# Structure of Superdeformed Bands in $^{193}$Pb Nucleus and Examination of ΔI = 1 Staggering

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**Abstract**

The nine superdeformed(SD) bands of $^{193}$Pb nucleus have been systematically analyzed in framework of Bohr-Mottelson two parameters formula. The first eight SD bands form four pairs of signature partners assigned to the configuration 3/2 [761], 3/2 [642], 9/2 [624] and 5/2 [752] neutron orbitals respectively. For each SD band, the level spins are suggested and the model parameters are determined by using a simulated search program to minimize the root mean square (rms) deviation of the calculated Bohr-Mottelson transition energies $E_\gamma$ from the observed values considering the spin value of the lowest level and the model parameters as free parameters. The calculated $E_\gamma$ agree very well with the experimental date for all nine SD bands. This indicates that the Bohr-Mottelson two parameters approach is powerful in describing the signature partner bands. The variation of the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia as a function of rotational frequency ħω have been examined. The $J^{(2)}$ for bands 1 and 2 is quite constant as a function of ħω due to pairing blocking effect. Bands 3 to 8 show an increase $J^{(2)}$ with increasing ħω. Band 9 has irregularity of $J^{(2)}$ at low ħω and does not show an increase with increasing ħω. The existence of the ΔI =1 staggering is investigated in four signature partner pairs by means of proposed staggering function depending on the dipole gamma ray transition linking the two signature partners and the quadruple transitions in each band. Zigzag pattern emerges in the two pairs (SD1, SD2) and (SD3, SD4) but very small staggering appear in the third and fourth pairs (SD5, SD6) and (SD7,SD8).

## 1. Introduction

The discovery of the high spin superdeformed (SD) bands is one of most exciting events in nuclear structure. These SD bands have much larger deformation and much higher spin than the normal - deformed (ND) bands, and usually are identified in the gamma ray spectra by the presence of a sequences of transitions almost equally spaced in energy. Experimentally more than 350 SD bands have been well established in A ~ 190,150,130,80 mass regions [1]. The first observation of superdeformation in the A ~ 190 mass region was in $^{191}$Hg [2], since then more than 85 SD bands were observed in 25 different nuclei. The region of SD nuclei with mass A ~ 190 region has been studied extensively by us in previous works over the last few years [3-11].

Level spins in most SD bands were not determined experimentally up to date because discrete linking transitions between the high-spin SD states and the lower spin ND states were not observed.

Several theoretical methods have been proposed for the absolute spin assignments in SD bands [3-19].

Most of SD bands in A ~ 190 exhibit a smooth increasing trend in the dynamical moment of inertia $J^{(2)}$ with increasing the rotational frequency ħω. This rise in $J^{(2)}$ results from the alignment of the angular momentum of paired nucleons in high-j low-Ω intruder orbital and from the gradual disappearance of pairing correlations with the collective rotation [20, 21]. The Pauli blocking of the high – N intruder orbitals which reduce the increase in $J^{(2)}$ is used to explain the behavior of SD bands in $^{193}$pb [22,23] where the $J^{(2)}$ curve is flatter (bands with almost constant values of $J^{(2)}$) than that of most of the bands in the A ~ 190 mass region. The blocking of the N = 7 quasineutron or N = 6 quasiproton alignment is thought to be responsible for the flatting.

It has been demonstrated that rotational sequences in some SD nuclei with nuclear spins differing by two may split into two branches [24,25]. This phenomenon is called ΔI=2 staggering or ΔI=4 bifurcation in the gamma ray transition energies. Thus the SD band can be viewed as two sequences of states with spin values I+4n and I+4n+2(n=1,2,3,-------) respectively in which spins differing by 4ħ from level to level and a small energy displacement occur between the two states. Several theoretical attempts have been make for the possible explanation of this ΔI=2 staggering phenomenon [26-33].

There is another kind of staggering phenomenon, the ΔI=1 staggering or signature splitting happens in SD odd-A nuclei. It was seen that most of SD bands observed in odd-A nuclei in mass region A ~ 190 are signature partners [34, 35].

Most of these signature partners show large amplitude ΔI=1, staggering and the bandhead moments of inertia of each pair are almost identical. To explore this ΔI=1 staggering in signature partners pairs, we used in previous works [36-38] staggering parameter represent the differences between the average transitions I+2 →I→I-2 energies in one band and the transition I+1→I-1 energy in the signature partner.

In this paper, to investigate the ΔI=1 staggering phenomenon in signature partner SD bands of $^{193}$Pb, we introduced a signature parameter depends on the dipole gamma ray transition linking the two signature partners and the quadrupole transitions in each band. The paper is organized as follows: In section 2, we describe the formulism of the Bohr - Mottelson approach for fitting the SD rotational bands. Section 3 concerns the origin of ΔI=1 staggering and the identical bands. In section 4, the calculated results and discussion are presented. Finally a conclusion and some remarks are given in section 5.

