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The Effect of Hall-Currents on MHD Flow Over an Oscillating Vertical Plate in a Rotating System

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ABSTRACT
An unsteady MHD flow of a viscous, incompressible and electrically conducting fluid past a vertical oscillatory plate with Hall current in a rotating system filled with a porous medium is studied. Exact solution of the energy equation is obtained in closed form by Laplace transform technique, and solution of momentum equation is obtained by defining a complex variable. The expression for the shear stress at the plate due to the primary and secondary flows is obtained. The results obtained are shown by graphs and table. Applications of the study arise in planetary and solar plasma fluid dynamics systems, rotating MHD induction machine energy generators, magnetic field control of materials processing systems etc.

KEYWORDS: Rotation, MHD, Porous medium, Hall current, Heat Transfer.

1. INTRODUCTION:

Free convection flow of incompressible viscous fluid past an infinite or semi-infinite vertical plate is studied since long because of its technological importance. To understand the behavior of the fluids in unsteady boundary layer a significant study was done by Steworten [15 & 16]. Similarity solution for a flow past over semi- infinite vertical plate was given by Ostrach [10]. Effect of magnetic field on such flow is of much significance in thermal insulation of buildings, enhanced recovery of petroleum products, geothermal energy extraction, sensible heat storage bed, plasma studies and on the performance of many engineering devices like, MHD energy generators, MHD accelerators, MHD pumps, MHD flow-meters, Plasma jet engines etc. Due to the importance of the study, many researchers worked on flow past on vertical plate with heat and mass transfer using different analytical and numerical methods. For instance, Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate was studied by Prasad et al. [11] and they solved the governing equations using an implicit finite-difference method of Crank–Nicolson type. And observed that, when the radiation parameter increases, the velocity and temperature decrease in the boundary layer. Chamkha et al. [2] find the similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through a porous medium. Soundalkar [14] analyzed the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate and his research shows that there is a rise in the velocity due to the presence of a foreign mass. But an increase in Sc (<1), Schmidt number, leads to a fall in the velocity. Further Mutthuswami et al. [9] studied the heat and mass transfer effect on flow past impulsively started vertical plate. Many other scholars [4, 5, 6, 7, 12 & 13] studied the effect of heat and mass transfer on impulsively started vertical flat plate in porous or non-porous medium under the influence of external transverse magnetic field. However, if we consider strong magnetic field, then the effect of Hall current can not be neglected. Due to abundant astrophysical and geophysical applications of rotating fluids (Greenspan [3]) many researches are carried out on the MHD fluid flow by taking different models of rotating fluids within porous or non-porous medium with Hall current, such as Agarwal et al. [1] analysed the combined influence of dissipation and Hall effect on free convective flow in a rotating fluid and they analysed that the primary shear stress increases and secondary shear stress decrease with increase in magnetic and hall parameters. Also, Mazumdar et al. [8] studied the hydrodynamic study flow with the effect of Hall current. The present study is carried out to examine the effects of Hall currents and rotation on an unsteady convective flow of a viscous, incompressible and electrically conducting fluid past a vertical oscillatory plate in a porous medium under the influence of transversely applied uniform magnetic. The problem is solved analytically using the Laplace Transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem is presented and discussed. The numerical values of skin-friction have been tabulated.

2. MATHEMATICAL ANALYSIS:

Consider an unsteady MHD flow of a viscous, incompressible, electrically conducting fluid past a vertical infinite flat plate with oscillation in a porous medium. Let *x*-axis be chosen along the plate in the direction of flow, the *z*-axis normal to the plate, the *y*-axis normal to the *x*-*z* plane and the plate is assumed to be lie in the *x*-*y* plane. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω about the *z*-axis. A uniform magnetic field B_o is applied along the *z*-axis and the plate is taken to be electrically non-conducting. Initially, at time $t \leq 0$, the fluid and the plate are at rest and at a uniform temperature T_∞. At time $t > 0$(small), the plate starts to oscillate with a velocity $u_o sin\omega t$ in its own plane and the temperature of the plate is raised to $T_\infty + (T_w - T_\infty)\frac{u_o^2}{\upsilon}t$. The fluid motion is induced due to the oscillation of the plate as well as the free convection due to heating of the plate. As the plate occupying the plane z = 0 is of infinite extent, all the physical quantities depend only on z and t only and hence continuity equation identically becomes zero. As the fluid is electrically conducting whose magnetic Reynolds number is very small and hence the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Also, the conservation of electric charge gives J_z (current density along *z* direction) constant. Since J_z = 0 at the plate which is assumed to be non-conducting and hence J_z can be assumed to be zero everywhere. So, under the above assumptions, the governing equations with Boussinesq's approximation are as follows:

$$\frac{\partial u}{\partial t} - 2\,\Omega\, v = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta(T - T_\infty) + \frac{B_o}{\rho} J_y - \frac{\upsilon}{K} u, \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\,\Omega\, u = \upsilon \frac{\partial^2 v}{\partial z^2} - \frac{B_o}{\rho} J_x - \frac{\upsilon}{K} v, \tag{2}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial z^2}. \tag{3}$$

The boundary conditions taken are as under:

$$\left.\begin{array}{l} t \le 0 : u = 0, v = 0, T = T_\infty \qquad \forall\ z, \\ t > 0 : u = u_o \sin \omega t, v = 0, T = T_\infty + (T_w - T_\infty)\dfrac{u_o^2}{\upsilon} t \ \ at\ z = 0, \\ u \to 0, v \to 0, T \to T_\infty \ \ as\ z \to 0. \end{array}\right\}$$

(4) where the symbols are: T — temperature of the fluid, T_∞ — temperature of the fluid far away from the plate, T_w —temperature at the plate, B_o — external magnetic field, u —primary velocity of the fluid along *x* direction, v —secondary velocity of the fluid along *y* direction, K–permeability parameter, β — volumetric coefficient of thermal expansion, α—thermal diffusivity, g— acceleration due to gravity, ρ— density of the fluid, υ —kinematic viscosity, m—Hall current parameter σ—Stefan-Boltzmann constant, J_x — current density along *x*-axis and J_y —current density along *y*-axis .

