Oblong Mean Prime Labeling of Some Path Graphs

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ABSTRACT
A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge(uv),the labels assigned to u and v are oblong numbers and for each vertex of degree at least 2, the g c d of the labels of the incident edges is 1. Here we characterize some path related graphs for oblong mean prime labeling.
Keywords: Graph labeling, oblong numbers, prime graphs, prime labeling, path graph.

1.INTRODUCTION

All graphs in this paper are finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3],[4] and [5] . Some basic concepts are taken from Frank Harary [1]. In this paper we investigated the oblong mean prime labeling of some path graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common divisor of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form n(n+1).The oblong numbers are 2, 6, 12, 20, --------------.

2.MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection

f : V(G) → {2,6,12,20 ,------------------,p(p+1)} by f(v_i) = $i(i+1)$, for every i from 1 to p and define a 1-1 mapping f^{*}_{ompl} : E(G) → set of natural numbers N by $f^{*}_{ompl}(uv) = \frac{f(u)+f(v)}{2}$. The induced function f^{*}_{ompl} is said to be an oblong mean prime labeling, if the g c d of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong mean prime labeling is called an oblong mean prime graph.

Theorem: 2.1 The path P_n admits oblong mean prime labeling.

Proof : Let G = P_n and let v_1,v_2,----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = n-1.

Define a function f : V → {2,6,12,-----------------,$n(n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^{*}_{ompl} is defined as follows

$f^{*}_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,n-1

Clearly f^{*}_{ompl} is an injection.

g c d of (v_{i+1}) = g c d of { $f^{*}_{ompl}(v_i\ v_{i+1})$, $f^{*}_{ompl}(v_{i+1}\ v_{i+2})$ }

= g c d of { $(i+1)^2$, $(i+2)^2$ }
= g c d of { (i+1) , (i+2) }
= 1, i = 1,2,---------------,n-2

So, g c d of each vertex of degree greater than one is 1.
Hence P_n admits, oblong mean prime labeling.

Theorem 2.2 The corona of a path P_n admits oblong mean prime labeling.

Proof : Let G = $P_n \odot K_1$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n-1.
Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by
f(v_i) = i(i+1) , i = 1,2,------,2n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,n+1
$f^*_{ompl}(v_{i+2}\ v_{2n-i+1})$ = $2n^2$-2ni+3n+i^2+i+4 i = 1,2,3,-----------,n-2

Clearly f^*_{ompl} is an injection.
g c d of (v_{i+1}) = 1, i = 1,2,---------------,n.
So, g c d of each vertex of degree greater than one is 1.
Hence $P_n \odot K_1$,admits oblong mean prime labeling.

Theorem 2.3 2-tuple graph of path P_n, admits oblong mean prime labeling, when n is not a multiple of 3.

Proof : Let G = $T^2(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 3n-2.
Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by
f(v_i) = i(i+1) , i = 1,2,------,2n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_{2i-1}\ v_{2i+1})$ = $4i^2$+2i+1, i = 1,2,------------,n-1
$f^*_{ompl}(v_{2i}\ v_{2i+2})$ = $4i^2$+6i+3, i = 1,2,------------,n-1
$f^*_{ompl}(v_{2i-1}\ v_{2i})$ = $4i^2$ i = 1,2,------------,n

Clearly f^*_{ompl} is an injection.

g c d of (v_{2i+1}) = g c d of { $f^*_{ompl}(v_{2i-1}\ v_{2i+1})$, $f^*_{ompl}(v_{2i+1}\ v_{2i+3})$, $f^*_{ompl}(v_{2i+1}\ v_{2i+2})$ }
= g c d of { $4i^2$+2i+1, $4i^2$+10i+7, $4i^2$+8i+4 }
= 1, i = 1,2,------------,n-2

g c d of (v_{2i+2}) = g c d of { $f^*_{ompl}(v_{2i}\ v_{2i+2})$, $f^*_{ompl}(v_{2i+2}\ v_{2i+4})$, $f^*_{ompl}(v_{2i+1}\ v_{2i+2})$ }
= g c d of { $4i^2$+6i+3, $4i^2$+14i+13, $4i^2$+8i+4 }
= 1, i = 1,2,------------,n-2

g c d of (v_1) = g c d of {4, 7} =1.
g c d of (v_2) = g c d of {4,13} =1.
g c d of (v_{2n-1}) = g c d of { $f^*_{ompl}(v_{2n-3}\ v_{2n-1})$, $f^*_{ompl}(v_{2n-1}\ v_{2n})$ }
= g c d of { $4n^2$-6n+3, $4n^2$}
= 1, since ‘n’ not a multiple of 3.

