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Mathematical approach on Peristaltic transport of Biofluids in reference to time dependent waves

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Abstract:

Mathematical approach of the interaction between Peristaltic transport and time dependent waves through the flexible circular cylindrical tube(artery) of biofluids has been studied in the present paper. The flow of Newtonian viscous incompressible fluid on which an axisymmetric travelling sinusoidal wave is imposed is considered. The initial flow is considered by an arbitrary periodic pressure gradient. The frequency of the wave is affected due to the interaction of peristaltic transport on the influence of time dependent wave so that the wall of the tube(artery) is under consideration of higher harmonic oscillating flow on the application of periodic pressure gradient. The effect of one flow(peristaltic) gets perturbed with the effect of another flow (time dependent) in the presence of impedance factor. The volumetric flow rate has been numerically computed. The geometry of the time dependent flow is assumed as sinusoidal wave so that we can estimate the flow parameters such as velocity, flux, and resistance of flow, frictional force and the impedance factor for time dependent and peristaltic flows using numerical method. Numerical results show considerable variation of velocity profiles with boundary waves in axial and radial direction.

Keywords: *Peristaltic, time dependent, Blood.*

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Introduction:

The study consists of peristaltic transport of two layered viscous incompressible fluid. Peristalsis comes from the Greek word peristaltikos, which means clasping and compressing. In physiology, it is described as a progressive wave of contraction seen in tubes provided with longitudinal and transverse muscular fibers. Normally intestine, ureter, movement of spermatozoa and the number of biomedical instruments such as some heart lung machines have been identified for the propelling action of the fluid and the fluid mixtures. By the unique pumping process, the fluid is transported by regular coordinated waves of muscular contractions along the wall of the vessel. The wave is sinusoidal due to the longitudinal and transverse moments produced by muscular fibers. Human aorta, carotid artery, capillaries and other vessels of the sizes constituted by connective tissues followed by smooth muscular fibers are contractile in nature. The amplitude of the traveling wave on the elastic wall is so large that at the narrowest point the wall pressed by each other.

Over past few years, L. M. Srivastava, et.al [4] proposed to study the flow of an incompressible viscous fluid through a cylindrical tube for the case of an axisymmetric sinusoidal wave travelling along the flexible wall when the initial motion is induced by an arbitrary periodic pressure gradient. Gianni Pedrizzetti et.al.[5] presented the specific application under study in the third section and the details of the application of the perturbative approach are given in the fourth section; the numerical method is briefly outlined in the fifth section. Alexander Yakhot et.al.[6] focused on fully developed periodic pipe flows with sinusoidally varying pressure gradients or flow rates!. Numerical investigations of pulsatile flows in a pipe have received considerable attention for decades. Somkid Amornsamankul. et.al.[7] aims to develop a finite element model to study the behavior of blood flow through stenotic arteries taking into account of blood transport through the porous arterial wall, and to investigate the effect of non-Newtonian viscosity of blood on pulsatile flow through an artery with an asymmetric or a symmetric stenosis over a cardiac cycle. H. David Hromádka et.al. [8] presents an approximate solution of the pulsatile flow of a Newtonian fluid in the laminar flow regime in a rigid tube of constant diameter and the model is represented by two ordinary differential equations also the first equation describes the time evolution of the total flow rate. Finite differences method with an implicit scheme is used in order to solve the dimensionless governing equations. Velocity and temperature profiles are presented for different Womersley and Hartmann numbers. we have considered the effect of time dependent flow on the peristaltic transport with a reference to compare the flow parameters.

Formulation.

Consider the flow of a Newtonian viscous incompressible fluid moving in an axisymmetric form in a flexible and extensible circular cylindrical tube. The fluid is set into motion by imposing an axisymmetric sinusoidal travelling wave of small amplitude a. Then we have appropriate equations describing the flow as,

Equations of motion,

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\vartheta\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \quad (1)$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+u\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\vartheta\left(\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial z^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right) \quad (2)$$

The equation of continuity,

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \tag{3}$$

In left hand side of equations (1) and (2) denote inertia forces with u and v as velocity, t– time, r- the radial coordinate and z as the axial coordinate. First terms in right hand side are pressure forces with p – pressure due to sinusoidal wave, ρ – as the density. Second terms in right hand side denote viscous Forces with ϑ as the kinetic viscosity.