## 2. Formulism of Bohr – Mottelson Approach

To characterize the collective rotational energies, Bohr and Mottelson [39] proposed the two parameter formula

$$E(I) = A\,[I(I+1)] + B\,[I(I+1)]^2 \qquad (1)$$

The first term represent the rigid rotational behavior which depends on the moment of inertia, while the second term includes the centrifugal stretching, Coriolis antiparing and gradual alignment.

Equation (1) leads to a form for the gamma ray transition energies

$$E_{\gamma_2}(I) = E(I) - E(I-2) = \left(I - \frac{1}{2}\right)[4I + 8B(I^2 - I + 1] \quad (2)$$

$$E_{\gamma_1}(I) = E(I) - E(I-1) = 2\,I\,(A + 2BI^2\,) \quad (3)$$

and so the ratios $E_{\gamma 2}$ to ( I - ½ ) and $E_{\gamma 1}$ to (2I) (E-Gamma Over Spin or EGOS) are given by

$$EGOS2 = \frac{E_{\gamma 2}(I)}{I - \frac{1}{2}} = 4A + 8B(I^2 - I + 1) \quad (4)$$

$$EGOS1 = \frac{E_{\gamma 1}(I)}{2I} = A + 2BI^2 \quad (5)$$

The rotational frequency ħω, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia are given by (we take ħ = 1 ) $\hbar\omega(I) = \frac{dE(I)}{d\hat{I}}$ , with $\hat{I} = \sqrt{I(I+1)}$

$$\hbar\omega(I) = 2A\hat{I} + 4B\hat{I}^3 \quad (6)$$

$$J^{(1)} = \frac{\hat{I}}{\hbar\omega} = \left(\frac{1}{\hat{I}}\frac{dE}{d\hat{I}}\right)^{-1} = J_0\left(1 + \frac{2B}{A}\hat{I}^2\right)^{-1} \quad (7)$$

$$J^{(2)} = \left(\frac{d^2E}{d\,\hat{I}^2}\right)^{-1} = J_0\left(1 + \frac{6B}{A}\hat{I}^2\right)^{-1} \quad (8)$$

$$\text{with} \quad J_0 = \frac{1}{2\,A} \quad (9)$$

The two moments of inertia $J^{(1)}$ and $J^{(2)}$ are obviously dependent ,one has

$$J^{(2)} = \left(\frac{d}{d\hat{I}}\frac{dE}{d\hat{I}}\right)^{-1} = \left(\frac{d}{d\hat{I}}\,\omega\right)^{-1} = \frac{d\hat{I}}{d\omega}$$

$$= \frac{d}{d\omega}\left(\omega\, J^{(1)}\right) = J^{(1)} + \omega\,\frac{d\,J^{(1)}}{d\omega} \quad (10)$$

For a rigid nuclear rotor, one should obtain $J^{(2)} = J^{(1)} = J^{rigid}$.

Empirically $J^{(1)}$ for normal deformed nuclei are systematically larger than that the rigid moment of inertia values, while for SD nuclei $J^{(1)}$ approaches very closely to the rigid rotor limit.

Experimentally, the gamma – ray transition energies are commonly translated into values of rotational frequency ħω and dynamical moment of inertia $J^{(2)}$

$$\hbar\omega = \frac{1}{4}[E_\gamma(I+2 \to I) + E_\gamma(I \to I-2)] \quad (11)$$

$$J^{(2)} = \frac{4}{\Delta E_\gamma} \quad (12)$$

where $\Delta E_\gamma$ is the difference between two consecutive gamma rays in the cascade

$$\Delta E_\gamma = E_\gamma(I+2 \to I) - E_\gamma(I \to I-2)] \quad (13)$$

After predicted the level spins, the kinematic moment of $J^{(1)}$ may be extracted by the experimental gamma transition energies themselves

$$J^{(2)} = \frac{2I-1}{E_\gamma(I \to I-2)} \quad (14)$$

**3. Analysis of ΔI=1 Staggering in transition Energies and Identical Bands**

In a previous works [36-38], we investigated the ΔI=1 staggering in transition energies of signature partners in odd-A SD nuclei by extracting the difference between the average transitions I + 2 → I → I - 2 energies in one band and the transitions I + 1 → I – 1 energies in its signature partner,

$$\Delta^2 E_\gamma\,(I) = \frac{1}{2}\left[E_{\gamma_2}(I+2\to I) + E_{\gamma_2}(I\to I-2)\right] - E_{\gamma_2}(I+1\to I-1) \qquad (15)$$

Also staggering parameter was introduced [39], which represent the finite difference approximation to the fourth order derivative of transition energies with respect to spin,

$$\Delta^4 E_\gamma\,(I) = \frac{1}{16}[E_{\gamma_1}(I+2\to I+1) - 4E_{\gamma_1}(I+1\to I) + 6E_{\gamma_1}(I\to I-1) - 4E_{\gamma_1}(I-1\to I-2) + E_{\gamma_1}(I-2\to I-3)] \qquad (16)$$