Taking Hall current into account and the electron pressure gradient (for partially ionised gas), the ion slip & the thermo-electric effects are neglected. As the plate is electrically non-conducting, so assuming there is no electric field. Hence by generalised Ohm's law, we have $J_x = \dfrac{\sigma B_o(v + mu)}{1 + m^2}, J_y = \dfrac{\sigma B_o(mv - u)}{1 + m^2}$ and equations (1) and (2) becomes

$$\frac{\partial u}{\partial t} - 2\,\Omega\, v = \upsilon \frac{\partial^2 u}{\partial z^2} + g\beta(T - T_\infty) + \frac{\sigma B_o^2}{\rho(1 + m^2)}(mv - u) - \frac{\upsilon}{K} u,$$

(5)

$$\frac{\partial v}{\partial t} + 2\,\Omega\, u = \upsilon \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_o^2}{\rho(1 + m^2)}(v + mu) - \frac{\upsilon}{K} v,$$

(6)

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$\left.\begin{array}{l} \bar{u} = \dfrac{u}{u_o}, \bar{v} = \dfrac{v}{u_o}, \bar{t} = \dfrac{u_o^2}{\upsilon} t, \bar{K} = \dfrac{u_o^2}{\upsilon^2} K, \bar{z} = \dfrac{u_o}{\upsilon} z, \theta = \dfrac{(T - T_\infty)}{(T_w - T_\infty)}, \\ \bar{\Omega} = \dfrac{\upsilon}{u_o^2}\Omega, \mathrm{P}_r = \dfrac{\upsilon}{\alpha}, \bar{\omega} = \dfrac{\upsilon}{u_o^2}\omega, M = \dfrac{\sigma B_o^2 \upsilon}{\rho u_o^2}, G_r = \dfrac{g\beta\upsilon(T_w - T_\infty)}{u_o^3} \end{array}\right\}$$

(7)

where $\bar{u}$ is dimensionless primary velocity of the fluid, $\bar{v}$ — dimensionless secondary velocity of the fluid, $\bar{z}$ —dimensionless spatial coordinate normal to the plate, ϑ— dimensionless temperature, P_r –Prandlt number G_r — Thermal Grashof number $\bar{t}$ —dimensionless time, $\bar{\Omega}$ —dimensionless rotation parameter, $\bar{\omega}$ —dimensionless frequency of oscillation, $\bar{K}$ — dimensionless porosity of the porous medium and M—magnetic field parameter. The equations (3), to (6) become:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - 2\,\bar{\Omega}\,\bar{v} = \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{M}{(1 + m^2)}(m\bar{v} - \bar{u}) + G_r\theta - \frac{\bar{u}}{\bar{K}},$$

(8)

$$\frac{\partial \bar{v}}{\partial \bar{t}}+2\,\bar{\Omega}\,\bar{u}=\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}-\frac{M}{(1+m^2)}(\bar{v}+m\bar{u})-\frac{\bar{v}}{\bar{\bar{K}}},$$

(9)

$$\frac{\partial \theta}{\partial \bar{t}}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial \bar{z}^2}$$

(10)

$$\left.\begin{aligned}&\bar{t}\le 0:\bar{u}=0,\bar{v}=0,\theta=0\quad(\forall \bar{z}),\\&\bar{t}>0:\bar{u}=\sin\overline{\omega t},\bar{v}=0,\theta=\bar{t}\quad(at\quad \bar{z}=0),\\&\bar{u}\to 0,\theta\to 0\quad(as\quad \bar{z}\to\infty).\end{aligned}\right\}$$

(11)

After dropping the bars, equations 8 –11 can be written as-

$$\frac{\partial u}{\partial t}-2\,\Omega\,v=\frac{\partial^2 u}{\partial z^2}+\frac{M}{(1+m^2)}(mv-u)+G_r\theta-\frac{u}{k},$$

(12)

$$\frac{\partial v}{\partial t}+2\,\Omega\,u=\frac{\partial^2 v}{\partial z^2}-\frac{M}{(1+m^2)}(v+mu)-\frac{v}{K},$$

(13)

$$\frac{\partial \theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}.$$

(14)

$$\left.\begin{aligned}&t\le 0:u=0,v=0,\theta=0\quad(\forall z),\\&t>0:u=\sin\omega t,v=0,\theta=t\quad(at\quad z=0),\\&u\to 0,\theta\to 0\quad(as\ z\to\infty).\end{aligned}\right\}$$

(15)

To solve above system, take $\mathbf{V}=u+iv.$ Then using equations (12) and (13), we get,

$$\frac{\partial \mathbf{V}}{\partial t}=\frac{\partial^2 \mathbf{V}}{\partial z^2}-b\mathbf{V}+G_r\theta,$$

