The Analysis of stability and existence of Critical permeability value in a 2D-Bio-Porous Convection (BPC) in the absence of gravity inclination

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Abstract

Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspensions of micro-organisms like Bacillus subtilis and algae. The term "bioconvection" refered to macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal sparsely packed porous fluid layer of finite depth subject to adverse temperature gradient. This problem is relevant to certain species of thermophilic micro-organisms that live in hot environment. Finally it is concluded that the permeability is a important parameter on the onset of bio-convection. Thus, by controlling these parameters it is possible to enhance or suppress bio-convection.

***Keywords* -** component; Bio-porous convection, gravity inclination microorganism,fluid saturated porous media critical Rayleigh number and critical wave number.

1. Introduction

Bioconvection is an exciting, complex, yet experimentally tractable, approach for studying the mutual interdependence of physics and biology, where the overall phenomenology greatly exceeds its primitive components. Bioconvection is the name given to pattern formation in suspensions of microorganisms, such as bacteria and algae, due to up-swimming of the micro-organisms (Pedley & Kessler 1992). Bioconvection has been observed in several bacterial species, including aerobic, anaerobic and magnetotactic organisms, as well as in algal and protozoan cultures (Kessler and Hill (1997)). All have in common the sudden appearance of a pattern when viewed from above. In all cases, the microorganisms are denser than water and on average they swim upwards (although the reasons for up-swimming may be different for different species). The algae (e.g. Chlamydomonas) are approximately 5%

denser than water, whereas the bacteria are nearly 10% denser than water. Microorganisms respond to certain stimuli by tending to swim in particular directions. These responses are called taxes. Gyrotaxis is swimming directed by the balance between the torque due to gravity acting on a bottom-heavy cell and the torque due to viscous forces arising from local shear flows. This chapter is concerned with the 2D stability analysis of bioconvection in a suspension of motile gyrotactic microoganisms in a fluid saturated porous medium subject to gravity inclination.

2. Mathematical formulation and analysis

In this section, the mathematical formulation of the problem together with the stability analysis are discussed .The major assumptions utilized in this chapter were: **(i)** the porous matrix does not absorb microorganisms **(ii)** the suspension is dilute **(iii)** the medium is an isotropic fluid saturated porous medium of uniform porosity **(iv)** no macroscopic motion of the fluid accurs and **(v)** all the microorganisms are swimming vertically upwards. The governing equations for a two dimensional unsteady flow in a porous medium are obtained by volume averaging the equations of Pedley et al., [1988] model by utilizing the volume averaging procedure described in Whitaker [1999]. This procedure resulted in the replacement of the Laplacian viscous terms with the Darcian terms that described viscous resistance in a porous medium (Nield and Bejan [1999]). The governing equations are :

The momentum equations in the component form,

$$c_a\rho_o\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}-\frac{\mu v}{K}-n\theta\Delta\rho g \quad \textbf{(1)}$$

$$c_a\rho_o\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}-\frac{\mu u}{K} \quad (2)$$

the continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad ..(3)$$

the conservation of cells

$$\frac{\partial n}{\partial t}=-div(n\vec{v}+nW_c\underset{\rightarrow}{\vec{p}}-D\Delta_n) \quad ...(4)$$

Nomenclature: C_a : The acceleration coefficient.

D : The diffusivity of the microorganisms (this assumes that all random motions of the microorganisms can be approximated by a diffusive process).

$\vec{g}$: The gravitational acceleration. n : The number density of motile microorganisms.

n_o : The number density of microorganisms in the basic state.

p : The excess pressure (above hydrostatic).

$\hat{p}$: The unit vector indicating the direction of swimming of microorganisms.

t : The time; u, v and w are the x-,y- and z-velocity components respectively.

$\vec{v}$:The velocity vector:(u,v,w) .

$W_c\,\hat{p}$: The vector of average swimming velocity of microorganisms relative to the fluid (W_c is assumed to be constant). x y and z : The Cartesian coordinates (y is vertical coordinate).

θ : The average volume of microorganisms.

$\Delta\rho$: The density difference $\rho_{cell} - \rho_0$. μ : The dynamic viscosity. assumed to be approximately the same as that of water. ρ_o : The density of water.K: permeability of porous medium

3. Stability analysis:

In order to obtain the stability criterion the perturbations of cell concentration, fluid velocity components and the unit vector $\hat{p}$ that indicates the direction of bacterial swimming are introduced as follows;

$$[n,u,v,\ \hat{p}](t,x,y)=[n_0 + \varepsilon n', \varepsilon u', \varepsilon v', n' + \hat{p}'](t,x,y)\} \qquad ..(5)$$

where $\hat{k}$ is the vector in the vertically upward y-direction, a prime denotes a perturbation quantity and ε is the small perturbation amplitude. After eliminating the pressure, the perturbations (3.5) are substituted into the governing equations which result in the following set of linearized equations:

$$c_a \rho_o \frac{\partial}{\partial t}\left(\frac{\partial u^i}{\partial y} - \frac{\partial v^i}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u^i}{\partial y} - \frac{\partial v^i}{\partial x}\right) + \theta \Delta \rho g \frac{\partial n^i}{\partial x} \qquad ..(6)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \qquad ..(7)$$

$$\frac{\partial n'}{\partial t} = -div[n_o(\vec{v}^1 + W_c \hat{p}') + n' w_c \hat{k} \;\; - D\nabla n') \qquad ..(8)$$

where $\vec{v}^1$ is the vector composed of perturbations of the corresponding velocity components, (u', v'). Pedly et al. [1988] analyzed the impact of the gyrotaxes on the direction of swimming of microorganisms. Then obtained an equation that relates the perturbation of the swimming direction $\hat{p}^1$ with the perturbation of velocity components. For a two dimensional problem, the results of pedly et al. [1988] can be presented as

$$\hat{p}^1 = (-B\xi, 0) \qquad ..(9)$$

where B is the time scale for the reorientation of microorganisms by the gravitational torque against viscous resistance. In Pedley and Kessler [1987] this parameter is called the "gyrotactic orientation parameter", which can be expressed as

$$B = \frac{\alpha_\perp \mu}{2h\rho_o g} \qquad ..(10)$$

where $\alpha_\perp$ is the dimensionless constant relating the viscous torque to the relative angular velocity of the cell and h is the displacement of centre of mass of the cell from the centre of buoyancy. The components ξ of vector $\underline{\hat{p}}^1$ in eqn.(9) is connected to perturbations of velocity component by the following equation:

$$\xi = (1+\alpha_o)\frac{\partial u^1}{\partial y} - (1-\alpha_o)\frac{\partial v^1}{\partial x} \qquad ..(11)$$

α_o is the cell eccentricity which is given by the equation.

$$\alpha_o = \frac{a^2 - b^2}{a^2 + b^2} \qquad ..(12)$$

where a and b are the semi-major and semi-minor axes of the spheroidal cell respectively.

By substituting eqn(9) into eqn.(8) and accounting eqn. (11), the following equation for n' is obtained.

$$\frac{\partial n^1}{\partial t} + W_c \frac{\partial n^1}{\partial y} + W_c B n_o \left(\frac{\partial \xi}{\partial x}\right) = D\nabla^2 n^1 \quad ..(13)$$

In order to study the stability of the system, the perturbation quantities are introduced in terms of individual Fourier modes: $[n', v']$ (t,x,y)=[N,U] exp[$\sigma t + i(lx + my)$] ..(14)

Again by substituting eqn.(14) into the continuity equation (7) for perturbation quantities, the following equation for u' is obtained. u' (t,x ,y)=-$\frac{\mathrm{Um}}{\mathrm{l}}$ exp[$\sigma t + i(lx + my)$] ..(15)

In order to determine the amplitude equations for U and N eqns.(14) and (15) are substituted into (6) and (13) which result in the following equations

$$k^2(Dk^2 + imw_c + \sigma)(\mu + c_a\rho_o K\sigma) = Bn_o gKw_c\Delta\ \rho\theta l^2[m^2(1+\alpha_o) + l^2(1-\alpha_o) \quad ..(16)$$

In a similar way, we get after simplification, $gKl^2 N\Delta\rho\theta + k^2 U(\mu + c_a\rho_o K\sigma)$ ..(17)

where $k^2 = l^2 + m^2$. Eliminating the amplitudes from eqs.(16) and (17) the following dispersion equation for the growth rate parameter σ is obtained:

$$k^2(Dk^2 + imw_c + \sigma)(\mu + c_a\rho_o K\sigma) = Bn_o gKl^2 w_c\Delta\rho\theta l[m^2(1+\alpha_o) + l^2(1-\alpha_o)] \quad (18)$$

The two roots for the growth rate parameter σ computed from eqn. (18) are

$$\sigma = -\frac{c_a\rho_o K(Dk^2 + imw_c) + \mu}{2c_a\rho_o K} \pm \frac{\{4Bc_a\rho_o n_o gK^2 l^2 w_c\Delta\rho\theta l[m^2(1+\alpha_o) + l^2(1-\alpha_o)] + k^2[\mu - c_a\rho_o K(Dk^2 + imw_c)]^2\}^{1/2}}{c_a\rho_o Kk}$$
(19)

In order to prove that critical permeability exists it is absolutely necessary to prove that **(i)** the system is stable, when the permeability of the porous medium is close to zero and **(ii)** the system becomes unstable when the permeability is sufficiently large. Accordingly instability appeared only when the real part of σ is positive. It is observed that since the root with the positive sign in front of the second term in eqn.(19) of first root has a greater real part, analysis is concentrated on this root itself. The Taylor series expansion of this root about the point K=0 is found. By neglecting the quadratic and higher order terms in this expansion, the following solution (valid only for the small values of permeability) is obtained;

$$\sigma = -(Dk^2 + imw_c) + \frac{\{Bn_o g\ l^2 w_c \Delta\rho\theta[m^2(1+\alpha_o) + l^2(1-\alpha_o)]}{k^2\mu} K + o(K^2) \qquad \ldots.(20)$$

From the above equation. it is observed that for K=0, σ has a negative real part $-k^2 D$. This suggests that for sufficiently small values of permeability the system is stable. Therefore in order to prove that the system becomes unstable with the increase of K, it is absolutely necessary to show that the real part of σ will be positive. Suppose m=o (which corresponds to the case of no vertical disturbances). In this case the root of the eqn. (20) with the greater real part is found to be

$$\sigma = -\frac{c_a\rho_o KDk^2 + \mu}{2c_a\rho_o K} \pm \frac{\{4Bc_a\rho_o n_o gK^2 l^2 w_c \Delta\rho\theta[m^2(1+\alpha_o) + l^2(1-\alpha_o)] + k^2[\mu - c_a\rho_o KDk^2]^2\}^{1/2}}{2c_a\rho_o Kk} \qquad (21)$$

The upper limit of the critical permeability is computed by solving the eq. (21) for σ =0

$$K_{cri}^{upper} = \frac{D\mu}{Bn_o g w_c \Delta\rho\theta(1-\alpha_o)} \qquad ..(22)$$

Conclusion:

The above equation clearly proves and predicts that the existence of critical permeability. The critical permeability is the minimum value of K for all allowable wave numbers.

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