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# Varieties of independent domination in a Fuzzy Graphs-I

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**Abstract:**

In this paper we introduce the concepts of end independent domination, Differentiation independent domination,locating independent domination, Irredundant domination, Irredundant independent domination in Fuzzy graphs. We determine end independent domination$\gamma_{eif}$(G ), Differentiation independent domination$\gamma_{dif}$(G ) ,locating independent domination$\gamma_{lif}$(G ), Irredundant domination$\gamma_{irf}$(G ), Irredundant independent domination$\gamma_{irif}$(G ).end independent domination $\gamma_{eif}$ (G).Some basic theorems related to the standard graphs have also been presented.

**Index terms:**

End independent domination, Differentiation independent domination ,locating independent domination, Irredundant domination, Irredundant independent domination, regular independent domination in Fuzzy graphs.

## 1. Introduction

A***fuzzy graph*** G = ($\sigma$,$\mu$) is a pair of function $\sigma$ :V→[0,1] and $\mu$:V ×V→[0,1], where $\mu$ (u ,v) ≤ $\sigma$(u) $\Lambda\sigma$ (v) for all u ,v $\in$V. The ***order*** *p* and ***size*** *q* of the fuzzy graph G = ($\sigma$ , $\mu$) are define by p = $\sum_{v\in V}\sigma(v)$and q = $\sum_{u,v\in E}\mu(u,v)$. The ***complement***of a fuzzy graph G = ($\sigma$ , $\mu$ ) where $\sigma$ = $\sigma$ and $\mu^c$ (u ,v) = $\sigma$(u) $\Lambda\sigma$ (v) -$\mu$ (u ,v) for all u ,v in V. The **fuzzy cardinality** of a fuzzy subset D of V is$|D|_f$= $\sum_{v\in D}\sigma(v)$. An edge e = {u , v} of a fuzzy graph is called an **effective edge** if $\mu$ (u ,v) = $\sigma$(u) $\Lambda\sigma$ (v). The **effective degree** of a vertex u is defined to be the sum of the weights of the effective edges incident at u and is denoted by $d_E(u)$. The **Minimum effective degree**$\delta_E$(G) = min{$d_E$(u) / u$\in$V(G)} and the **maximum effectivedegree**$\Delta_E$(G) = max{$d_E$(u) / u$\in$V(G)}.**Upper domination number** equals the maximum cardinality of a mimimaldominationg set of G.A graph G is called **well – covered** if every maximal independent set of vertices is also maximum. An independent set S is called a **maximal independent set** if any vertex set properly containing S is not independent. A set D of vertices in a graph G is **irredundant** if every vertex v D has atleast one private neighbour.Agraph G is said to be **regular** if every vertex of G has the same degree.The **girth g(G)** of a graph G is the length of a shortest cycle in G. The **vertex independence number $\beta_0$(G)** of G is the maximum cardinality among the independent sets of vertices.Let x , y $\in$V. We say that x **dominates** y in G if $\mu$ (u ,v) = $\sigma(u) \Lambda \sigma$ (v). A subset S of V is called a **dominating set** in G if for every v $\notin$S, there exists u $\in$ S such that u dominates v. The minimum cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma$(G).Let G = ($\sigma,\mu$) be a fuzzy graph. A subset D of V is said to be **fuzzy dominating set** of G if for every v$\in$V–D there exists u $\in$D such that (u, v) is a strong arc. A dominating set D of a graph G is called **minimal dominating set** of G if for every node v$\in$ D, D–{v} is not a dominating set of the domination number $\gamma$(G) is the minimum cardinalities taken over all minimal dominating sets of G.A set S $\subseteq$V in a fuzzy graph G is said to be **independent** if $\mu$ (u ,v) <$\sigma(u)\Lambda\sigma$ (v) for all u ,v $\in$ S. A dominating set is called an **independent dominating set** if D is independent. An independent dominating set S of a fuzzy graph G is said to be a **maximal independent dominating set** if there is no independent dominating set $S^1$ of G such that $S^1 \subset$ S. An independent dominating set S of a fuzzy graph G is said to be a **maximum independent dominating set** if there is no independent dominating set $S^1$ of G such that $|S^1|>|S|$.The minimum scalar cardinality of an maximum independent dominating set of G is called the **independent domination number** of G and is denoted by i(G).

$\gamma_f$ – fuzzy domination number

$\gamma_{if}$ – independent fuzzy domination number

$\gamma_{eif}$– end independent fuzzy domination number

$\gamma_{lif}$– locating independent fuzzy domination number

$\gamma_{rif}$– regular independent fuzzy domination number

$\gamma_{irif}$– irredundant independent fuzzy domination number.

## 2.**End independentdomination in fuzzy graphs**

**Definition:2.1**

An independent dominating set S is called an end independent dominating set of a fuzzy graph G if S contains all the nodes are end nodes in a fuzzy graph G. The end independent domination number$\gamma_{eif}$(G ) of G is the minimum cardinality of an end independent dominating set of G.

**Example:2.1**

Consider the fuzzy graph G($\sigma,\mu$) where $\sigma$ = {$v_1$/0.7,$v_2$/0.5,$v_3$/0.4,$v_4$/0.2,$v_5$/0.5,$v_6$/0.6} and $\mu$ = {($v_1$,$v_3$)/0.4,($v_2$,$v_3$)/0.4,($v_3$,$v_4$)/0.2,($v_4$,$v_5$)/0.2,($v_4$,$v_6$)/0.2}.S = { $v_1$ $v_2$ $v_5$,$v_6$}. $\gamma_{eif}$(G ) = 2.3.

**3.Differentiation independentdomintion in fuzzy grphs**

**Definition:3.1**

An independent dominating set S is called an differentiation independent dominating set of a fuzzy graph G if for every pair of nodes u and v in V, $N[u]\cap S\neq N[v]\cap S$ in a fuzzy graph G.TheDifferentiation independentdomination number$\gamma_{dif}(G)$ of G is the minimum cardinality of a differentiation independent dominating set of G.

**Example:3.1**

Consider the fuzzy graph $G(\sigma,\mu)$ where $\sigma = \{v_1/0.3,v_2/0.4,v_3/0.6,v_4/0.5,v_5/0.7\}$ and $\mu = \{(v_1,v_2)/0.3,(v_1,v_5)/0.3,(v_2,v_3)/0.4,(v_2,v_5)/0.4,(v_3,v_4)/0.5, ,(v_4,v_5)/0.5\}$.S = $\{ v_3, v_5\}$.$\gamma_{dif}(G) = 1.3$.

**4.Locating independent domination in fuzzy grphs**

**Definition:4.1**

An independent dominating set S is called a locating independent dominating set if for any two nodes $v,w \in V- S$, $N(v)\cap S\neq N(w)\cap S$ in a fuzzy graph G.The locating independent domination number$\gamma_{lif}(G)$of G is the minimum cardinality of a locating independentdominating set of G.

**Example:4.1**

Consider the fuzzy graph $G(\sigma,\mu)$ where $\sigma = \{v_1/0.7,v_2/0.2,v_3/0.4,v_4/0.3,v_5/0.6,v_6/0.5\}$ and $\mu = \{(v_1,v_2)/0.2,(v_1,v_3)/0.4,(v_3, v_5)/0.4,(v_2,v_4)/0.2,(v_4,v_6)/0.3\}$.S=$\{v_5,v_6,v_1\}$. $\gamma_{lif}(G) = 1.8$.

**Theorem:4.1**

For any fuzzy tree T, $\gamma_{lif}(T) > p/3$.

**Proof:**

Assume that G is a fuzzy graph with P nodes andatmost q – 1 edgesand let D be a node set of order $|D|\leq p-3$. If D is a$\gamma_{lif}(T)$- set then atmost p/ 3 nodes v of V-D have $|N_D(v)| = 1$.Thus,atleast p/3 nodes of V-D have $|N_D(v)|\geq 2$.But this implies that G has atleast q edges, a contradiction.

**Theorem:4.2**

If a fuzzy graph G has degree sequence $(d_1,d_2,\ldots\ldots.d_n)$ with $d_i \geq d_{i+1}$ , then$\gamma_{lif}(G)\geq \min\{k: (3k+d_1+d_2+\ldots\ldots\ldots+d_k)/2 \geq P\}$.

**Theorem:4.3**

If a fuzzy graph G is an r –regular graph,then$\gamma_{Iif}$(G )≥ 2p/( r+3).

**Theorem:4.4**

If a fuzzy graph G has girth g(G) ≥ 5,then G is well – covered if and only if every independent dominating set of G is locating.

**5.Irredundant domination in fuzzy graphs**

**Definition:5.1**

A set of nodes in a fuzzy graph G is called an irredundant dominating set if for every node v ∈S there exists a node w∈ N[v ]such that w∉ N[ S-{ v}].The irredundant domination number$\gamma_{irf}$(G) of G is the minimum cardinality of a Irredundant dominating set of G.

**Example:5.1**

Consider the fuzzy graph G(σ,μ) where σ = {$v_1$/0.7,$v_2$/0.6,$v_3$/0.5,$v_4$/0.8,$v_5$/0.3,$v_6$/0.4} and μ = {($v_1$,$v_2$)/0.6,($v_1$,$v_3$)/0.5,($v_1$,$v_4$)/0.7,($v_2$,$v_5$)/0.3,($v_2$,$v_3$)/0.5, ,($v_3$,$v_6$)/0.4,($v_4$,$v_6$)/0.4,($v_4$,$v_5$)/0.3}.D = {$v_1$,$v_4$} and $D_1$ = {$v_1$,$v_2$,$v_3$} are maximal irredundant sets.$\gamma_{irf}$(G) = 1.5, $\gamma_{IRf}$(G) = 1.8, $\gamma_f$(G) = 1.5.