g c d of (v_{2n}) = g c d of { $f^*_{ompl}(v_{2n-2}\ v_{2n})$, $f^*_{ompl}(v_{2n-1}\ v_{2n})$ }
= g c d of { $4n^2$-2n+1, $4n^2$}

So, g c d of each vertex of degree greater than one is 1.

Hence $T^2(P_n)$,admits oblong mean prime labeling.

Theorem 2.4 P_n^2 , admits oblong mean prime labeling.

Proof : Let G = P_n^2 and let v_1,v_2,----------------,v_n are the vertices of G.

Here |V(G)| = n and |E(G)| = 2n-3.

Define a function f : V → {2,6,12,-----------------,$n(n+1)$ } by

$f(v_i)$ = i(i+1) , i = 1,2,------,n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,n-1

$f^*_{ompl}(v_i\ v_{i+2})$ = i^2+3i+3, i = 1,2,------------,n-2

Clearly f^*_{ompl} is an injection.

g c d of (v_1) = g c d of {4,7} = 1.

g c d of (v_n) = g c d of { $f^*_{ompl}(v_{n-1}\ v_n)$, $f^*_{ompl}(v_{n-2}\ v_n)$ }
= g c d of { n^2, n^2 - n +1 }
= g c d of { n-1, n^2 - n +1 } = 1.

g c d of (v_{2i+1}) = 1, i = 1,2,------------,n-2

So, g c d of each vertex of degree greater than one is 1.

Hence P_n^2 ,admits oblong mean prime labeling.

Theorem 2.5 Middle graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = M(P_n) and let v_1,v_2,----------------,v_{2n-1} are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-4.

Define a function f : V → {2,6,12,-----------------,$(2n-1)2n$ } by

$f(v_i)$ = i(i+1) , i = 1,2,------,2n-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,2n-2

$f^*_{ompl}(v_{2i}\ v_{2i+2})$ = $4i^2$+6i+3, i = 1,2,------------,n-2

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-3

So, g c d of each vertex of degree greater than one is 1.

Hence M(Pn) ,admits oblong mean prime labeling.

Theorem 2.6 Total graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = T(P_n) and let v_1,v_2,----------------,v_{2n-1} are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 4n-5.

Define a function f : V → {2,6,12,-----------------,$(2n-1)2n$ } by

$f(v_i)$ = i(i+1) , i = 1,2,------,2n-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,2n-2

$f_{ompl}^{*}(v_{2i}\ v_{2i+2})$ = $4i^2+6i+3$, i = 1,2,------------,n-2

$f_{ompl}^{*}(v_{2i-1}\ v_{2i+1})$ = $4i^2+2i+1$, i = 1,2,------------,n-1

Clearly f_{ompl}^{*} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-3

g c d of (v_1) = g c d of {4,7} = 1.

g c d of (v_{2n-1}) = g c d of { $f_{ompl}^{*}(v_{2n-2}\ v_{2n-1})$, $f_{ompl}^{*}(v_{2n-1}\ v_{2n-3})$ }

= g c d of { $4n^2-6n+3$, $4n^2-4n+1$}

= g c d of { $4n^2-6n+3$, 2n-2}

= 1.

So, g c d of each vertex of degree greater than one is 1.

Hence $T(P_n)$,admits oblong mean prime labeling.

Theorem 2.7 The semi total point graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = $R(P_n)$ and let v_1,v_2,----------------,v_{2n-1} are the vertices of G.

Here |V(G)| = 2n-1 and |E(G)| = 3n-3.