(16)

where $b=\frac{Mi}{m+i}+\frac{1}{K}+2i\Omega,$

The boundary conditions (15) are reduced to:

$$\left.\begin{aligned}&t\le 0:\mathbf{V}=0,\theta=0\quad(\forall z),\\&t>0:\mathbf{V}=\sin\omega t,\theta=t\quad(at\quad z=0),\\&\mathbf{V}\to 0,\theta\to 0\quad(as\quad z\to\infty).\end{aligned}\right\}$$

(17)

The governing non-dimensional partial differential equations (14) and (16) subject to the above boundary conditions prescribed in equation (17) is solved using the Laplace Transform technique. The solution is as under:

$$\mathbf{V}(z,t)=2\alpha_1\left(-e^{i\omega t}\cosh(b_1z)+e^{-i\omega t}\cosh(b_2z)\right)-\alpha_1e^{-i\omega t}\left(e^{-b_2z}erf(\eta-b_2\sqrt{t})+e^{b_2z}erf(\eta+b_2\sqrt{t})\right)$$
$$+\alpha_1e^{i\omega t}\left(e^{-b_1z}erf(\eta-b_1\sqrt{t})+e^{b_1z}erf(\eta+b_1\sqrt{t})\right)-2(\alpha_3t-\alpha_2)\cosh(a_1z)$$
$$-(\alpha_3t-\alpha_2)\left(e^{-a_1z}erf(a_1\sqrt{t}-\eta)-e^{a_1z}erf(a_1\sqrt{t}+\eta)\right)-2\alpha_2e^{-tB_1}\cosh(a_1z)$$
$$+\alpha_2e^{-tB_1}\left(e^{-a_2z}erf(\eta-a_2\sqrt{t})+e^{a_2z}erf(\eta+a_2\sqrt{t})\right)+\alpha_4z\left(e^{-a_1z}erf(a_1\sqrt{t}-\eta)+e^{a_1z}erf(a_1\sqrt{t}+\eta)\right)$$
$$-2\alpha_4z\sinh(a_1z)-\alpha_5z\left(2e^{-a_4^2\eta^2}\sqrt{t}-za_4\sqrt{\pi}erfc(a_4\eta)\right)+2\alpha_2e^{-tB_1}\cosh(a_5z)$$
$$+\alpha_2e^{-tB_1}\left(e^{-a_5z}erf(a_6\sqrt{t}-a_4\eta)-e^{-a_5z}erf(a_6\sqrt{t}+a_4\eta)\right)+2\alpha_2(B_1t-1)erfc(a_4\eta)$$

(18)

$$\theta(z,t)=-\frac{e^{-P_r\eta^2}z\sqrt{P_rt}}{\sqrt{\pi}}+\frac{1}{2}erfc(\eta\sqrt{P_r})\left(2t+z^2P_r\right)$$

(19)

Skin Friction:

The dimensionless form of skin-friction components τ_x and τ_y are obtained as:

$$\tau_x+i\tau_y=-\left.\frac{\partial\mathbf{V}}{\partial z}\right|_{z=0}$$

3. RESULT AND DISCUSSION:

In order to get a physical insight of the problem, a representative set of numerical results is shown graphically in Figures 1–8. It is noticed from figures 1 to 7 that primary velocity *u* and secondary velocity *v* attain a distinctive maximum value near the surface of the plate and then decrease properly on increasing boundary layer coordinate *z* to approach free stream value. Figure 1 and 2 shows the influence of Hall current on primary and secondary velocities at the low rotation. Primary velocity *u* increases rapidly near the surface of the plate whereas secondary velocity *v* decreases throughout the boundary layer region on increasing Hall current parameter *m.* This shows that Hall current tends to accelerate primary velocity in the region near the surface of the plate whereas it tends to retard secondary velocity throughout the boundary layer region. It is also noticed that the influence of Hall current on fluid velocities is significant only if a strong magnetic field is applied. Effect of applied magnetic field can be seen from figures 3, 4 and 7 at different instant of time for different rotation. It is found that primary velocity decreases and secondary velocity increase as we increase magnetic field. This happens because a transverse applied magnetic field generates a Lorentz force perpendicular to the direction of primary flow and hence results in a decrease in the primary velocity and increase in secondary velocity. We can also see that this effect is rich as the time increases. Figures 5 and 6 depicts the effect of oscillation frequency on the fluid velocity at different instant of time and it is observed that both the component of velocity accelerated by the frequency of oscillation. On the other hand, figure-8 shows that the thermal boundary layer thickness increases with time. Also, the solution (18) obtained is valid only for $P_r \neq 1$. Prandtl number P_r is the dimensionless number which is the ratio of momentum diffusivity to the thermal diffusivity of fluid. For the case when $P_r = 1$ the momentum and thermal boundary layer thicknesses have values of the same order of magnitude and the fluids which belong to this category have some practical interest.

Table -1 and 2 shows the variation of skin friction coefficient with the magnetic field, Hall current and rotation parameter at the different oscillation frequency. It is found that the value of τ_x increases when the values of *M* and *Ω* are increased, while it gets decreased with increase in *m*. On the other hand the τ_y decrease with *M* and *Ω* and it is increased *m*.