In the present paper, we will investigate the Δ I =1 staggering by considering a staggering parameter Y(I) depending on the dipole transition energies linking the two signature partners and the quadrupole transition energies within the bands, Y(I) takes the form

$$Y(I) = \left(\frac{2I-1}{I}\right)\frac{E(I)-E(I-1)}{E(I)-E(I-2)} - 1 = \left(\frac{2I-1}{I}\right)\frac{E_{\gamma 1}}{E_{\gamma 2}} - 1 \qquad (17)$$

An unexpected discovery observed in different nuclei, that several pairs of SD bands have almost identical transition energies [41, 42]. Stephens et al [43] introduced the incremental alignment which depends only on gamma-ray energies to compare the SD bands in neighboring nuclei and also as a tool to predict gamma-ray energies [17, 44]. To illustrate the similarities between two identical bands (IB's), the difference between the gamma – ray energies observed in the IB's in the pairs of nuclei Δ $E_\gamma$ is plotted versus the transition energy, on average the deviation is less than one KeV. Since the gamma ray energy for a given spin is dependent on the kinematic moment of inertia $J^{(1)}$, a 1-part in -1000 difference in the gamma ray energies corresponds to a difference in $J^{(1)}$ of the same order.

**4. Calculations and Discussion**

For the odd-A $^{193}$Pb nucleus, nine SD bands have been established, none of these bands are connected to the low lying normal deformed states and consequently, their excitation energies, spins and positions are not known. With equation (2), we have been calculated the gamma ray transition energies $E_\gamma(I)$ by using fitting search program to the experimental gamma ray energies considering the spin value of the lowest level and the model parameters as free parameters. For each band, the best fitted model parameters and the bandhead spin value $I_0$ are listed in Table (1).

In fitting procedure, we employed the common definition of $\chi^2$

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{E_\gamma^{exp}(i) - E_\gamma^{cal}(i)}{\delta E_\gamma^{exp}}\right]^2$$

where, N is the number of data points and $\delta E_\gamma^{exp}$ is the experimental error in the gamma ray energies.

The comparison with experimental data[1]are listed in Table (2). Avery good agreement between the calculated and the corresponding experimental values is obtained which gives good support to the model. Using equations (6-8), the rotational frequency ħω, the kinematic $J^{(1)}$ and dynamic $J^{(2)}$ moments of inertia of the above nine bands are also obtained. The obtained results and comparison with experimental data are illustrated in Figure (1). The $J^{(2)}$ for bands 1 and 2 is quite constant as a function of ħω due to paring blocking effect. Bands 3 to 8 show an increase $J^{(2)}$ with increasing ħω. This rise in $J^{(2)}$ is consistent with the assumption that pair correlations are present. Band 9 has irregularity of $J^{(2)}$ at low ħω and does not show an increase with increasing ħω.

In previous papers [36-38], we investigated the ΔI=1 staggering effect in gamma ray energies for signature partners in odd-A SD nuclei by calculating the differences between the average transitions. $E\gamma_2$ ( I + 2 → I ) and $E\gamma_2$ ( I → I -2 ) in one band and the transition $E\gamma_2$ ( I +1→ I -1 ) in the other signature partner band. In this paper a staggering parameter Y(I) equation (17) is introduced which depends on the dipole transitions $E\gamma_1$ ( I → I - 1 ) linking the pairs of signature partners and the quadrapole transitions $E\gamma_2$ ( I → I - 2 ) within each band. The results for the four signature partner pairs of $^{193}$Pb are listed in Table (3) and plotted versus spin in Figure(2). Zigzag pattern emerges in the two pairs (SD1,SD2) and (SD3,SD4) but very small staggering appear in the third and fourth pairs (SD5,SD6) and (SD7,SD8). Another result in the present work is the observation of identical bands between the signature partners $^{193}$ Pb (SD3,SD4) and $^{191}$Hg (SD2,SD3). The difference in gamma ray energies $\Delta E_\gamma$ between transitions in $^{193}$Pb (SD3) &$^{191}$Hg (SD2) and $^{193}$Pb (SD4) &$^{191}$Hg (SD3) are plotted versus transition energy in Figure(3).

**5. Conclusion**

The nine SD bands of the nucleus $^{193}$Pb have been described in framework of Bohr- Mottelson two term formula. Eight SD bands are signature partners. For all nine bands, the bandhead spins are obtained by spin fitting method. We calculated the transition energies, the rotational frequencies, the kinematic and dynamic moments of inertia. Excellent agreement between theory and experiment are obtained. The transition energies of band 8 lie at the midpoint of the energies of band 7 along most of the observed range, suggesting that these bands are a pair of strongly coupled signature partners. The dynamic moment of inertia of bands 5 to 8 shows a fairly smooth increase with increasing rotational frequency.

