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MHD CONVECTIVE FLOW ALONG VERTICAL EXPONENTIALLY ACCELERATED PLATE WITH VARIABLE TEMPERATURE IN THE PRESENCE OF HALL CURRENT WITH ROTATION

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ABSTRACT

The present study is carried out to examine the unsteady MHD free convective flow along an infinite non-conducting vertical exponentially accelerated flat plate through a porous medium with Hall current in a rotating system. The dimensionless governing equations are solved by using Laplace transform method. The results obtained are discussed with the help of graphs. The numerical values of the shear-stress at the plate are shown in tables. It is observed that the flow pattern is affected significantly with plate acceleration, Hall current, and porous medium.

Keywords: Porous medium; MHD; Rotation; Hall Current.

1. INTRODUCTION

The flow past a semi-infinite flat plate is one of the classical problems in the fluid dynamics The application of MHD viscous incompressible flow through porous medium involving radiative heat transfer under the heat source have been found in many areas of science and engineering. Steworten [1, 2] had done significant study to understand the behaviour of the fluids in unsteady boundary layer. His study was completely based within the context of boundary layer equations. The influence of magnetic field on such a flow within porous and non-porous medium with thermal radiation and heat generation has significance in the designing of heat exchangers, MHD pumps, MHD generator, nuclear reactors, oil exploration, space vehicle propulsion etc. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate was studied by Prasad et al. [3]. They solved the governing equations using finite-difference method, and observed that the velocity of the flow decreases in the boundary layer when the radiation parameter is increased. Sharidan et al. [4] worked on the combined effects of radiative heat and mass transfer on unsteady MHD free convective flow in a porous medium past an infinite inclined plate with ramped wall temperature. They obtained analytical solutions for the velocity, temperature and concentration fields by using Laplace transform technique and they found that increasing the angle of inclination and radiation parameters, the fluid velocity along an inclined plate decreases. MHD flow of an uniformly stretched vertical permeable surface in the presence of heat generation/absorption and chemical reaction was studied by Chamka [5]. He solved the problem analytically and observed that the Prandlt number, Schmidt number and the strength of magnetic field retard the fluid velocity. Makinde and Mhone [6] analyzed the combined effects of a transverse magnetic field and radiative heat transfer to unsteady flow of a conducting optically thin fluid through a channel filled with saturated porous medium and non uniform wall temperature. They solved the problem analytically and used to compute rate of heat transfer and shear stress at the wall. Further, if the strength of applied magnetic field is very strong, the effect of Hall current is also significant. Also the rotating flow of viscous, incompressible and electrically conducting fluid has attracted attention of researchers due to their abundant geophysical and astrophysical applications. Many scholars have studied such models, for instance, Agarwal et al. [7] analyzed the combined influence of dissipation and

Hall effect on free convective flow in a rotating fluid. It was observed by Agarwal et al. [7] that the primary and secondary shear-stresses increases and decreases, respectively, with the increase in magnetic field and Hall parameters. Mazumdar et al. [8] worked on flow with heat transfer in the hydrodynamic Ekman layer on a porous plate with Hall effects. Further, