ROTATION AND CHEMICAL REACTION EFFECTS ON MHD FLOW THROUGH POROUS MEDIUM PAST AN IMPULSIVELY STARTED INCLINED PLATE

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ABSTRACT
In the present paper, we study the effects of rotation and chemical reaction on unsteady flow through porous medium past an impulsively started inclined plate with variable wall temperature and mass diffusion in the presence of transversely applied uniform magnetic field and Hall current. The fluid considered is viscous, incompressible and electrically conducting. The medium of the flow is taken as porous. The plate temperature and the concentration level near the plate increase linearly with time. The fluid model under consideration has been solved by Laplace transform technique. The results obtained are discussed with the help of graphs. The numerical values obtained for skin-friction and Sherwood number have been tabulated.
Keywords: MHD flow, rotation, chemical reaction, Hall current.

1 INTRODUCTION
The MHD flow past a flat plate through porous medium is one of the classical problems in the fluid dynamics. Applications of the study arise in magnetic field controlled materials processing systems, planetary and solar plasma fluid dynamics systems and rotating MHD induction machine energy generators etc. Hall effect on free and forced convective flow in a rotating channel was analyzed by Prasad et al. [5]. Muthucumarswamy along with and Ganesan [4] have studied first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Further, Muthucumarswamy [3] has worked on effect of chemical reaction on a moving isothermal vertical surface with suction. Hall current and heat transfer effects on MHD flow in a channel partially filled with a porous medium in a rotating system was developed by Chauhan and Rastogi [1]. Sahin and Chamkha [9] have examined effects of chemical reaction, radiation, heat and mass transfer on the MHD flow along a vertical porous wall in the presence of induced magnetic field. Mahdy [2] has discussed effect of chemical reaction and heat generation or absorption on double-diffusive convection from a vertical truncated cone in porous media with variable viscosity. Hall effect on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation was explained by Srinivas and Naikoti [8]. Rajput and Shareef [6] have presented effect of rotation and Hall current on unsteady MHD flow past an impulsively started vertical plate with heat and mass transfer in porous medium. Chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current was studied by us [7]. The present study is carried out to examine the effects of rotation and chemical reaction on unsteady MHD flow through porous medium past an impulsively started inclined plate with variable wall temperature and mass diffusion in the presence of Hall current. The problem is solved by the Laplace transform method. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction and Sherwood number have been tabulated.

2 MATHEMATICAL ANALYSIS

The geometrical model of the problem is shown in Figure-1

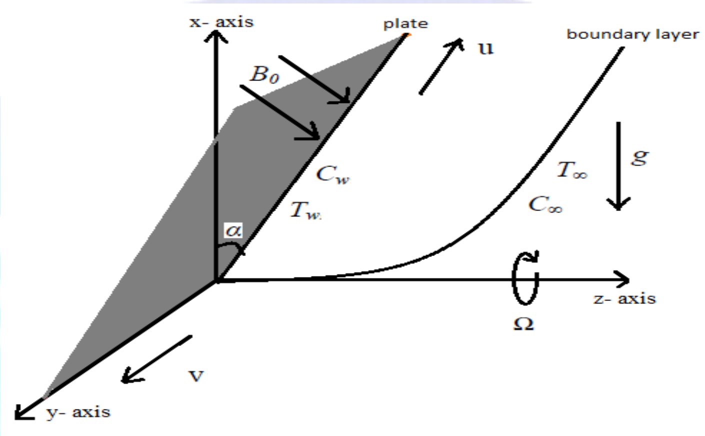

Figure-1. Physical model

The x axis is taken along the vertical plane and z axis is normal to it. Thus the z axis lies in the horizontal plane. The plate is inclined at angle α from vertical. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω about *z-* axis. The magnetic field B_0 of uniform strength is applied perpendicular to the flow. Since the fluid is electrically conducting whose magnetic Reynolds number is very small, therefore the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At time t>0, the plate starts moving with a velocity u_0 in its own plane, and temperature of the plate is raised to T_w. The concentration C_w near the plate is raised linearly with respect to time. The flow model is as under:

Equations of motion

$$\frac{\partial u}{\partial t}-2\Omega v=\upsilon\frac{\partial^2 u}{\partial z^2}+g\beta Cos\alpha(T-T_\infty)+g\beta^* Cos\alpha(C-C_\infty)-\frac{\sigma B_0^2(u+mv)}{\rho(1+m^2)}-\frac{\upsilon u}{K} \quad (1)$$

$$\frac{\partial v}{\partial t}+2\Omega u=\upsilon\frac{\partial^2 v}{\partial z^2}+\frac{\sigma B_0^2(mu-v)}{\rho(1+m^2)}-\frac{\upsilon v}{K} \quad (2)$$

Diffusion equation

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial z^2}-K_c(C-C_\infty) \quad (3)$$

Equation of energy

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} \tag{4}$$

The initial and boundary conditions are

$$\left.\begin{array}{l} t \le 0 : u = 0, v = 0, T = T_\infty, C = C_\infty, \forall\ z \\ t > 0 : u = u_0, v = 0, T = T_\infty + (T_w - T_\infty) A_0, C = C_\infty + (C_w - C_\infty) A_0, \text{ at z=0} \\ u \to 0, v \to 0, T \to T_\infty, C \to C_\infty \quad \text{as} \quad z \to 0. \end{array}\right\} \tag{5}$$

Here u and v are the primary and secondary velocities along x and z directions respectively, *C*- species concentration in the fluid, ν- the kinematic viscosity, ρ- the density, C_p- the specific heat at constant pressure, *g*-the acceleration due to gravity, *β*-volumetric coefficient of thermal expansion, t-time, m -the Hall current parameter, *K*- the permeability parameter, T_w - temperature of the plate at z= 0, *T*-temperature of the fluid, β^*- volumetric coefficient of concentration expansion, *k*- thermal conductivity of the fluid, *D*-the mass diffusion coefficient, K_c chemical reaction, C_w -species concentration at the plate z= 0, B_0- the uniform magnetic field, σ - electrical conductivity.

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (4) into dimensionless form:

$$\left.\begin{array}{l} \bar{z} = \dfrac{z u_0}{\upsilon}, \bar{u} = \dfrac{u}{u_0}, \bar{v} = \dfrac{v}{u_0}, \theta = \dfrac{(T - T_\infty)}{(T_w - T_\infty)}, S_c = \dfrac{\upsilon}{D}, \mu = \rho\upsilon, P_r = \dfrac{\mu c_p}{k}, \bar{C} = \dfrac{(C - C_\infty)}{(C_w - C_\infty)}, \\ G_r = \dfrac{g\beta\upsilon(T_w - T_\infty)}{u_0^{\ 3}}, G_m = \dfrac{g\beta^*\upsilon(C_w - C_\infty)}{u_0^{\ 3}}, M = \dfrac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \bar{t} = \dfrac{t u_0^2}{\upsilon}, \bar{\Omega} = \dfrac{\upsilon\Omega}{u_0^2}, \bar{K} = \dfrac{u_0}{\upsilon^2} K \end{array}\right\} \tag{6}$$

The symbols in dimensionless form are as under:
$\bar{u}$-primary velocity, $\bar{v}$-secondary velocity, Ω- rotation parameter, P_r- Prandtl number, S_c- Schmidt number, *R*-Radiationparameter, $\bar{t}$ - time, θ- temperature, $\bar{C}$ - concentration, G_r-thermal Grashof number, G_m- mass Grashof number, μ- coefficient of viscosity, *M*- magnetic parameter.