The ΔI=1 staggering has been investigated by considering a staggering parameter depending on the dipole gamma ray transition linking the two signature partners and the quadrupole transitions in each band. Bands 1 and 2 display a large signature splitting while bands 3 and 4 show a small signature splitting at high rotational frequency. The identical bands in $^{193}$Pb (SD3) &$^{191}$Hg (SD2) and $^{193}$Pb (SD4) & $^{191}$Hg(SD3) are investigated, the difference between the calculated relative transition energies in the two pairs are within 3 KeV.

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Table (1) The fitting optimized best model parameters A, B, the theoretical bandhead spin assignment $I_0$and the bandhead moment of inertia $J_0$ for the nine SD bands in $^{193}$Pb nucleus The lowest transition energies $E_\gamma$ ($I_0$+2→$I_0$) are also indicated.

| SD band | $E_\gamma$ ($I_0$+2→$I_0$) (KeV) | A (KeV) | B ($10^{-4}$KeV) | $I_0$ (ħ) | $J_0$ ($ħ^2 MeV^{-1}$) |
|---|---|---|---|---|---|
| $^{193}$Pb (SD1) | 277.0 | 5.32791 | -1.153253 | 11.5 | 93.8454 |
| (SD2) | 190.2 | 5.81514 | -2.381224 | 6.5 | 85.9824 |
| (SD3) | 251.5 | 5.25921 | -1.376125 | 10.5 | 95.0713 |
| (SD4) | 272.0 | 5.297401 | -1.942720 | 11.5 | 94.3859 |
| (SD5) | 213.2 | 5.34524 | -1.989457 | 8.5 | 93.5411 |
| (SD6) | 234.6 | 5.34149 | -1.931692 | 9.5 | 93.6068 |
| (SD7) | 260.6 | 5.07078 | -1.332902 | 11.5 | 98.6041 |
| (SD8) | 281.8 | 5.10068 | -1.544456 | 12.5 | 98.0261 |
| (SD9) | 212.9 | 5.34643 | -1.523548 | 8.5 | 93.5203 |

Table (2): The calculated gamma – ray transition energies $E_\gamma$ (I+2→I) of the superdeformed bands in $^{193}$Pb. Experimental date are taken from Ref [1]. The fitting model parameters and the bandhead spins are listed in Table (1)

| (SD1) | | | (SD2) | | | (SD3) | | |
|---|---|---|---|---|---|---|---|---|
| $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) |
| 227.0 | 275.0153 | 13.5 | 190.2 | 185.0977 | 8.5 | 251.5 | 250.5298 | 12.5 |
| 317.3 | 316.5504 | 15.5 | 232.6 | 230.6863 | 10.5 | 291.5 | 291.4333 | 14.5 |
| 357.3 | 357.7533 | 17.5 | 275.2 | 275.8177 | 12.5 | 332.4 | 332.0669 | 16.5 |
| 397.5 | 398.5798 | 19.5 | 317.9 | 320.4005 | 14.5 | 372.1 | 372.2278 | 18.5 |
| 437.8 | 438.9856 | 21.5 | 360.9 | 364.3433 | 16.5 | 411.9 | 411.9130 | 20.5 |
| 477.4 | 478.9265 | 23.5 | 403.5 | 407.5545 | 18.5 | 450.6 | 451.0699 | 22.5 |
| 517.3 | 518.3580 | 25.5 | 445.9 | 449.9427 | 20.5 | 488.9 | 489.6455 | 24.5 |
| 556.1 | 557.2360 | 27.5 | 488.3 | 491.4166 | 22.5 | 526.6 | 527.5869 | 26.5 |
| 594.8 | 595.5161 | 29.5 | 528.0 | 531.8847 | 24.5 | 563.4 | 564.8414 | 28.5 |
| 633.4 | 633.1541 | 31.5 | 569.8 | 571.2554 | 26.5 | 599.9 | 601.3561 | 30.5 |
| 671.8 | 670.1057 | 33.5 | 610.5 | 609.4375 | 28.5 | 637.0 | 637.0781 | 32.5 |
| 708.2 | 706.3266 | 35.5 | 650.0 | 646.3394 | 30.5 | 762.2 | 671.9547 | 34.5 |
| | | | 689.8 | 681.8698 | 32.5 | 709.2 | 705.9329 | 36.5 |