Define a function f : V → {2,6,12,-----------------,$(2n-1)2n$ } by

$f(v_i)$ = i(i+1) , i = 1,2,------,2n-1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ompl}^{*} is defined as follows

$f_{ompl}^{*}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,2n-2

$f_{ompl}^{*}(v_{2i-1}\ v_{2i+1})$ = $4i^2+2i+1$, i = 1,2,------------,n-1

Clearly f_{ompl}^{*} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-3.

g c d of (v_1) = g c d of {4,7} = 1.

g c d of (v_{2n-1}) = 1.

So, g c d of each vertex of degree greater than one is 1.

Hence $R(P_n)$, admits oblong mean prime labeling.

Theorem 2.8 Duplicate graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = $D(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n-2.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

$f(v_i)$ = i(i+1) , i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f_{ompl}^{*} is defined as follows

$f_{ompl}^{*}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,n-1

$f_{ompl}^{*}(v_{n+i}\ v_{n+i+1})$ = $(n+i+1)^2$, i = 1,2,------------,n-1

Clearly f_{ompl}^{*} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,n-2.

g c d of (v_{n+i+1}) = 1, i = 1,2,------------,n-2.

So, g c d of each vertex of degree greater than one is 1.

Hence $D(P_n)$,admits oblong mean prime labeling.

Theorem 2.9 Strong duplicate graph of path P_n, admits oblong mean prime labeling, when n is not a multiple of 3.

Proof: Let G = SD(P_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-2.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_{2i-1}\ v_{2i})$ = $(2i)^2$, i = 1,2,------------,n

$f^*_{ompl}(v_{2i}\ v_{2i+1})$ = $(2i+1)^2$, i = 1,2,------------,n-1

$f^*_{ompl}(v_{2i-1}\ v_{2i+2})$ = $(2i+1)^2$ +2 i = 1,2,------------,n-1

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

g c d of (v_1) = g c d of {4,11} =1

g c d of (v_{2n}) = g c d of {$f^*_{ompl}(v_{2n}\ v_{2n-1})$, $f^*_{ompl}(v_{2n}\ v_{2n-3})$}

= g c d of { $4n^2$-4n+3, $4n^2$}

= g c d of {2n, $4n^2$-4n+3}

= g c d of { 3, 2n}

= 1 , since n is not a multiple of 3.

So, g c d of each vertex of degree greater than one is 1.

Hence SD(P_n) ,admits oblong mean prime labeling.

Theorem 2.10 Strong shadow graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = $SD_2(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 5n-4.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_{2i-1}\ v_{2i})$ = $(2i)^2$, i = 1,2,------------,n

$f^*_{ompl}(v_{2i}\ v_{2i+1})$ = $(2i+1)^2$, i = 1,2,------------,n-1

$f^*_{ompl}(v_{2i-1}\ v_{2i+2})$ = $(2i+1)^2$ +2, i = 1,2,------------,n-1

$f^*_{ompl}(v_{2i}\ v_{2i+2})$ = $(2i+1)^2$ +(2i+2), i = 1,2,------------,n-1

$f^*_{ompl}(v_{2i-1}\ v_{2i+1})$ = $(2i+1)^2$ -2i, i = 1,2,------------,n-1

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

g c d of (v_1) = g c d of {4,11} =1

g c d of (v_{2n}) = g c d of {$f^*_{ompl}(v_{2n}\ v_{2n-1})$, $f^*_{ompl}(v_{2n}\ v_{2n-3})$, $f^*_{ompl}(v_{2n}\ v_{2n-2})$ }

= g c d of { $4n^2$-4n+3, $4n^2$,$4n^2$-2n+1}

= 1 .

So, g c d of each vertex of degree greater than one is 1.

Hence $SD_2(P_n)$,admits oblong mean prime labeling.

Theorem 2.11 Shadow graph of path P_n, admits oblong mean prime labeling, when n is not a multiple of 3.

Proof: Let G = $D_2(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 4n-4.