The flow model in dimensionless form is

$$\frac{\partial \bar{u}}{\partial \bar{t}} - 2\bar{\Omega}\bar{v} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + G_r Cos\,\alpha\theta + G_m Cos\,\alpha\bar{C} - \frac{M(\bar{u} + m\bar{v})}{(1+m^2)} - \frac{1}{\bar{K}}\bar{u} \tag{7}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + 2\bar{\Omega}\bar{u} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{M(m\bar{u} - \bar{v})}{(1+m^2)} - \frac{1}{\bar{K}}\bar{v} \tag{8}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{S_c}\frac{\partial^2 \bar{C}}{\partial \bar{z}^2} - K_0\bar{C} \tag{9}$$

$$\frac{\partial\theta}{\partial\bar{t}} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial\bar{z}^2} \tag{10}$$

The corresponding boundary conditions (5) become:

$$\left.\begin{array}{l} \bar{t} \le 0 : \bar{u} = 0, \bar{v} = 0, \theta = 0, \overline{C} = 0, \forall\, \bar{z}, \\ \bar{t} > 0 : \bar{u} = 1, \bar{v} = 0, \theta = \bar{t}, \overline{C} = \bar{t}, \text{ at } \bar{z} = 0, \\ \bar{u} \to 0, \bar{v} \to 0, \theta \to 0, \overline{C} \to 0 \text{ as } \bar{z} \to \infty. \end{array}\right\} \tag{11}$$

Dropping bars in the above equations, we get

$$\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} + G_r Cos\alpha\theta + G_m Cos\alpha C - \frac{M(u+mv)}{(1+m^2)} - \frac{1}{K}u \tag{12}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu-v)}{(1+m^2)} - \frac{1}{K}v \tag{13}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} - K_0 C \tag{14}$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2} \tag{15}$$

The boundary conditions become

$$\left.\begin{array}{l} t \le 0 : u = 0, v = 0, \theta = 0, C = 0, \forall\, z, \\ t > 0 : u = 1, v = 0, \theta = t, C = t, \text{ at } z = 0, \\ u \to 0, v \to 0, \theta \to 0, C \to 0, \text{ as } z \to \infty \end{array}\right\} \tag{16}$$

Writing the equations (12) and (13) in combined form (using *q= u + i v*)

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G_r Cos\alpha\theta + G_m Cos\alpha C - \frac{M(1-im)q}{1+m^2} - \frac{q}{K} - 2i\Omega q \tag{17}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial z^2} - K_0 C \tag{18}$$

$$\frac{\partial\theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2} \tag{19}$$

Finally, the boundary conditions become:

$$\left.\begin{array}{l} t \le 0 : q = 0, \theta = 0, C = 0, \forall\ z, \\ t > 0 : q = 1, \theta = t, C = t, \ \text{at } z=0, \\ q \to 0, \theta \to 0, C \to 0, \ \text{as } z \to \infty \end{array}\right\}. \qquad (20)$$

The dimensionless governing equations (17) to (19), subject to the boundary conditions (20), are solved by the usual Laplace transform technique. The solution obtained is as under:

$$q = \frac{e^{-\sqrt{az}}A_{15}}{4} + \frac{G_r Cos\alpha}{4a^2}[zA_{11} + 2e^{-\sqrt{az}}A_2P_r + 2A_{14}A_4(1-P_r)] + \frac{G_m Cos\alpha}{4(a-K_0S_c)^2}[zA_{11}$$

$$+ 2A_{13}A_5(1-S_c) + 2e^{-\sqrt{az}}A_2S_c(1-tK_0) - \frac{ze^{-\sqrt{az}}A_3K_0S_c}{\sqrt{a}}] + \frac{G_r Cos\alpha}{2a^2\sqrt{\pi}}[2zae^{\frac{-z^2P_r}{4t}}\sqrt{tP_r}$$

$$+ \sqrt{\pi}A_{14}(A_6 + A_7P_r) + \sqrt{\pi}A_{12}(az^2P_r - 2 + 2at + 2P_r)] + \frac{G_m Cos\alpha}{4\sqrt{\pi}(a-K_0S_c)^2}$$

$$[\frac{e^{-\sqrt{K_0S_c}}\sqrt{\pi}A_9\sqrt{S_c}}{2\sqrt{K_0}}(S_cK_0 - az) + A_{13}\sqrt{\pi}A_{10}(S_c - 1) + e^{-\sqrt{K_0S_c}}\sqrt{\pi}A_8(1 - at - S_c + tK_0S_c)]$$

$$C = \frac{e^{-z\sqrt{S_cK_0}}}{4\sqrt{K_0}}\{erfc[\frac{z\sqrt{S_c} - 2t\sqrt{K_0}}{2\sqrt{t}}](-z\sqrt{S_c} + 2t\sqrt{K_0}) + e^{2z\sqrt{S_cK_0}}erfc[\frac{z\sqrt{S_c} + 2t\sqrt{K_0}}{2\sqrt{t}}](z\sqrt{S_c} + 2t\sqrt{K_0})\}$$

$$\theta = t\left\{(1 + \frac{z^2P_r}{2t})erfc[\frac{\sqrt{P_r}}{2\sqrt{t}}] - \frac{z\sqrt{P_r}}{\sqrt{\pi}\sqrt{t}}e^{\frac{z^2}{4t}}P_r\right\},$$

The expressions for the symbols involved in the above solutions are given in the appendix.

3 SKIN FRICTION

The dimensionless skin friction at the plate is

$$\left(\frac{dq}{dz}\right)_{z=0} = \tau_x + i\tau_y \cdot$$

The numerical values of τ_x and τ_y, for different parameters are given in table-1.

4 SHERWOOD NUMBER

The dimensionless Sherwood number at the plate is

$$S_h = \left(\frac{\partial C}{\partial z}\right)_{z=0} = erfc[-\sqrt{tK_0}](-\frac{1}{4\sqrt{K_0}}\sqrt{S_c} - \frac{t\sqrt{S_cK_0}}{2}) + \sqrt{S_c}erfc[\sqrt{tK_0}](\frac{1}{4\sqrt{K_0}} + t\sqrt{K_0}) - \frac{e^{-tK_0}\sqrt{tS_cK_0}}{\sqrt{\pi K_0}}$$

The numerical values of S_h for different parameters are given in table-2.

5 RESULTS AND DISCUSSION

In this paper we have studied the effects of rotation,chemical reaction and permeability of porous medium on the flow. The behavior of other parameters like magnetic field, Hall current and thermal buoyancy is almost similar the earlier model studied by us [7]. The analytical results are shown graphically in figures 2 to 9. The numerical values of skin-friction and Sherwood number are presented in Table–1 and 2 respectively. Effect of rotation on flow behavior is shown by figures 2 and 3. It is observed that an increase in rotation parameter primary velocity decreases throughout the boundary layer region whereas secondary velocity increases continuously near the surface of the plate. This implies that rotation tends to accelerate secondary velocity whereas it retards primary velocity in the boundary layer region. Chemical reaction effect on fluid flow behavior is shown by figures 4 and 5. It is seen here that when chemical reaction parameter increases, primary and secondary velocities decreases throughout the boundary layer region. From figures 6 and 7, it is observed that primary and secondary velocities increase with permeability parameter. This result is due to the fact that increases in the value of permeability parameter results in reducing the drag force, and hence increasing the fluid velocity. Further, it is observed that the concentration of the fluid near the plate decreases when chemical reaction and Schmidt number parameters increased (figures 8 and 9).

Skin friction is given in table 1. The values of τ_x and τ_y increase with the increase in chemical reaction parameter. It decreases with rotation parameter and permeability.

Sherwood numberis given in table 2. The value of S_h decreases with the increase in the chemical reaction parameter, Schmidt number and time.