Table (2) continued

| (SD4) | | | (SD5) | | | (SD6) | | |
|---|---|---|---|---|---|---|---|---|
| $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) |
| 273.0 | 272.0351 | 13.5 | 213.2 | 212.2060 | 10.5 | 234.6 | 232.9559 | 11.5 |
| 313.4 | 312.5812 | 15.5 | 254.6 | 253.8069 | 12.5 | 275.5 | 274.3472 | 13.5 |
| 353.1 | 352.5677 | 17.5 | 296.2 | 294.9494 | 14.5 | 316.2 | 313.2564 | 15.5 |
| 391.9 | 391.9202 | 19.5 | 336.1 | 335.5572 | 16.5 | 355.9 | 355.6092 | 17.5 |
| 430.0 | 430.5639 | 21.5 | 375.1 | 375.5537 | 18.5 | 394.4 | 395.3316 | 19.5 |
| 467.1 | 468.4244 | 23.5 | 413.5 | 414.8628 | 20.5 | 432.8 | 434.3493 | 21.5 |
| 503.9 | 505.4269 | 25.5 | 451.2 | 453.4078 | 22.5 | 470.6 | 472.5881 | 23.5 |
| 539.5 | 541.4969 | 27.5 | 488.6 | 491.1125 | 24.5 | 507.4 | 509.9738 | 25.5 |
| 575.1 | 576.5599 | 29.5 | 526.5 | 527.9005 | 26.5 | 543.5 | 546.4324 | 27.5 |
| 610.0 | 610.5411 | 31.5 | 562.2 | 563.6954 | 28.5 | 579.7 | 581.8895 | 29.5 |
| 644.5 | 643.3660 | 33.5 | 596.2 | 598.4207 | 30.5 | 614.6 | 616.2712 | 31.5 |
| 676.4 | 674.9600 | 35.5 | 631.3 | 632.0001 | 32.5 | 649.5 | 649.5030 | 33.5 |
| 707.2 | 705.2485 | 37.5 | 666.8 | 664.3571 | 34.5 | 684.0 | 681.5109 | 35.5 |
| | | | 700.5 | 695.4155 | 36.5 | 717.9 | 712.2220 | 37.5 |

Table (2) continued

| (SD7) | | | (SD8) | | | (SD9) | | |
|---|---|---|---|---|---|---|---|---|
| $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) | $E_\gamma^{exp}$ (KeV) | $E_\gamma^{cal}$ (KeV) | Assigned I(ħ) |
| 260.6 | 261.3274 | 13.5 | 281.8 | 282.2347 | 14.5 | 212.9 | 212.6292 | 10.5 |
| 299.8 | 300.6359 | 15.5 | 321.5 | 321.3678 | 16.5 | 255.8 | 254.5115 | 12.5 |
| 340.1 | 339.5606 | 17.5 | 360.9 | 360.0264 | 18.5 | 297.3 | 296.0427 | 14.5 |
| 378.9 | 378.0501 | 19.5 | 398.5 | 398.1513 | 20.5 | 336.6 | 337.1645 | 16.5 |
| 417.2 | 416.0535 | 21.5 | 435.9 | 435.6831 | 22.5 | 375.8 | 377.8182 | 18.5 |
| 454.0 | 453.5194 | 23.5 | 472.3 | 472.5625 | 24.5 | 415.9 | 417.9454 | 20.5 |
| 490.1 | 490.3967 | 25.5 | 508.1 | 508.7303 | 26.5 | 455.5 | 457.4875 | 22.5 |
| 526.1 | 526.6342 | 27.5 | 543.2 | 544.1270 | 28.5 | 495.6 | 496.3861 | 24.5 |
| 561.3 | 562.1807 | 29.5 | 578.0 | 578.6935 | 30.5 | 535.4 | 534.5826 | 26.5 |
| 596.4 | 596.9851 | 31.5 | 612.0 | 612.3703 | 32.5 | 575.3 | 572.0186 | 28.5 |
| 631.1 | 630.9961 | 33.5 | 646.8 | 645.0983 | 34.5 | | | |
| 664.2 | 664.1626 | 35.5 | | | | | | |