The flow is assumed to be axisymmetric then the stream functions are given by,

$$u = -\frac{1}{r}\frac{\partial\psi}{\partial r}, \qquad v = \frac{1}{r}\frac{\partial\psi}{\partial z} \tag{4}$$

In view of equation (4), equations (1), (2) and (3) assume the form, after elimination of pressure as,

$$\frac{\partial}{\partial t}\left(\frac{\partial^2\Psi}{\partial z^2} + \frac{\partial^2\Psi}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r}\frac{\partial\psi}{\partial z}\left[\frac{\partial^3\psi}{\partial r\,\partial z^2} + \frac{\partial^3\psi}{\partial r^3} - \frac{1}{r}\frac{\partial^2\psi}{\partial r^2} - \frac{2}{r}\left(\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial\psi}{\partial r}\right] -$$

$$-\frac{1}{r}\left(\frac{\partial\psi}{\partial r}\frac{\partial^3\psi}{\partial z^3} + \frac{\partial^3\psi}{\partial z\,\partial r^2} - \frac{1}{r}\frac{\partial^2\psi}{\partial z\,\partial r}\right) = \vartheta\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r}\right) \tag{5}$$

Then at the wall the fluid particles give the velocity components and the displacement in r- direction as,

$$u = 0,\ v = \frac{\partial\eta}{\partial t},\ \frac{\eta}{a} = \cos\frac{2\pi}{\lambda}(z - ct) \tag{6}$$

The new boundary conditions due to no slip term are

$$\frac{\partial\psi}{\partial r} = 0, \frac{\partial\psi}{\partial z} = \frac{2\pi acr}{\lambda}\sin\frac{2\pi}{\lambda}(z - ct) \tag{7}$$

For the successive axisymmetric(sinusoidal) waves we consider the pressure gradient towards z - direction as,

$$\frac{\partial p}{\partial z} = p_0 \cos\Omega t \tag{8}$$

At various amplitudes we can define pressure gradient as,

$$\frac{\partial p}{\partial z} = \left(\frac{\partial p}{\partial z}\right) + a\left(\frac{\partial p}{\partial z}\right) + a^2\left(\frac{\partial p}{\partial z}\right) + \cdots \ldots\ldots\ldots\ldots \text{at } z_0, z_1, z_2, \ldots\ldots \tag{9}$$

The terms on RHS of equation (9) will vary accordingly r, z and t vary in the range of peristaltic action.

Nondimensional quantities are,

$$r' = \frac{r}{R}, z' = \frac{z}{R}, u' = \frac{u}{C}, v' = \frac{v}{C}, \eta' = \frac{\eta}{R},\ \psi' = \frac{\psi}{R^2C}, t' = \frac{Ct}{R}, p' = \frac{p}{\rho C^2 R/\Lambda}, \overline{\Lambda} = \frac{2\pi R}{\Lambda}, \tag{10}$$

Introducing $\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} = \nabla^2$, then

$$\frac{\partial}{\partial t}(\nabla^2\psi)+\frac{1}{r}\frac{\partial\psi}{\partial z}\left[\nabla^2\left(\frac{\partial\psi}{\partial r}\right)-\frac{2}{r}\nabla^2\psi+\frac{1}{r^2}\frac{\partial\psi}{\partial r}\right]-\frac{1}{r}\frac{\partial\psi}{\partial r}\nabla^2\left(\frac{\partial\psi}{\partial z}\right)=\frac{1}{R_e}\nabla^2\nabla^2\psi \quad (11)$$

$$\eta=\varepsilon\cos\alpha(z-t) \quad (12)$$

$$\left.\begin{matrix}\frac{\partial\psi}{\partial r}=0\\ \frac{\partial\psi}{\partial z}=\alpha\varepsilon r\sin\alpha(z-t)\end{matrix}\right\}\text{at } r=1+\eta \quad (13)$$

$$\frac{\partial p}{\partial z}=\left(\frac{\partial p}{\partial z}\right)+\varepsilon\left(\frac{\partial p}{\partial z}\right)+\varepsilon^2\left(\frac{\partial p}{\partial z}\right)+\cdots\ldots\ldots\ldots\ldots.\text{at } z_0,z_1,z_2,\ldots\ldots \quad (14)$$

Where ε, α and R_e as,

$$\varepsilon=\frac{a}{R},\ \alpha=\frac{2\pi R}{\lambda}\ ,\ R_e=\frac{R\,c}{\vartheta} \quad (15)$$

The first term in equation (14) represents the imposed time dependent pressure gradient.