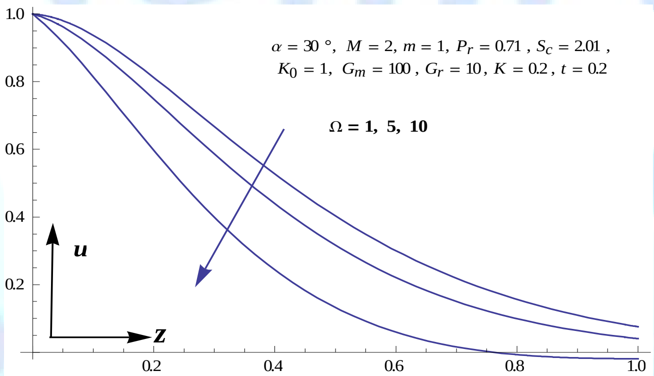

Figure 2: velocity *u* for different values of *Ω*

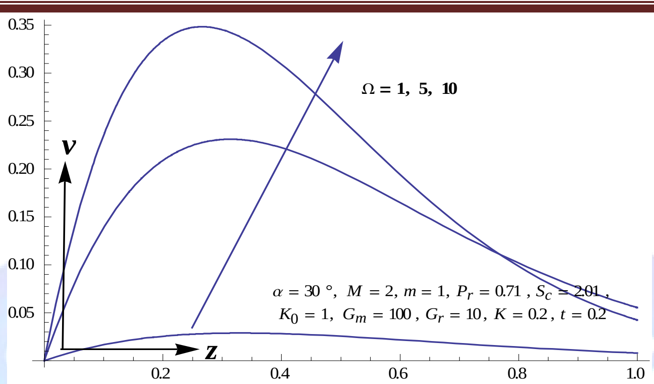

Figure 3: velocity v for different values of Ω

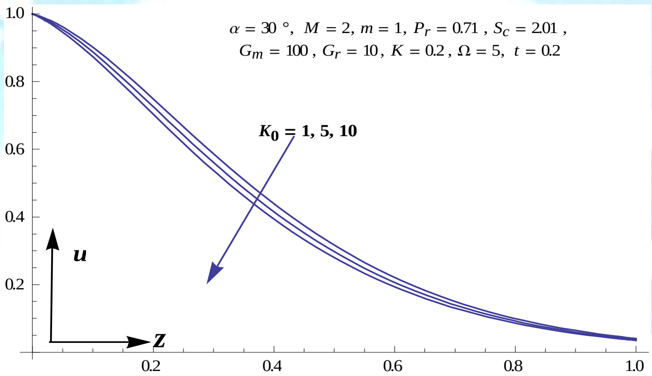

Figure 4: velocity u for different values of K_0

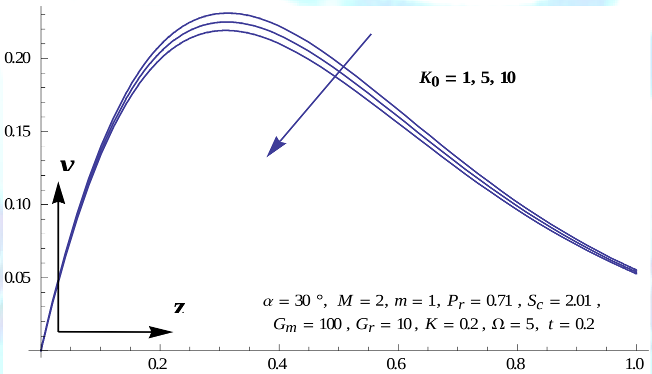

Figure 5: velocity v for different values of K_0

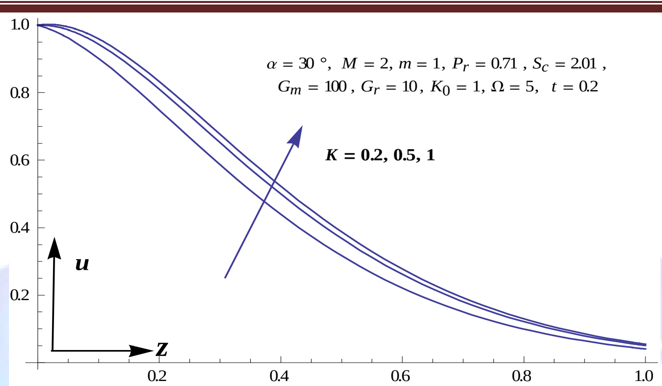

Figure 6: velocity *u* for different values of *K*

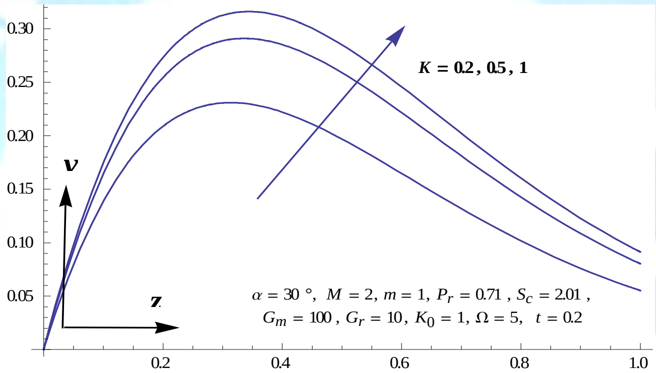

Figure 7: velocity *v* for different values of *K*

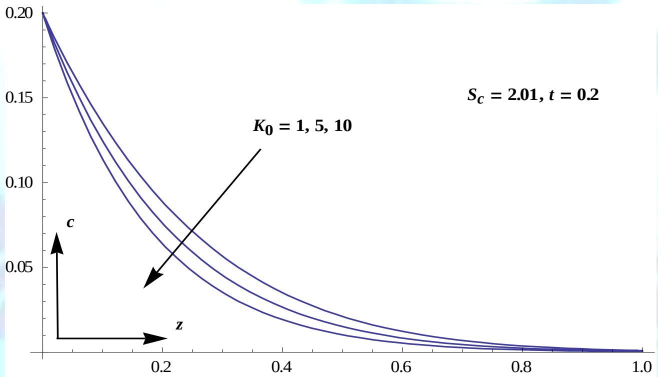

Figure 8: concentration *C* for different values of K_0

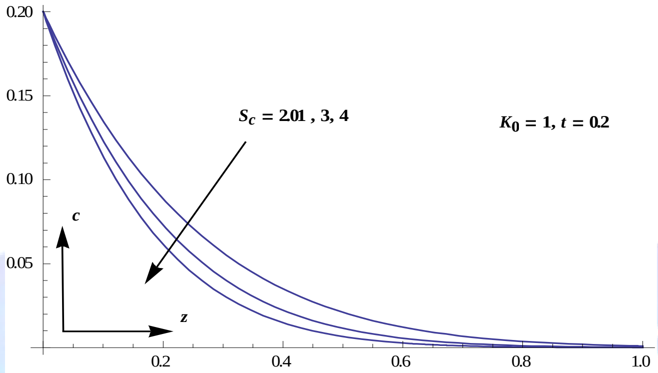

Figure 9: concentration *C* for different values of *t*

Table 1: Skin friction for different parameters

($\alpha=30^0$, $M=2$, $m=1$, $Pr=0.71$, $Sc=2.01$)

Gm	*Gr*	Ω	K_0	*K*	*t*	τ_x	τ_y
100	10	01	01	0.2	0.2	8.0729	-2.8944
100	10	05	01	0.2	0.2	0.2176	-4.4669
100	10	10	01	0.2	0.2	-1.2560	-4.5076
100	10	05	05	0.2	0.2	-6.6887	-2.6603
100	10	05	10	0.2	0.2	-2.3820	0.8097
100	10	05	01	0.5	0.2	-0.1893	-4.9271
100	10	05	01	1.0	0.2	-0.3196	-4.9404

Table 2. Sherwood number for different Parameters.

K_0	*Sc*	*t*	S_h
01	2.01	0.2	-0.76220
05	2.01	0.2	-0.93304
10	2.01	0.2	-1.11824
01	3.00	0.2	-0.93117
01	4.00	0.2	-1.07523
01	2.01	0.3	-0.96132
01	2.01	0.4	-1.14157

6 CONCLUSION

The present study is carried out to examine the unsteady MHD flow through porous medium past a moving inclined plate with heat and mass transfer in the presence of Hall current, rotation and chemical reaction. The results obtained are in agreement with the usual flow. It has been found that the velocity in the boundary layer region increases with the values of permeability parameter. But trend is reversed with chemical reaction parameter. That is the velocity decreases when chemical reaction parameter is increased. The rotation parameter retards the primary flow whereas it accelerates the secondary flow. It is also observed that chemical reaction parameter increases the drag at the plate surface, and decreases with the permeability and rotation parameter. Sherwood number and Nusselt number decrease with increase in radiation and chemical reaction parameter. The results obtained will have applications in the research related to the solar physics dealing with the sunspot development, the structure of rotating magnetic stars, cooling of electronic components of a nuclear reactor, bed thermal storage and heat sink in the turbine blades.