Define a function f : V → {2,6,12,-----------------,$(2n+1)2n$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,2n-1

$f^*_{ompl}(v_{4i-2}\ v_{4i+1})$ = $16i^2+2$, i = 1,2,------------,$\frac{n-2}{2}$

$f^*_{ompl}(v_{4i}\ v_{4i+3})$ = $16i^2+16i+6$, i = 1,2,------------, $\frac{n-2}{2}$

$f^*_{ompl}(v_{4i-3}\ v_{4i})$ = $16i^2-8i+3$, i = 1,2,------------, $\frac{n}{2}$

$f^*_{ompl}(v_{4i-2}\ v_{4i+3})$ = $16i^2-8i+7$, i = 1,2,------------, $\frac{n-2}{2}$

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

g c d of (v_1) = g c d of {4,11} =1

g c d of (v_{2n}) = g c d of {$f^*_{ompl}(v_{2n}\ v_{2n-1})$, $f^*_{ompl}(v_{2n}\ v_{2n-3})$ }

= g c d of { $4n^2-4n+3$, $4n^2$}

= g c d of { $4n^2-4n+3$, 4n-3}

= g c d of { 3n, 4n-3}= g c d of { 3n, n-3} = g c d of { 9, n-3} = 1

So, g c d of each vertex of degree greater than one is 1.

Hence $D_2(P_n)$,admits oblong mean prime labeling.

Theorem 2.12 Z graph of path P_n, admits oblong mean prime labeling, when n is even.

Proof: Let G = $Z(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 3n-3.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,2n-1

$f^*_{ompl}(v_{4i-2}\ v_{4i+1})$ = $16i^2+2$, i = 1,2,------------,$\frac{n-2}{2}$

$f^*_{ompl}(v_{4i}\ v_{4i+3})$ = $16i^2+16i+6$, i = 1,2,------------, $\frac{n-2}{2}$

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,2n-2.

So, g c d of each vertex of degree greater than one is 1.

Hence $Z(P_n)$,admits oblong mean prime labeling.

Theorem 2.13 H graph of path P_n, admits oblong mean prime labeling.

Proof: Let G = $H(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G.

Here |V(G)| = 2n and |E(G)| = 2n-1.

Define a function f : V → {2,6,12,-----------------,$2n(2n+1)$ } by

$f(v_i) = i(i+1)$, i = 1,2,------,2n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1})$ = $(i+1)^2$, i = 1,2,------------,n-1
$f^*_{ompl}(v_{n+i}\ v_{n+i+1})$ = $(n+i+1)^2$, i = 1,2,------------,n-1
Case (i) n is odd.
$f^*_{ompl}(v_{\frac{n+1}{2}}\ v_{\frac{3n+1}{2}})$ = $\frac{5n^2+8n+3}{4}$
Case (ii) n is even
$f^*_{ompl}(v_{\frac{n+2}{2}}\ v_{\frac{3n}{2}})$ = $\frac{5n^2+6n+4}{4}$
Clearly f^*_{ompl} is an injection.
g c d of (v_{i+1}) = 1, i = 1,2,------------,n-2,n+1,-----------,2n-2
So, g c d of each vertex of degree greater than one is 1.
Hence H(P_n) ,admits oblong mean prime labeling.

Theorem 2.14 Centipede graph C(2,n), admits oblong mean prime labeling.

Proof: Let G = C(2,n) and let v_1,v_2,----------------,v_{3n} are the vertices of G.
Here |V(G)| = 3n and |E(G)| = 3n-1.
Define a function f : V → {2,6,12,-----------------,$(3n+1)3n$ } by
f(v_i) = i(i+1) , i = 1,2,------,3n.
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows
$f^*_{ompl}(v_{3i-2}\ v_{3i-1})$ = $(3i-1)^2$, i = 1,2,------------,n
$f^*_{ompl}(v_{3i-1}\ v_{3i})$ = $(3i)^2$, i = 1,2,------------,n
$f^*_{ompl}(v_{3i-1}\ v_{3i+2})$ = $9i^2+6i+3$ i = 1,2,------------,n-1
Clearly f^*_{ompl} is an injection.
g c d of (v_{3i-1}) = g c d of {$f^*_{ompl}(v_{3i-2}\ v_{3i-1})$, $f^*_{ompl}(v_{3i-1}\ v_{3i})$ }
= g c d of { $(3i-1)^2$, $(3i)^2$}
= g c d of { 3i-1, 3i} = 1, i = 1,2,------------,n
So, g c d of each vertex of degree greater than one is 1.
Hence C(2,n) ,admits oblong mean prime labeling.

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