Analysis

The parameter $(\frac{\Omega\,R}{c})$ = A represents a decreasing frequency, which is equal to the wave number. And thus equation (15) becomes,

$$\left(\frac{\partial p}{\partial z}\right)_{\text{at } z_0}=P_0e^{iAt} \quad (16)$$

Where $A=(\frac{\Omega\,R}{c})$

In order to seek the solution of the problem under small amplitude ratio assumption. Expanding the stream function ψ as,

$$\psi=\psi_0+\varepsilon\psi_1+\varepsilon^2\psi_2+\cdots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

Substitution of equation (17) into equation (11) yields the following

$$\frac{r}{R_e}\nabla^2\nabla^2\psi_0=r\frac{\partial}{\partial t}\nabla^2\psi_0+\frac{\partial\psi_0}{\partial z}\left[\nabla^2\left(\frac{\partial\psi_0}{\partial r}\right)-\frac{2}{r}\nabla^2\psi_0+\frac{1}{r^2}\frac{\partial\psi_0}{\partial r}\right]-\frac{\partial\psi_0}{\partial r}\nabla^2\left(\frac{\partial\psi_0}{\partial z}\right) \quad (18)$$

$$\frac{r}{R_e}\nabla^2\nabla^2\psi_1=r\frac{\partial}{\partial t}\nabla^2\psi_1+\frac{\partial\psi_1}{\partial z}\left[\nabla^2\left(\frac{\partial\psi_0}{\partial r}\right)-\frac{2}{r}\nabla^2\psi_0+\frac{1}{r^2}\frac{\partial\psi_0}{\partial r}\right]-$$

$$-\frac{\partial\psi_1}{\partial r}\nabla^2\left(\frac{\partial\psi_0}{\partial z}\right)+\frac{\partial\psi_0}{\partial z}\left[\nabla^2\left(\frac{\partial\psi_1}{\partial r}\right)-\frac{2}{r}\nabla^2\psi_1+\frac{1}{r^2}\frac{\partial\psi_1}{\partial r}\right]-\frac{\partial\psi_0}{\partial r}\nabla^2\left(\frac{\partial\psi_1}{\partial z}\right) \quad (19)$$

The solution in the complex plane is then given as,

$$u_0(r,t)=i\,f_0(r)(\cos At+i\sin At)=i\ f_0(r)e^{iAt} \quad (20)$$

Where $f_0(r)$ is given by

$$f_0(r) = \frac{i\,P_0 e^{iAt}}{2\pi} - \frac{i\,P_0}{2\pi}\left[\frac{J_1\left[(1+i)\sqrt{AR_e/2}.r\right]}{J_1\left[(1+i)\sqrt{AR_e/2}\right]}\right] \quad (21)$$

Where J_1 is the modified Bessel function of the first kind.

From equation (19)

$$\frac{\partial^4\psi_{1,1}}{\partial r^4} - \frac{2}{r}\frac{\partial^3\psi_{1,1}}{\partial r^3} + \left(-2\alpha^2 + \frac{3}{r^2} + R_e.\alpha f_0(r)e^{iAt}\right)\frac{\partial^2\psi_{1,1}}{\partial r^2} + \left(\frac{2\alpha^2}{r} - \frac{3}{r^3} - \frac{R_e.\alpha\, f_0(r)e^{iAt}}{r}\right)\frac{\partial\psi_{1,1}}{\partial r} + \left(\alpha^4 - R_e.\alpha^3 f_0(r)e^{iAt} - R_e\ \alpha f_0''(r)e^{iAt} + \frac{R_e.\alpha f_0'(r)e^{iAt}}{r}\right)\psi_{1,1} = 0 \quad (22)$$

And the mixed derivative is,

$$R_e\left\{\left(\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\psi_{1,1}}{\partial r^2} - \frac{1}{r}\frac{\partial\psi_{1,1}}{\partial r} - \alpha^2\psi_{1,1}\right)\right\} = 0 \quad (23)$$