Appendix

$$a=\frac{M(1-im)}{1+m^2}+\frac{1}{K}+2i\Omega.\ A_0=\frac{u_0^2 t}{\upsilon}\ A_1=1+A_{16}+e^{2\sqrt{az}}(1-A_{17}),\ A_2=-A_1,\ A_3=A_{16}-A_1,$$

$$A_4=-1+A_{22}+A_{18}(A_{23}-1),\ A_5=-1+A_{24}+A_{19}(A_{25}-1),$$

$$A_6=-1-A_{26}+A_{18}(A_{27}-1),\ A_7=-A_6,\ A_8=-1-A_{20}+A_{30}(A_{21}-1),\ A_9=A_8+2(A_{20}+1),$$

$$A_{10}=-1-A_{28}+A_{19}(A_{29}-1),\ A_{11}=\frac{e^{-\sqrt{az}}}{z}(2A_1+2atA_2+\sqrt{a}A_3),\ A_{12}=-1+erf\left[\frac{z\sqrt{P_r}}{2\sqrt{t}}\right],$$

$$A_{13}=e^{\frac{at}{-1+S_c}-z\sqrt{\frac{(a-K_0)S_c}{-1+S_c}}-\frac{tK_0S_c}{-1+S_c}},\ A_{14}=e^{\frac{at}{-1+P_r}-z\sqrt{\frac{(a)P_r}{-1+P_r}}},\ A_{15}=1+A_{16}+e^{2\sqrt{az}}A_{17},\ A_{16}=erf\,[\frac{2\sqrt{at}-z}{2\sqrt{t}}],$$

$$A_{17}=erf\,[\frac{2\sqrt{at}+z}{2\sqrt{t}}],\ A_{18}=e^{-2z\sqrt{\frac{aP_r}{-1+P_r}}},\ A_{19}=e^{-2z\sqrt{\frac{(a-K_0)S_c}{-1+S_c}}},\ A_{20}=erf\left[\sqrt{tK_0}-\frac{z\sqrt{S_c}}{2\sqrt{t}}\right],$$

$$A_{21}=erf\left[\sqrt{tK_0}+\frac{z\sqrt{S_c}}{2\sqrt{t}}\right],\ A_{22}=erf\,[\frac{z-2t\sqrt{\frac{aP_r}{-1+P_r}}}{2t}],\ A_{23}=erf\,[\frac{z+2t\sqrt{\frac{aP_r}{-1+P_r}}}{2t}],$$

$$A_{24}=erf\,[\frac{z-2t\sqrt{\frac{(a-K_0)S_c}{-1+S_c}}}{2t}],\ A_{25}=erf\,[\frac{z+2t\sqrt{\frac{(a-K_0)S_c}{-1+S_c}}}{2t}],\ A_{26}=erf\,[\frac{2t\sqrt{\frac{a}{-1+P_r}}-z\sqrt{P_r}}{2\sqrt{t}}],$$

$$A_{27}=erf\,[\frac{2t\sqrt{\frac{a}{-1+P_r}}+z\sqrt{P_r}}{2\sqrt{t}}],\ A_{28}=erf\,[\sqrt{t}\sqrt{\frac{(a-K_0)}{-1+S_c}}-\frac{zS_c}{2\sqrt{t}}],\ A_{29}=erf\,[\sqrt{t}\sqrt{\frac{(a-K_0)}{-1+S_c}}+\frac{zS_c}{2\sqrt{t}}],$$

$$A_{30}=erf\,[e^{2z\sqrt{K_0\sqrt{S_c}}}],$$

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