_2} \gamma_j - \sum_{j=m_2+1}^{q_2} \delta_j; \quad (2)$$

$$V = \sum_{j=1}^{n_1} A_j + \sum_{j=1}^{m_1} B_j + \sum_{j=1}^{n_3} E_j + \sum_{j=1}^{m_3} F_j$$

$$- \sum_{j=n_1+1}^{p_1} A_j - \sum_{j=m_1+1}^{q_1} B_j - \sum_{j=n_3+1}^{p_3} E_j - \sum_{j=m_3+1}^{q_3} F_j, \quad (3)$$

where | arg x | < ½ Uπ, | arg y | < ½ Vπ.

In our investigation we shall need the following result:

From Dixon [1]:

$$\int_{-\infty}^{\infty} \frac{\sin(cx)}{\Gamma(\alpha+x)\Gamma(\beta-x)} dx = \frac{[2\cos(\frac{c}{2})]^{\alpha+\beta-2} \sin[\frac{1}{2}c(\beta-\alpha)]}{\Gamma(\alpha+\beta+1)}, \quad (4)$$

provided that Re (α + β) < 1, 0 < c <π.

2. INTEGRALS:

In this section, we shall establish following integrals:

$$\int_{-\infty}^{\infty} \sin(cx)\, H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+2;p_3,q_3}$$

$$\left[{}^{\zeta}_{\eta}\Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1-\alpha-x,u),(1-\beta+x,u):(f_j,F_j)_{1,q_3}}\right] dx$$

$$= [2\cos(\tfrac{c}{2})]^{\alpha+\beta-2} \sin[\tfrac{1}{2}c(\beta-\alpha)] H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}$$

$$\left[{}^{(2\cos\frac{c}{2})^{2u}\zeta}_{\eta} \Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(-\alpha-\beta,2u):(f_j,F_j)_{1,q_3}} \right], \qquad (5)$$

provided that Re (α + β) < 1, 0 < c <π, |argζ| < ½Uπ, |argη| < ½Vπ, where U and V are given in (2) and (3) respectively.

$$\int_{-\infty}^{\infty} \sin(cx)\, H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2+1,q_2+1;p_3,q_3}$$
$$\left[{}^{\zeta}_{\eta} \Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2},(\beta-x,-u):(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1-\alpha-x,u):(f_j,F_j)_{1,q_3}} \right] dx$$
$$= [2\cos(\tfrac{c}{2})]^{\alpha+\beta-2} \sin[\tfrac{1}{2}c(\beta-\alpha)] H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3}$$
$$\left[{}^{(2\cos\frac{c}{2})^{2u}\zeta}_{\eta} \Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(-\alpha-\beta,2u):(f_j,F_j)_{1,q_3}} \right], \qquad (6)$$

provided that Re (α + β) < 1, 0 < c <π, |argζ| < ½Uπ, |argη| < ½Vπ, where U and V are given in (1.2.43) and (1.2.44) respectively.

Proof of (5):

The result (5) can be established by replacing the generalized H–function of two variables on the left hand side as contour integral (1), we get

$$\int_{-\infty}^{\infty} \sin(cx)\Big[\frac{1}{(2\pi\omega)^2} \int_{L_1}\int_{L_2} \phi_1(\rho,\sigma)\theta_2(\rho)\theta_3(\sigma)$$
$$\frac{1}{\Gamma(\alpha+x+u\rho)\Gamma(\beta-x+u\rho)} \zeta^{\rho}\eta^{\sigma}\, d\rho\, d\sigma\Big] dx$$

interchanging the order of integral involved in the process, evaluating the inner integral with the help of (4) and applying (1) the definition of generalized H–function of two variables, the value of the integral is obtained. On using the same procedure as above, the integral (6) is established.

3. PARTICULAR CASE:

On choosing c = π/2 in (5), we get following result, which are useful in space science and used in explanation of quantum gravitational:

$$\int_{-\infty}^{\infty} \sin(\frac{\pi}{2}x)\, H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+2;p_3,q_3} \left[{}^{\zeta}_{\eta}\Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(1-\alpha-x,u),(1-\beta+x,u):(f_j,F_j)_{1,q_3}}\right]dx$$

$$= [\sqrt{2}]^{\alpha+\beta-2}\sin[\frac{\pi}{4}(\beta-\alpha)]H^{m_1,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2+1;p_3,q_3} \left[{}^{(\sqrt{2})^{2u}\zeta}_{\eta}\Big|^{(a_j,\alpha_j;A_j)_{1,p_1}:(c_j,\gamma_j)_{1,p_2}:(e_j,E_j)_{1,p_3}}_{(b_j,\beta_j;B_j)_{1,q_1}:(d_j,\delta_j)_{1,q_2},(-\alpha-\beta,2u):(f_j,F_j)_{1,q_3}}\right], \quad (7)$$

provided that Re $(\alpha + \beta) < 1$, $|\arg\zeta| < ½ U\pi$, $|\arg\eta| < ½ V\pi$, where U and V are given in (2) and (3) respectively.

REFERENCES

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2. Srivastava, H. S. P.: H-function of two variables I, Indore Univ., Res. J Sci. 5(1-2), p.87-93, (1978).