Then equation (22) becomes,

$S_4 - \frac{2}{r}S_3 + \left(-2\alpha^2 + \frac{3}{r^2} + R_e.\alpha f_0(r)e^{iAt}\right)S_2 + \left(\frac{2\alpha^2}{r} - \frac{3}{r^3} - \frac{R_e.\alpha f_0(r)e^{iAt}}{r}\right)S_1 + \left(\alpha^4 - R_e.\alpha^3 f_0(r)e^{iAt} -\right.$
Re α f0′′reiAt+Re.αf0′reiAtrψ=0 (24)

$$\psi_{1\,1} = 0.05 + (r-0.2)(0.1) + (r-0.2)^2(0.075) + (r-0.2)^3(0.0333) + (r-0.2)^4(1.1771) + \cdots \quad (25)$$

Again from equation (23) solution of mixed derivative is,

$$-R_e\left\{\left(\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\psi_{1,1}}{\partial r^2} - \frac{1}{r}\frac{\partial\psi_{1,1}}{\partial r} - \alpha^2\psi_{1,1}\right)\right\} = 0$$

$$\frac{\partial^3\psi_{1\,1}}{\partial t\,\partial r^2} - \frac{1}{r}\frac{\partial^2\psi_{1\,1}}{\partial t\ \partial r} - \alpha^2\frac{\partial\psi_{1\,1}}{\partial t} = 0 \qquad \psi_{1\,1} = \psi_{1\,1}(r,t)$$

Let $\psi_{1\,1} = RT$ where $R = R(r)$ and $T = T(t)$ $R''T' - \frac{1}{r}R'T' - \alpha^2 RT' = 0$

Reducible to Bessel's differential equation, $r^2R'' + (-r)R' + (-\alpha^2r^2)R = 0$

The series solution gives,

$$R(r) = a_0\alpha^n r^n\left[1+\frac{\alpha^2r^2}{n(n+2)}+\frac{\alpha^4r^4}{n(n+2)^2(n+4)}+\frac{\alpha^6r^6}{n(n+2)^2(n+4)^2(n+6)}+\cdots.\right] \quad (26)$$

From equations (25) and (26) we get the final equation for stream function as,

$$\psi_{1\,1} = 0.05+(r-0.2)(0.1)+(r-0.2)^2(0.075)+(r-0.2)^3(0.0333)+(r-0.2)^4(1.1771)+\cdots+ \\ +a_0\alpha^n r^n\left[1+\frac{\alpha^2r^2}{n(n+2)}+\frac{\alpha^4r^4}{n(n+2)^2(n+4)}+\frac{\alpha^6r^6}{n(n+2)^2(n+4)^2(n+6)}+\cdots.\right] \quad (27)$$

Average flux(Q_{AF}) is the average of time dependent flow flux(Q_{PL}) and time independent flow flux(Q_{PR}) then,

$$Q_{AF} = \frac{Q_{PL}+Q_{PR}}{2} \quad (28)$$

Where $Q_{PL} = \int_0^h 2\pi rv dr$ (29)

Where the velocity v is given by $v = \frac{1}{r}\frac{\partial\psi_{11}}{\partial r}$

$$Q_{PR} = \int_0^h 2\pi r\left(\frac{v_1^f+v_2^f}{2}\right)dr \quad (30)$$

From equation (29),

$$Q_{PL} = \left\{\begin{array}{c} 2\pi\left[\begin{array}{c}(0.1)r+(r-0.2)^2(0.075)+(r-0.2)^3(0.075)+ \\ (r-0.2)^4(0.075)+\cdots\end{array}\right] \\ +a_0\alpha^n r^n\left[1+\frac{\alpha^2r^2}{n(n+2)}+\frac{\alpha^4r^4}{n(n+2)^2(n+4)}+\frac{\alpha^6r^6}{n(n+2)^2(n+4)^2(n+6)}+\cdots.\right]\end{array}\right\}_0^h$$

$h = 0.4$, $a_0 = 0.8$, $n = 1$, $\alpha = 0.1$, $r = 0.4, 0.8, 1.2, 1.6, 2.0$

From equation (30),

$$Q_{PR} = \pi\left\{\begin{array}{c} -r^2+C_2^{k_1}\left[\frac{r^{mk_1+3}}{(mk_1+3)(mk_1+1)}-\frac{h_1^{mk_1+1}r^2}{(mk_1+1)2}\right] \\ +C_2^{k_2}\left[\frac{h_1^{mk_2+1}r^2}{(mk_2+1)2}-\frac{[a+\eta\cos(c-\lambda z)]^{mk_2+1}r^2}{(mk_2+1)2}\right] \\ +C_2^{k_2}\left[\frac{r^{mk_2+3}}{(mk_2+1)(mk_2+3)}-\frac{[a+\eta\cos(c-\lambda z)]^{mk_2+1}r^2}{(mk_2+1)2}\right]\end{array}\right\} \quad (31)$$