**Theorem:5.1**

For any fuzzy graph G,$\gamma_{IRf}$(G)≤ p - δ(G)

**Proof:**

LetS be an irredundant set in G and let v∈ S.Assume that v is adjacent to K nodes in S. since the degreeof v isatleastδ(G), v must be adjacent to at least δ(G) – K nodes in V-S. If K=o then│V-S│ ≥δ(G),ie,│S│≤ p - δ(G), as required. If K > o, then each neighbor in V-S and these K nodes must be distinct.Hence│V-S│ ≥ (δ(G) – k ) + k = δ(G). ie, │S│≤ p -δ(G).

**6. Irredundant independent domination in fuzzy graphs**

**Definition:6.1**

An independent dominating set of nodes S in a fuzzy graph G is an irredundant independent dominating set if N[S -{v}]≠ N[S] for every node v ∈ S.The irredundant independent domination number$\gamma_{irif}$(G) of G is the minimum cardinality of a irredundant independent dominating set of G.**Theorem :6.1**

A dominating set D is a minimal dominating set in a fuzzy graph G if and only if it is independent dominating and irredundant.

**Proof:**

Let D be a minimal dominating set in a fuzzy graph G. Then every node v∈ D, there exists a node w ∈ V – (D - {v}).Therefore w is a private neighbor of v with respect to D. Hence for every node v∈ D has atleast one neighbour. Thus D is irredundant. Also D is an independent dominating set in a fuzzy graph G.

Conversely, suppose a set D is both independent dominating and irredundant in a fuzzy graph G. we now prove that D is minimal dominating set in G. Assume that D is not a minimal dominating set. Then it is sufficient to say that there exists a node v∈ D, for which D - {v} is not a dominating set in G. But since D is irredundant. pn[v,D]≠φ.Let w∈pn[v,D]. Then by definition, w is not adjacent to any node in D - {v} . Thus D - {v} not a dominating set, which is a contradiction. Hence D is a minimal dominating set in a fuzzy graph G.

**Proposition:6.1**

$\gamma_{irf}(G) \le \gamma_f(G) \le i_f(G) \le \beta_0(G) \le \Gamma(G) \le \gamma_{IRf}(G)$.

**Theorem:6.2**

For any fuzzy graph G, $P/(2\Delta(G)-1) \le \gamma_{irf}(G)$

**Theorem:6.3**

For any fuzzy graph G, $2P/(3\Delta(G)) \le \gamma_{irf}(G)$

**Example:6.1**

Consider the fuzzy graph G(σ,μ) where σ = {$v_1$/0.2,$v_2$/0.3,$v_3$/0.7,$v_4$/0.5,$v_5$/0.4,$v_6$/0.5,$v_7$/0.2,$v_8$/0.7} and

μ = {($v_1$,$v_2$)/0.2,($v_1$,$v_3$)/0.2,($v_2$,$v_3$)/0.3,($v_3$,$v_5$)/0.4,($v_3$,$v_6$)/0.5, ,($v_4$,$v_5$)/0.4}.S = { $v_3$, $v_4$,$v_8$}.$\gamma_{irif}$(G) = 1.9.

## 7. Regular Independent Domination in Fuzzy Graphs

**Definition:7.1**

An independent dominating set in a fuzzy graph G is said to be regular independent fuzzy dominating set if all the nodes of S has the same degree. The Regular independent domination number $\gamma_{rif}(G)$ of G is the minimum cardinality of a Regular independent dominating set of G.

**Definition:7.2**

A set S is maximal regular independent fuzzy dominating set if there does not exist node u in V-S such that $S \cup \{u\}$ is independent fuzzy dominating set.

**Theorem :7.1**

A regular independent fuzzy dominating set S is regular maximal independent fuzzy dominating set if and only if it is regular independent dominating set.

**Proof:**

Assume S is regular maximal independent fuzzy dominating set. Then for each $u \in V-S$ there is a node $v \in S$, such that u is adjacent to v. Hence S is dominating .Clearly S is regular independent. Hence S is regular independent fuzzy dominating set. Conversely suppose S is regular independent fuzzy dominating set. Claim: S is regular maximal independent. Suppose S is not maximal independent then there exist a node $u \in V - S$ for which $S \cup \{u\}$ is independent. If $S \cup \{u\}$ is independent then $u \in V-S$ is not adjacent to any node in S. Which is contradiction to S is dominating set. Hence S is maximal independent. Thus S is regular maximal independent set.

**Theorem :7.2**

Every regular maximal independent set is regular minimal dominating set.

**Proof:**

Let S be regular maximal independent fuzzy dominating set. Then by known theorem it is independent dominating set. So S is independent fuzzy dominating set. Claim: S is minimal. Suppose S is not minimal then there exists at least one node $v \in S$ such that S-{v} is dominating set. But if S-{v} dominating V-(S-{v}) then at least one node in S-{v} is adjacent to v which is contradiction to S is independent. So S is minimal. Hence S is regular minimal dominating set.

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