4. CONCLUSION:

The conclusions are as follows: It is found that Hall current has a tendency to accelerate the fluid flow in the primary direction and retard the flow in the secondary direction. Magnetic field retards primary flow whereas it accelerates secondary flow. Also, the flows in both the directions are accelerated by the frequency of oscillation and the thickness of thermal boundary layer increases with time

- Skin friction coefficient-
    - τ_x increases when *M* and *Ω* are increased but it decreases with *m*.
    - τ_y increases when m is increased but it decreases if *M* and *Ω* are increased.

(Velocity and Temperature profiles at $P_r = 0.71, G_r = 5$ and $K = 0.5$)

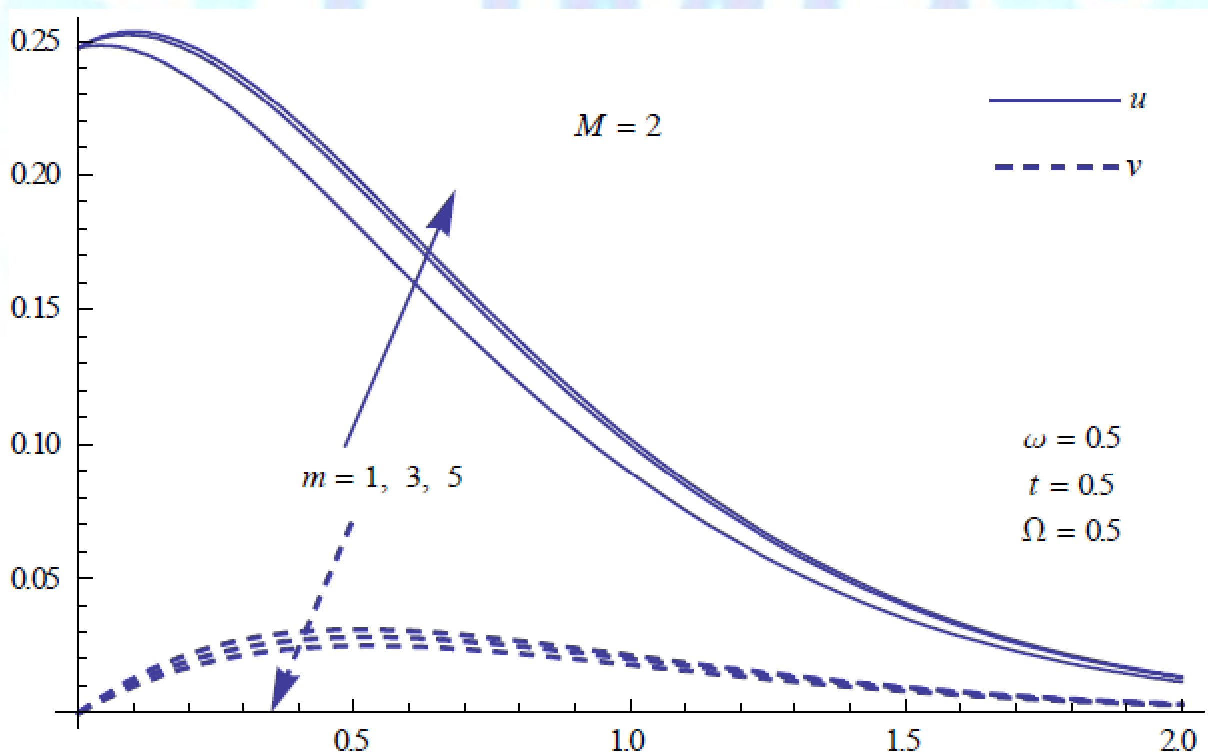

Fig.1: velocity profile for *m* at *M* = 2

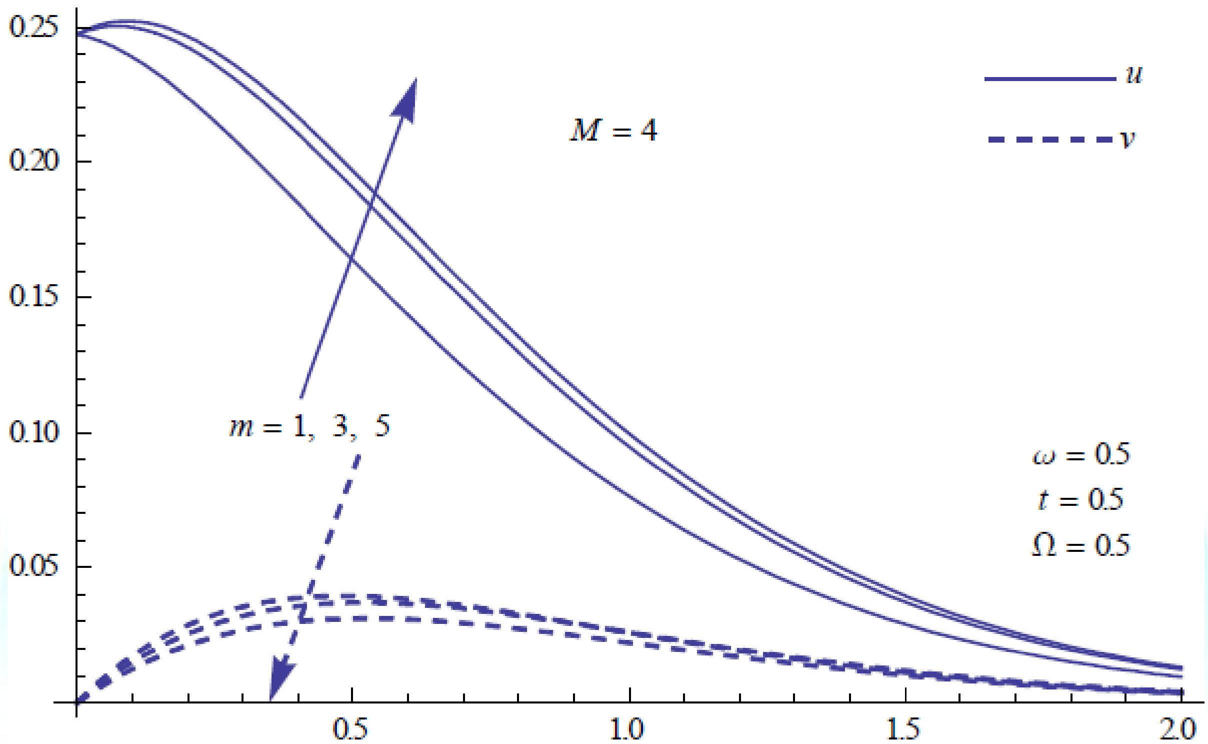

Fig.2: velocity profile for *m* at *M* = 4

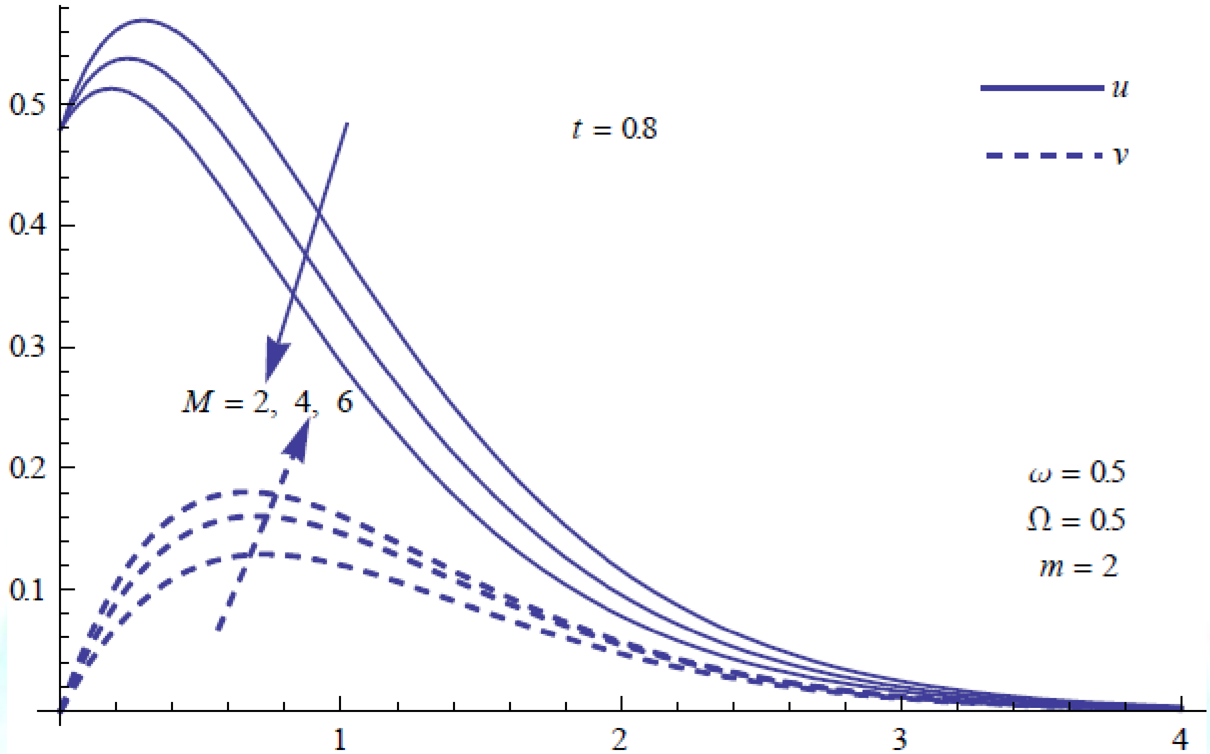

Fig.3: velocity profile for *M* at *t* = 0.8

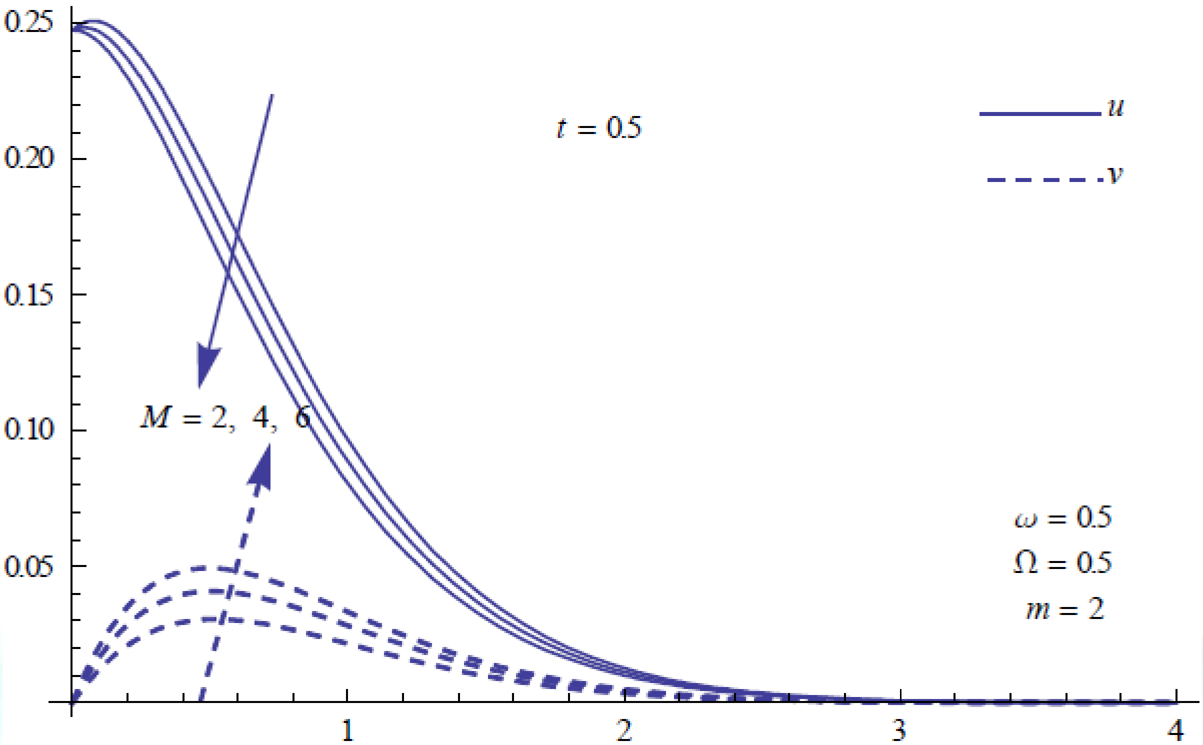