Induced pressure gradient is computed as,

$$\frac{\partial p}{\partial z} = \left[\frac{-(n+1)^n 2^n m^n [q+(a+\eta\cos(c-\lambda z))]^2}{[a+\eta\cos(c-\lambda z)]^4}\right] \quad (32)$$

$$\text{Where } q = -[a+\eta\cos(c-\lambda z)]^2 - C_2^{k_1}\left[\frac{h_1^{mk_1+3}}{mk_1+3}\right] + C_2^{k_2}\left[\frac{h_1^{mk_2+3}}{mk_2+3} - \frac{[a+\eta\cos(c-\lambda z)]^{mk_2+3}}{mk_2+3}\right] \quad (33)$$

Resistance of flow (R_f) of the time dependent flux is computed as the ratio of average induced pressure gradient to average flux in the direction of flow,

$$R_f = -0.43298$$

The impedance value (IF) reflects the pressure opposing at the time dependent wave and average flux further as,

$$IF = \frac{-\partial p/\partial z}{Q_{AF}} \quad (34)$$

$$IF = \frac{-\left[\frac{-(n+1)^n 2^n m^n [q+(a+\eta\cos(c-\lambda z))]^2}{[a+\eta\cos(c-\lambda z)]^4}\right]}{\left\{\begin{array}{c} 2\pi\left[\begin{array}{c}(0.1)r+(r-0.2)^2(0.075)+(r-0.2)^3(0.075)+\\ (r-0.2)^4(0.075)+\cdots\end{array}\right] \\ +a_0\alpha^n r^n\left[1+\frac{\alpha^2 r^2}{n(n+2)}+\frac{\alpha^4 r^4}{n(n+2)^2(n+4)}+\frac{\alpha^6 r^6}{n(n+2)^2(n+4)^2(n+6)}+\cdots\right]\end{array}\right\}_0^h} + \frac{-\left[\frac{-(n+1)^n 2^n m^n [q+(a+\eta\cos(c-\lambda z))]^2}{[a+\eta\cos(c-\lambda z)]^4}\right]}{\pi\left\{\begin{array}{l} -r^2+C_2^{k_1}\left[\frac{r^{mk_1+3}}{(mk_1+3)(mk_1+1)}-\frac{h_1^{mk_1+1}r^2}{(mk_1+1)2}\right] \\ +C_2^{k_2}\left[\frac{h_1^{mk_2+1}r^2}{(mk_2+1)2}-\frac{[a+\eta\cos(c-\lambda z)]^{mk_2+1}r^2}{(mk_2+1)2}\right] \\ +C_2^{k_2}\left[\frac{r^{mk_2+3}}{(mk_2+1)(mk_2+3)}-\frac{[a+\eta\cos(c-\lambda z)]^{mk_2+1}r^2}{(mk_2+1)2}\right]\end{array}\right\}} \quad (35)$$

Results and discussion

The Flow applicability between time dependent flow on peristaltic transport has been studied to estimate the variation in profiles of velocity and the flux in reference to artery diameter ranging from 0.4cm to 2.64cm. Numerical computations using series form solution indicate that the flow appears to be insignificant within the amplitude ratio $[\epsilon: 0.2 - 0.8]$. The yield stress changes with time at the initial point peristaltic transport with narrow region whereas at the region with $h \approx 1.75\text{cm}$ variations of amplitude are too significant. The fluid under consideration is assumed to be non-Newtonian shows the mean flow rate which does not change with induced amplitude and frequency. But it increases or decreases with the overlaping of two flows (time dependent on peristaltic transport). The effect of individual factors like tube radius, pressure gradient, the average pressure gradient give the qualitative and quantitative changes in velocity(Fig.1), flux(Fig.2), limitation of flux with varying yield stress. The shape of the profiles obtained are changed considerably. As a result the non uniform flow behavior appears in the axial direction which signifies that peristaltic transport is independent of time dependent flow with impedance value correction.

