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Some Properties and Applications of Local Fractional Fourier Transform

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Abstract:
The purpose of this paper, to study new properties of Local Fractional Fourier Transform of new form and also as applications of it, we have solved the Local Fractional Differential Equation.

Keywords:
Fractional Fourier Transform, Local Fractional Fourier Transform,

1 Introduction

The idea of transformations [6] is generally started with the need of converting problems from one form into another form which is rather simpler to solve and then by using inversion formula again coming back to the original form with the solution.In recent years [3, 4, 9, 10], many linear boundary value problems, initial value problems are effectively solved by these transformations [4, 5, 6, 7] like Laplace, Fourier, Mellin, Wavelet and other transformations with applications increasing rapidly in daily life and branches of science like Bio – engineering, Computational Fluid Dynamics, Abel's integral equations, Bio – mathematics, Capacitor theory, conductance of biological systems [7, 8].

2 Definitions of Fractional Derivatives and Fractional Integrals

Definition 2.1: (Riemann-Lowville): If f(x) ∈ C [a, b] and a < x < b then, $D_{a^+}^{\propto} f(x) = \frac{1}{\Gamma(1-\propto)} \frac{d}{dx} \int_a^x f(t)(x-t)^{-\propto} dt,$ where, $\propto \in (0\,,1)$

Definition 2.2: (M. Caputo (1967)): If f(x) ∈ C[a, b] and a < x < b then the Caputo Fractional derivative of order ∝ is defined as follows [1,5,7],

$$ {}_aD_x^{\propto} f(x) = \frac{1}{\Gamma(n-\propto)} \int_a^x f^n(\tau)(x-\tau)^{n-\propto-1} d\tau\,, \qquad Where, \propto \in (n-1\,,n) $$

3 Definitions of local fractional Fourier Transform

This section is devoted for the definition of Local Fractional Fourier Transform.

Definition 3.1:

The complex number z = x + iy can also be written in the polar form as $z = re^{i\theta}$, so that fractional order of a complex number z can be defined as $z^{\propto} = (re^{i\theta})^{\propto}, \propto \in (0\,,1]$ .The above definition can also have equivalent formula in the form of [9] trigonometric function defined by the expression, $z^{\propto} = \left(\sqrt{x^2+y^2}\right)^{\propto} (\cos(\theta\alpha) + isin(\alpha\theta)),$
$where, \propto \in (0\,,1].$

Definition 3.2:

Local Fractional Fourier Transform: The Local Fractional Fourier transform of function *f(t)* is defined as follows [9], $\widehat{f_{\propto}(\xi)} = \int_{-\infty}^{+\infty} f(t) e^{-2\pi i t \xi^{\propto}} dt\,, \propto \in (0\,,1]$

Where, ∝→ 1, the Local fractional Fourier transform tends to be an ordinary Fourier Transform. To find the Local fractional Fourier transform of given function *f (t)* at a point ξ.The sufficient condition for convergence is $\int_{-\infty}^{+\infty} |f(t)| dt < \infty$

Definition 3.3:

Inverse Local Fractional Fourier Transform: The Local Fractional Fourier transform of function $\widehat{f(\xi)}$ is defined [9] as follows, $f(t) = \int_{-\infty}^{+\infty} \widehat{f(\xi)} e^{2\pi i t \xi^{\propto}} dt\,, \propto \in (0\,,1]$

Where, $\propto \to 1$, the Inverse Local fractional Fourier transform tends to be an ordinary Inverse Fourier Transform.

Definition 3.4:

Local Fractional Derivative: Let, f: [a; b] $\rightarrow \mathbb{R}$ be any function, if the limit $D^{\propto} f =_{\mp} \lim_{x \to y\mp} \frac{d^{\propto}(f(x)-f(y))}{d\mp(x-y)^{\propto}}$ exist and is finite, then f is said have left (right) LFD [3] of order $\propto$ at x = y.

4. Properties and Applications of Local Fractional Fourier Transform

(A) Convolution Property for Local Fractional Fourier Transform

Let *f(t)* and *g(t)* be two functions which are in L^1(R) then Local FrFT of convolution for these two functions can be evaluated as follows:

$$(f * \widehat{g)_{\propto}}(\xi) = \int_{-\infty}^{+\infty} [f * g](t) e^{-2\pi i t \xi^{\propto}} dt\,, \propto \in (0\,,1]$$

$$= \int_{-\infty}^{+\infty} [\int_{-\infty}^{\infty} f(u) g(t-u)] e^{-2\pi i t \xi^{\propto}} dt\,, \propto \in (0\,,1]$$

Applying Fubini's theorem on the RHS of the above equation we get,

$$= \int_{-\infty}^{+\infty} f(t) e^{-2\pi i t \xi^{\propto}} dt \int_{-\infty}^{+\infty} g(t) e^{-2\pi i t \xi^{\propto}} dt$$

$$= \widehat{f_{\propto}(\xi)}\, \widehat{g_{\propto}(\xi)}$$

i.e., $(f * \widehat{g)_{\propto}}(\xi) = \widehat{f_{\propto}(\xi)}\, \widehat{g_{\propto}(\xi)} \propto \in (0\,,1]$

(B) Local Fractional Fourier Transform of some Functions

Consider the function which is define in the following way,

$$g(t) = \begin{cases} D^{\propto}(\pi + \sqrt{t}), & 0 \le t < 1 \\ 0 \quad , & otherwise \end{cases}$$

with initial condition $g(o) = \pi$ and $\propto = \frac{1}{2}$ then by using the definition (2.2), (3.2) and (3.4) $\widehat{g_{\propto}}(\xi) = \int_{-\infty}^{+\infty} g(t) e^{-2\pi i t \xi^{\propto}} dt$

$$= \int_0^1 [D^{\propto}(\pi + \sqrt{t})] e^{-2\pi i t \xi^{\propto}} dt$$

By using the given initial condition, we get

$$\widehat{g_{\propto}}(\xi) = \int_0^1 [\lim_{t \to 0} \frac{d^{0.5}}{dt^{0.5}} (\sqrt{t})] e^{-2\pi i t \xi^{\propto}} dt$$

$$= \int_0^1 [\lim_{t \to 0} (\frac{1}{\Gamma(0.5)} \frac{d^{0.5}}{dt^{0.5}} \int_a^t \tau^{0.5} (t-\tau)^{-0.5} dt)] e^{-2\pi i t \xi^{\propto}} dt$$

$$= \int_0^1 [\lim_{t \to 0} (\frac{1}{\sqrt{\pi}} \frac{d^{0.5}}{dt^{0.5}} \int_a^t \tau^{0.5} (t-\tau)^{-0.5} dt)] e^{-2\pi i t \xi^{\propto}} dt$$

$$= \int_0^1 [\lim_{t \to 0} \frac{1}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2}] e^{-2\pi i t \xi^{\propto}} dt$$

$$= \frac{1}{2} \int_0^1 e^{-2\pi i t \xi^{\propto}} dt$$

$$= [\frac{-1}{4\pi i \xi^{\alpha}} (e^{-2\pi i \xi^{\propto}} - 1)]$$

Conclusion:

Property of convolution and finding the Local FrFT of Fractional Derivative of a function has been calculated as an application.

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