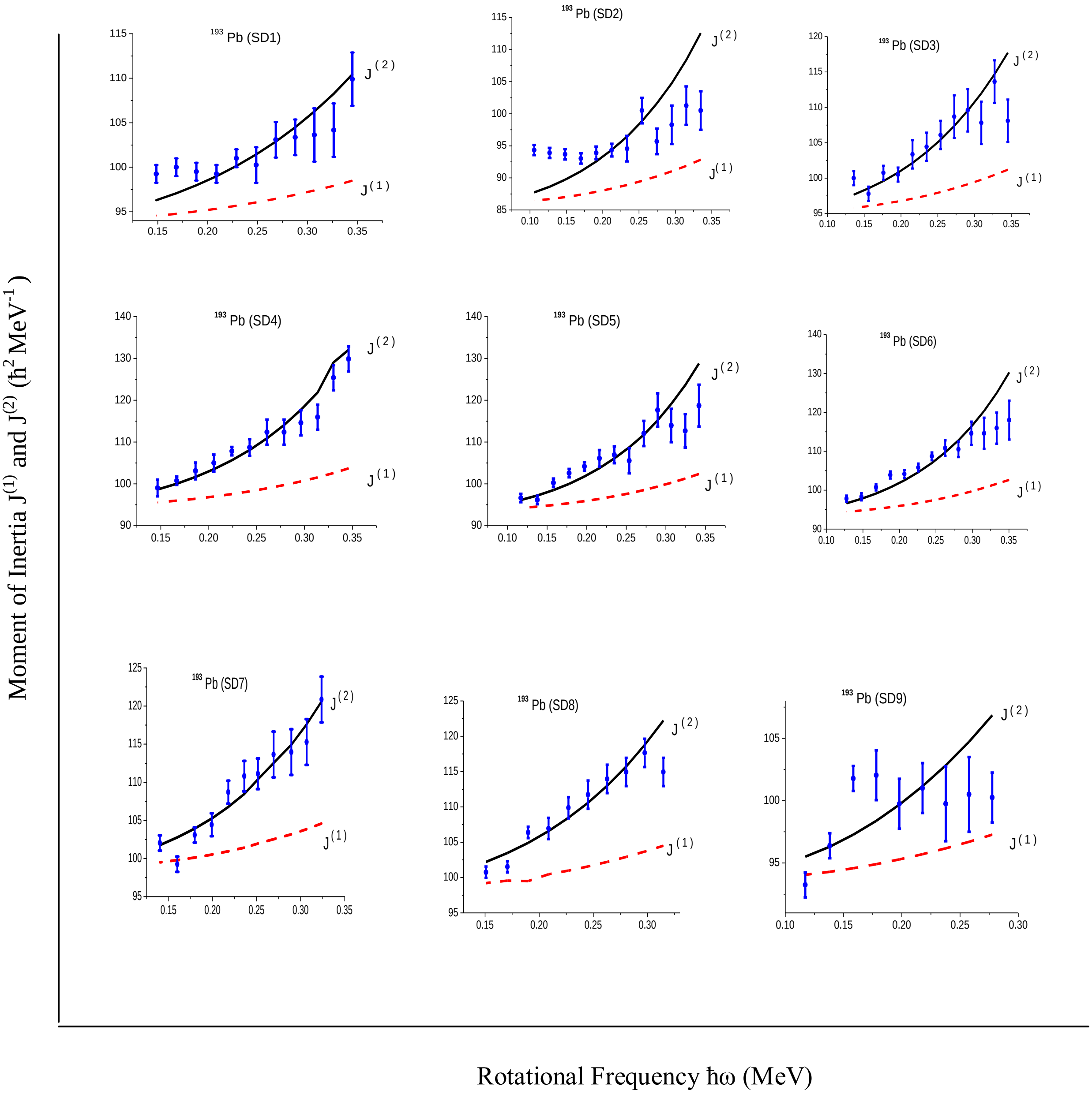


Figure (1). The calculated results of the dynamic moment of inertia $J^{(2)}$ (solid curve) as a function of the rotational frequency ħω of all nine SD bands of $^{193}$pb and comparison with experiment [1] (closed circles with error bars). The kinematic moment of inertia $J^{(1)}$ (dashed curve) is also given assuming spin values for SD states discussed in the text.

Table (3): the calculated Δ I =1 staggering parameter Y (I) for signature partner four pairs of $^{193}$Pb nucleus

| $^{193}$Pb (SD1,SD2) | | | $^{193}$Pb (SD3,SD4) | | |
|---|---|---|---|---|---|
| I (ħ) | E(I) (KeV) | Y(I) (KeV) | I (ħ) | E(I) (KeV) | Y(I) (KeV) |
| 10.5 | 608.7061 | | 10.5 | 633.0431 | |
| 11.5 | 763.5039 | | 11.5 | 757.4869 | |
| 12.5 | 974.5238 | 0.4689 | 12.5 | 883.5729 | - 0.0337 |
| 13.5 | 1038.5192 | - 0.5518 | 13.5 | 1029.5220 | 0.0332 |
| 14.5 | 1294.9243 | 0.5453 | 14.5 | 1175.0562 | - 0.0358 |
| 15.5 | 1355.0696 | - 0.6322 | 15.5 | 1342.1032 | 0.0342 |
| 16.5 | 1659.2676 | 0.6192 | 16.5 | 1507.1231 | - 0.0362 |
| 17.5 | 1712.8229 | - 0.7091 | 17.5 | 1694.6709 | 0.0334 |
| 18.5 | 2066.8221 | 0.6902 | 18.5 | 1879.3509 | - 0.0345 |
| 19.5 | 2111.4027 | - 0.7820 | 19.5 | 2086.5911 | 0.0304 |
| 20.5 | 2516.7648 | 0.7578 | 20.5 | 2291.2639 | - 0.0304 |
| 21.5 | 2550.3883 | - 0.8503 | 21.5 | 2517.1550 | 0.0248 |
| 22.5 | 3008.1814 | 0.8217 | 22.5 | 2742.3338 | - 0.0237 |
| 23.5 | 3029.3148 | - 0.9136 | 23.5 | 2985.5794 | 0.0164 |
| 24.5 | 3540.0661 | 0.8813 | 24.5 | 3231.9793 | - 0.0140 |
| 25.5 | 3547.6728 | - 0.9712 | 25.5 | 3491.0063 | 0.0048 |
| 26.5 | 4111.3215 | 0.9361 | 26.5 | 3759.5662 | - 0.0011 |
| 27.5 | 4104.9088 | - 1.0225 | 27.5 | 4032.5032 | - 0.0102 |

| | | | | | |
|---|---|---|---|---|---|
| 28.5 | 4720.7590 | 0.9855 | 28.5 | 4324.4076 | 0.0154 |
| 29.5 | 4700.4249 | - 1.0671 | 29.5 | 4609.0631 | - 0.0293 |
| 30.5 | 5367.0984 | 1.0291 | 30.5 | 4925.7637 | 0.0360 |
| 31.5 | 5333.5790 | - 1.1042 | 31.5 | 5219.6042 | - 0.0614 |
| 32.5 | 6048.9682 | 1.0660 | 32.5 | 5562.8418 | 0.0609 |
| 33.5 | 6003.6847 | - 1.1331 | 33.5 | 5562.9702 | - 0.0809 |
| | | | 34.5 | 6234.7965 | 0.0906 |
| | | | 35.5 | 6537.9302 | - 0.01144 |
| | | | 36.5 | 6940.7294 | 0.1255 |
| | | | 37.5 | 7243.1787 | - 0.0537 |

Table (3): continued

| $^{193}$Pb (SD5,SD6) | | | $^{193}$Pb (SD7,SD8) | | |
|---|---|---|---|---|---|
| I (ħ) | E(I) (KeV) | Y(I) (KeV) | I (ħ) | E(I) (KeV) | Y(I) (KeV) |
| 8.5 | 430.3308 | | 11.5 | 726.1703 | |
| 9.5 | 530.8915 | | 12.5 | 856.3416 | |
| 10.5 | 642.5368 | 2.1286 | 13.5 | 987.4977 | - 0.0334 |
| 11.5 | 763.8474 | - 3.7923 | 14.5 | 1138.5763 | 0.0336 |
| 12.5 | 896.3437 | 2.3088 | 15.5 | 1288.1336 | - 0.0371 |

| | | | | | |
|---|---|---|---|---|---|
| 13.5 | 1038.1946 | - 4.2022 | 16.5 | 1459.9441 | 0.0368 |
| 14.5 | 1191.2931 | 2.3362 | 17.5 | 1627.6942 | - 0.0401 |
| 15.5 | 1353.4510 | - 4.4484 | 18.5 | 1819.9705 | 0.0392 |
| 16.5 | 1526.8503 | 2.1824 | 19.5 | 2005.7443 | - 0.0424 |
| 17.5 | 1709.0602 | - 4.5032 | 20.5 | 2218.1218 | 0.0407 |
| 18.5 | 1902.4040 | 1.8183 | 21.5 | 2421.7978 | - 0.0436 |
| 19.5 | 2104.3918 | - 4.3364 | 22.5 | 2653.8049 | 0.0413 |
| 20.5 | 2317.2668 | 1.2125 | 23.5 | 2875.3122 | - 0.0439 |
| 21.5 | 2538.7411 | - 4.1244 | 24.5 | 3126.3674 | 0.0408 |
| 22.5 | 2770.6746 | 0.3331 | 25.5 | 3365.7139 | - 0.0430 |
| 23.5 | 3011.3292 | - 3.2153 | 26.5 | 3635.0977 | 0.0390 |
| 24.5 | 3261.7871 | - 0.8541 | 27.5 | 3892.3481 | - 0.0408 |
| 25.5 | 3521.3030 | - 2.1944 | 28.5 | 4179.2247 | 0.0359 |
| 26.5 | 3789.6876 | - 2.3849 | 29.5 | 4454.5288 | - 0.0371 |
| 27.5 | 4067.7354 | - 0.8191 | 30.5 | 4757.9182 | 0.0313 |
| 28.5 | 4353.3830 | - 4.2982 | 31.5 | 5051.5199 | - 0.0310 |
| 29.5 | 4649.6249 | 0.9489 | 32.5 | 5370.2885 | 0.0250 |
| 30.5 | 4951.8030 | - 6.6351 | 33.5 | 5682.5600 | - 0.0251 |
| 31.5 | 5265.8961 | 3.1518 | 34.5 | 6015.3868 | 0.0170 |
| 32.5 | 5583.8038 | - 9.4406 | 35.5 | 6346.6727 | - 0.0163 |
| 33.5 | 5915.3991 | 5.8340 | | | |
| 34.5 | 6248.1609 | - 12.7000 | | | |
| 35.5 | 6596.9100 | 9.0437 | | | |
| 36.5 | 6943.5764 | -16.6000 | | | |
| 37.5 | 7309.1320 | 12.8000 | | | |