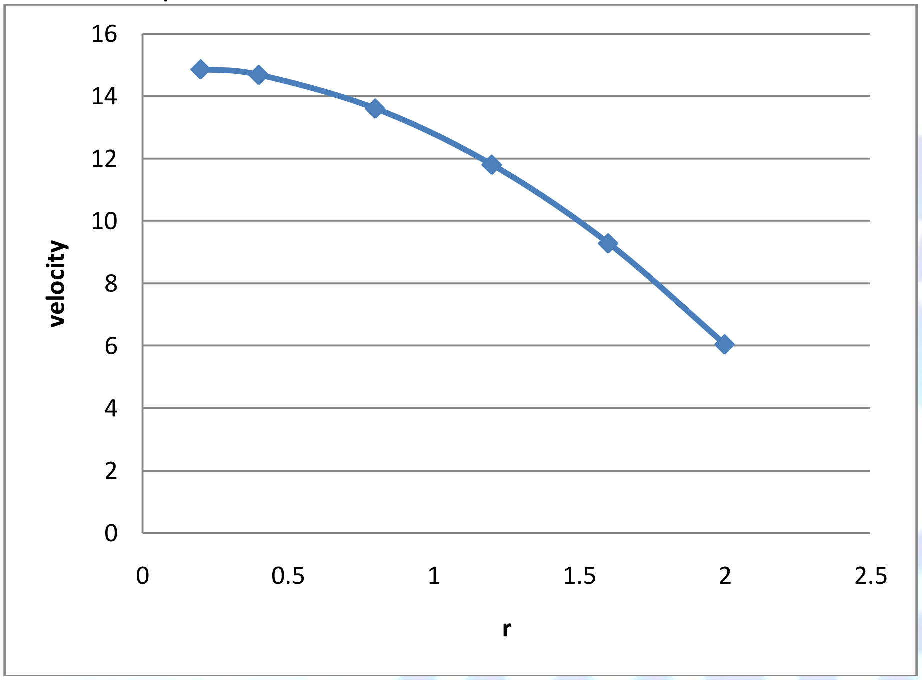


Fig (1): velocity profile

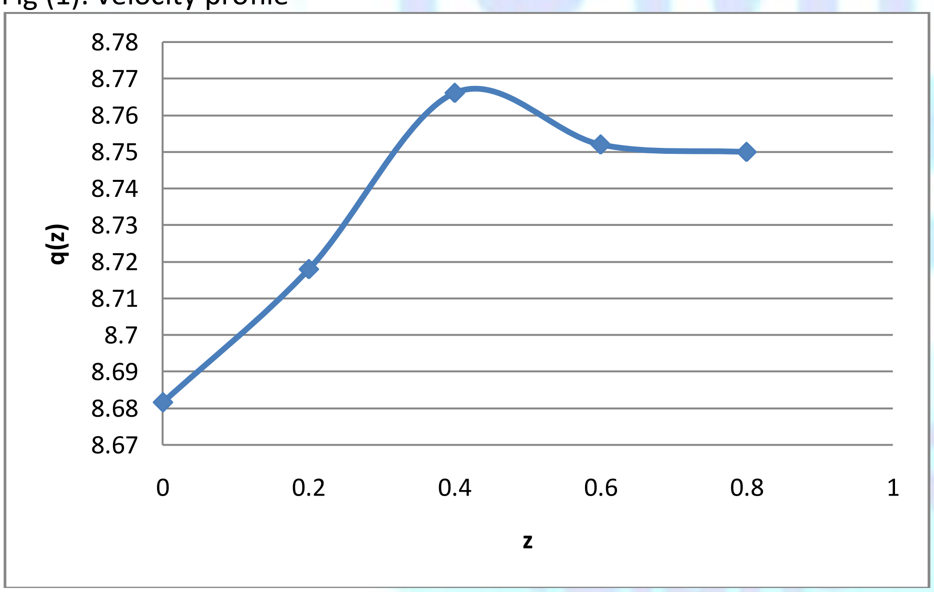


Figure (2): flux in axial direction

Induced pressure gradient

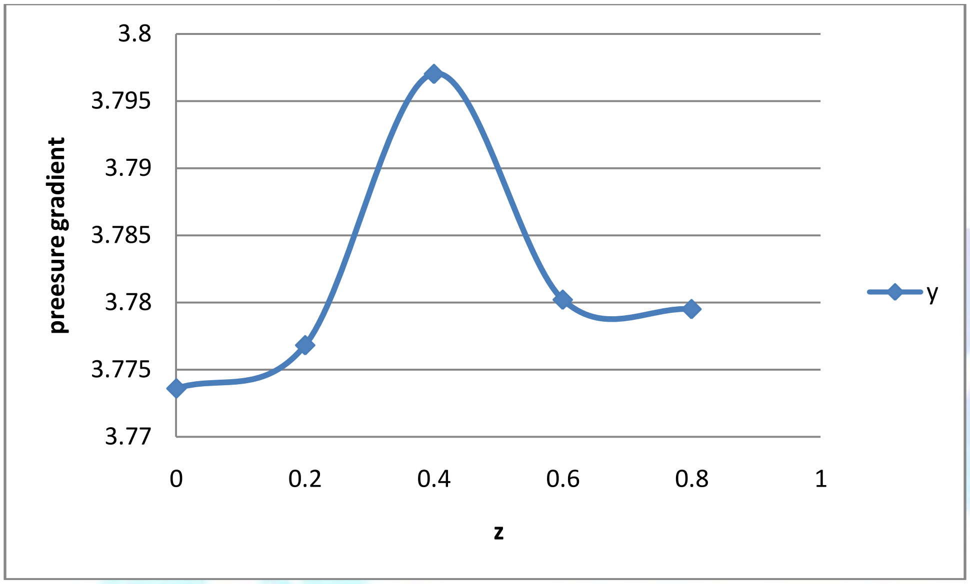
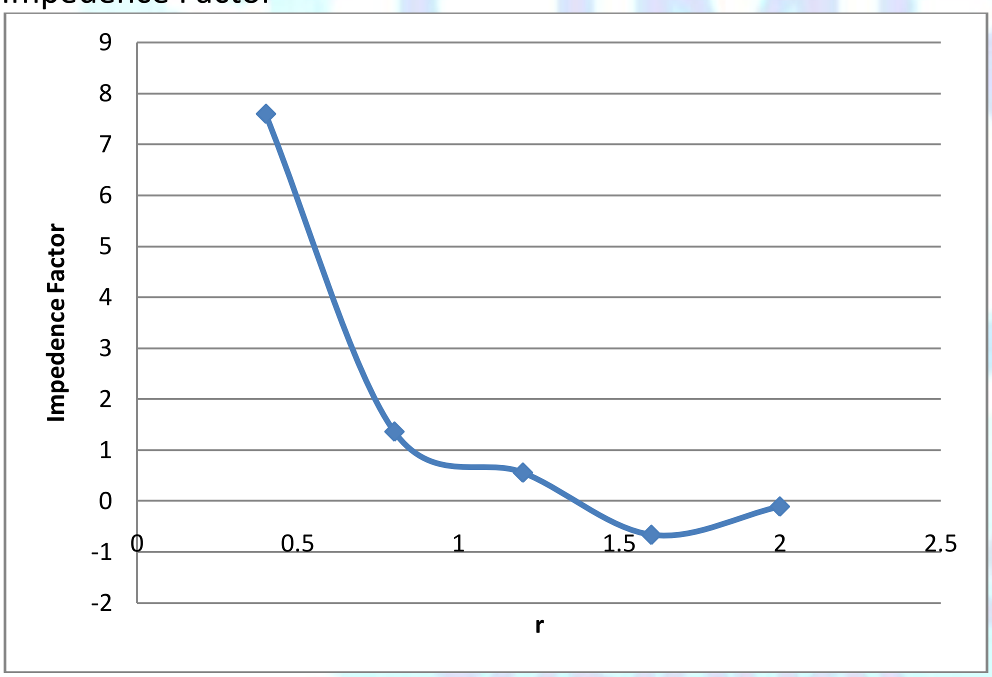


Figure (5): Induced pressure gradient along axial direction

Impedence Factor



Nomenclature

a- amplitude of boundary wave.

c- boundary wave speed

P_0- amplitude of imposed pressure wave

C-pressure gradient wave speed

R- radius of tube.
p- fluid pressure.
R_e- Reynolds number.
α - boundary wave number.
A- pressure wave number.
λ- boundary wavelength.
Λ- pressure wavelength.
$\varepsilon = \frac{a}{R}$- amplitude ratio.
η - radial displacement of tube.
$i = \sqrt{-1}$ - imaginary unit
ρ - density of the fluid.
ϑ - kinematic viscosity.
ψ - stream function.
$u = v_z$ – velocity in the axial direction.
$v = v_r$ - velocity in the radial direction.
z - axial coordinate
r - radial coordinate.
t - time.
$\nabla^2 \equiv \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r}$ - Laplace operator.
J_0, J_1 - modified Bessel function of first kind.

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