Fig.4: velocity profile for *M* at *t* = 0.5

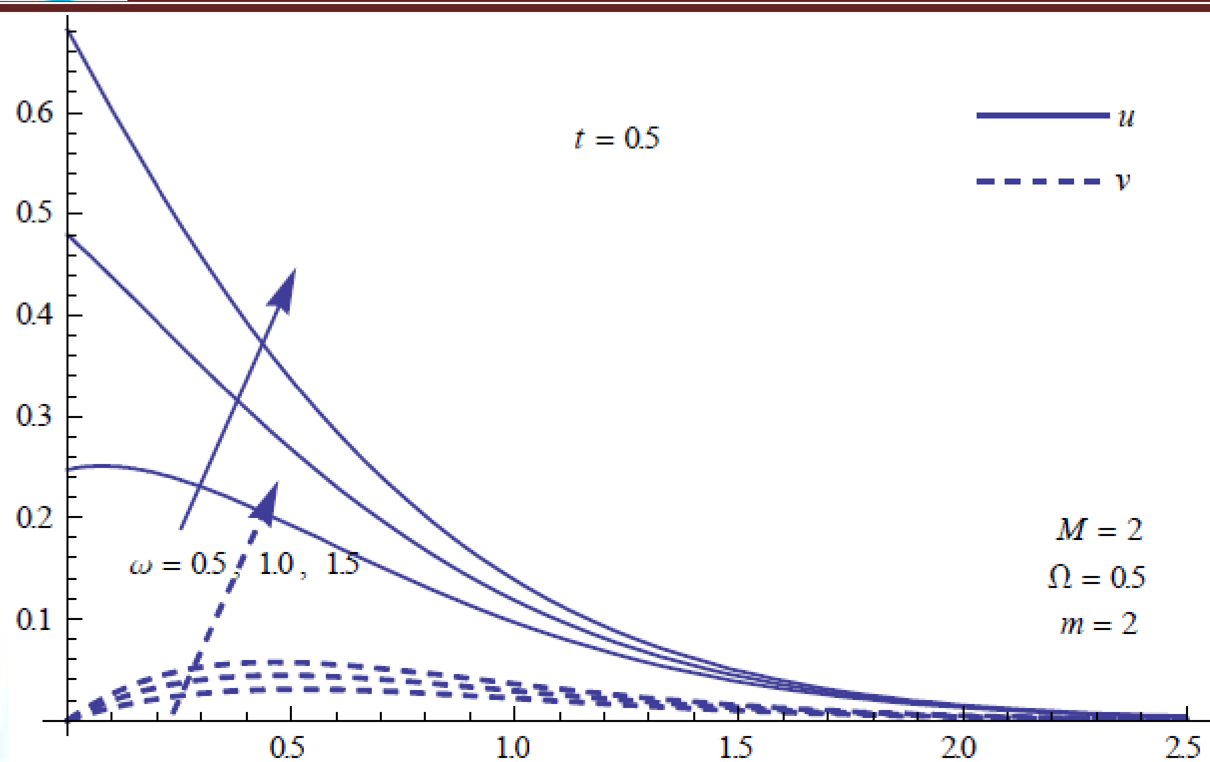

Fig.5: velocity profile for ω at t = 0.5

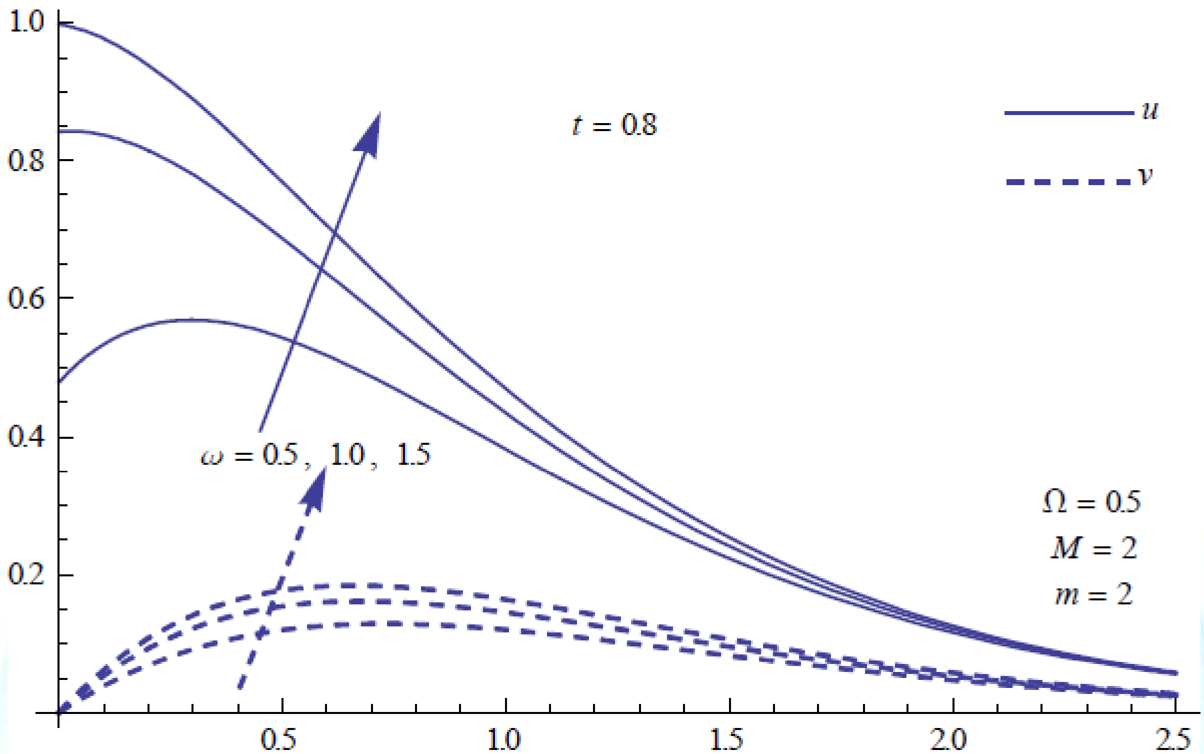

Fig.6: velocity profile for ω at t = 0.8

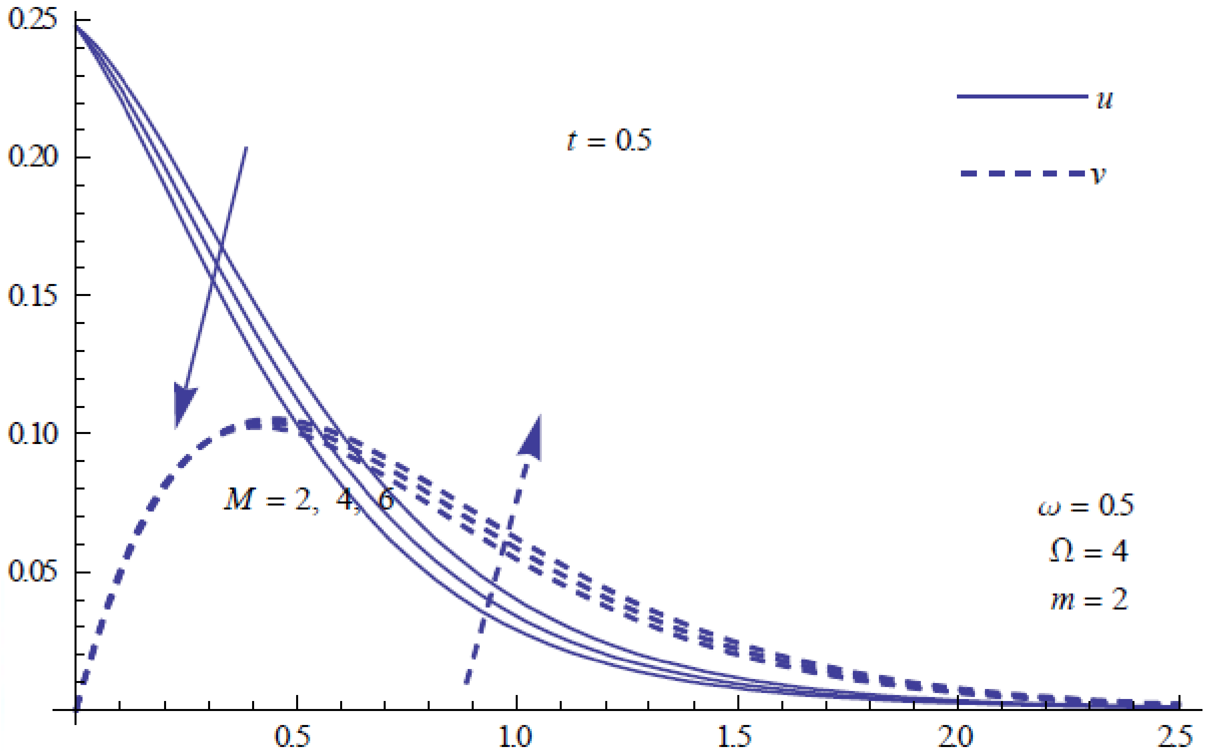

Fig.7: velocity profile for M at t = 0.5 and Ω = 4

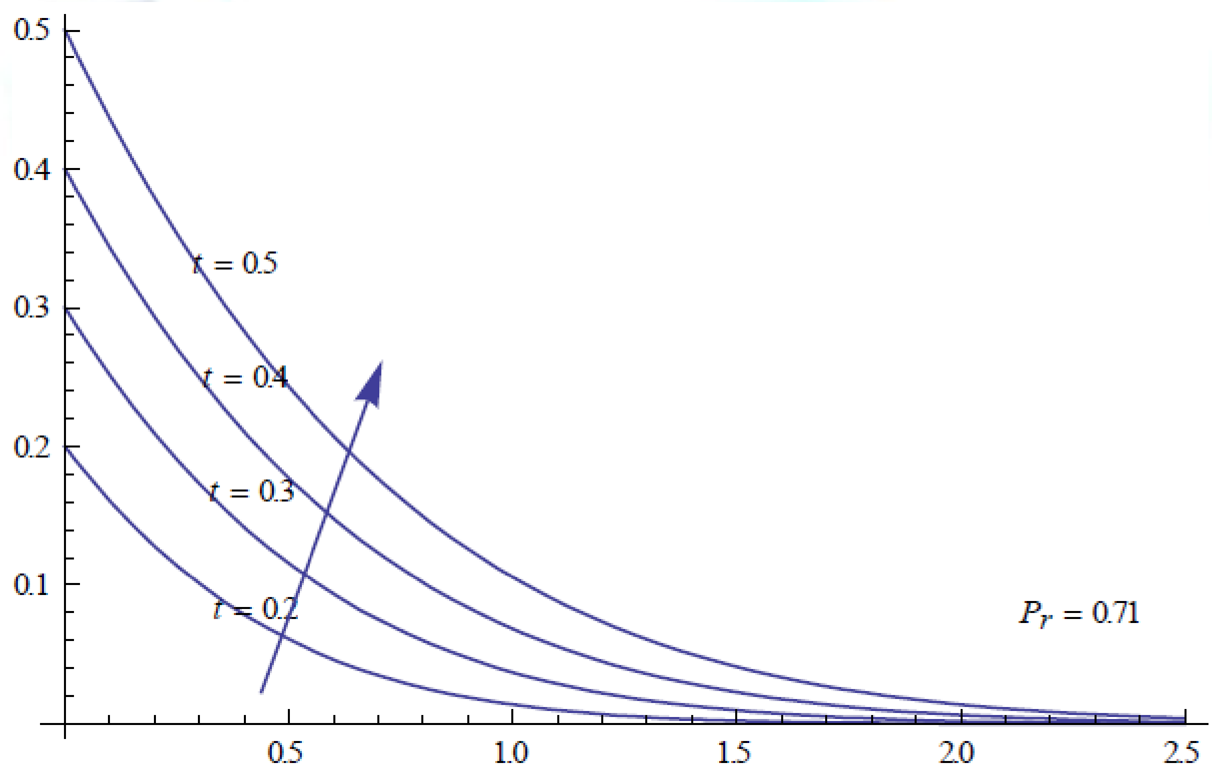

Fig.8: Temperature profile for *t*

Table -1

ω = 0.5	m	$\Omega = 0.5$		$\Omega = 1.5$	
		τ_x	$-\tau_y$	τ_x	$-\tau_y$
$M = 1$	1	0.1192	0.0281	0.1228	0.0651