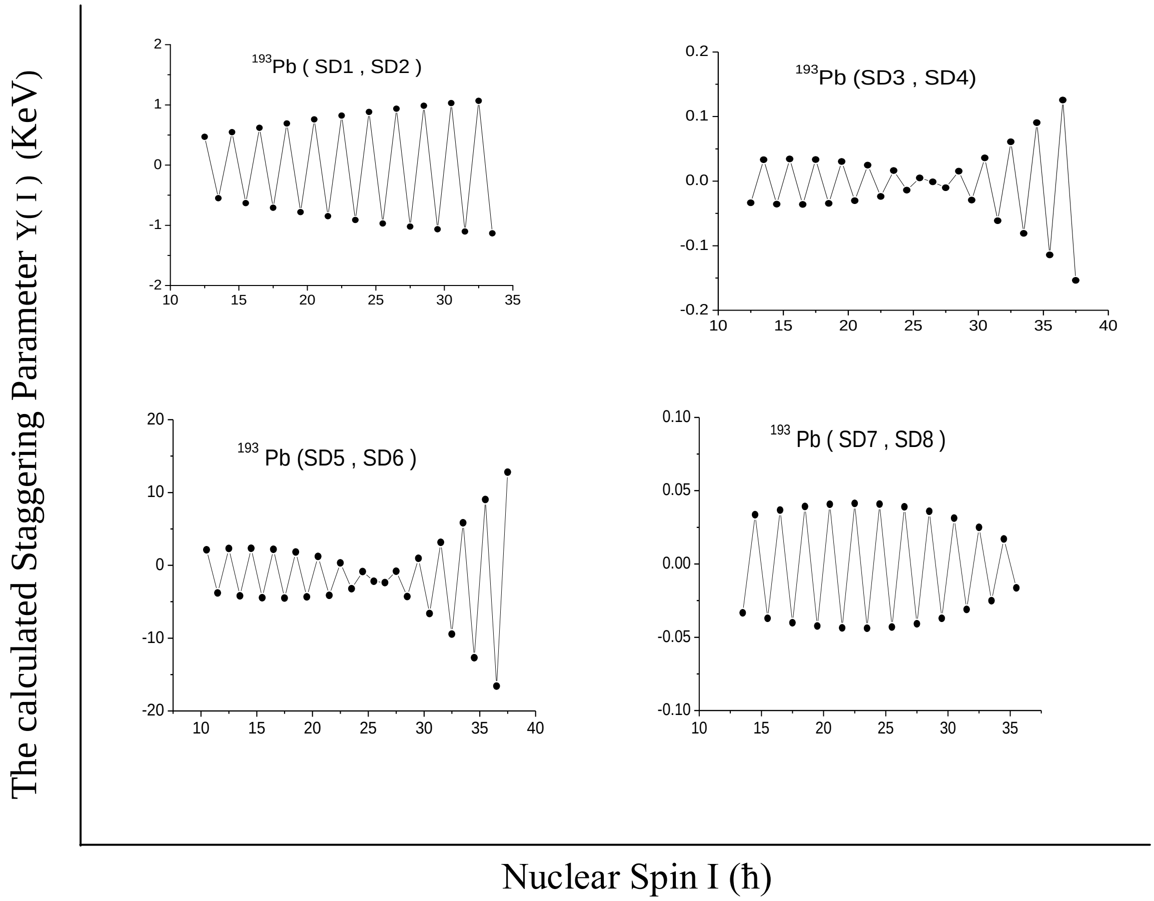


Figure (2). The calculated Staggering parameter Y (I) as a function of nuclear spin I for the signature partner pairs in $^{193}Pb$

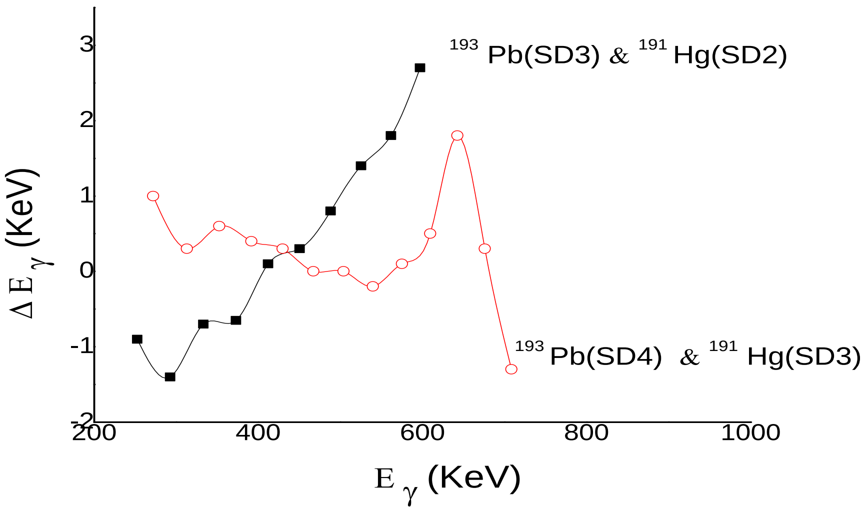


Figure (3). Difference in gamma ray transition energies $\Delta E_{\gamma}$ viruses Eγ between the SD bands $^{193}Pb$ (SD3) & $^{191}Hg$ (SD2) (closed circles) and $^{193}Pb$ (SD4) & $^{191}Hg$ (SD3) (open circles)