	2	0.1134	0.0265	0.1171	0.0641
	3	0.1114	0.0247	0.1149	0.0624
$M = 3$	1	0.1388	0.0449	0.1435	0.8022
	2	0.1221	0.0409	0.1266	0.0776
	3	0.1159	0.0358	0.1202	0.0731

(Skin Friction coefficient at $P_r = 0.71, G_r = 5, t = 0.2$ and $K = 0.5$)

Table -2

ω = 1	m	$\Omega = 0.5$		$\Omega = 1.5$	
		τ_x	$-\tau_y$	τ_x	$-\tau_y$
$M = 1$	1	0.4071	0.0508	0.4135	0.1179
	2	0.3968	0.048	0.403	0.1159
	3	0.3932	0.0447	0.3992	0.1129
$M = 3$	1	0.4427	0.0814	0.4508	0.1456
	2	0.4122	0.0741	0.4202	0.1406
	3	0.4012	0.0648	0.4087	0.1322

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Appendix:

$$b=\frac{M\ i}{m+i}+2i\Omega+\frac{1}{K},\ A_1=\frac{G_r}{1-P_r},\ B_1=\frac{b}{1-P_r},a_1=\sqrt{b}\ ,$$

$$a_2=\sqrt{b-B_1},\ a_3=\sqrt{-P_r},\ a_4=\sqrt{P_r},\ a_5=\sqrt{-B_1P_r},\ a_6=\sqrt{-B_1},$$

$$b_1=\sqrt{b+i\omega},\ b_2=\sqrt{b-i\omega},\ \alpha_1=\frac{i}{4},\alpha_2=\frac{A_1}{2B_1^2},\alpha_3=\frac{A_1}{2B_1},\alpha_4=\frac{A_1}{4a_1B_1},\alpha_5=\frac{A_1\sqrt{P_r}}{2B_1\sqrt{\pi}},\eta=\frac{z}{2\sqrt{t}}.$$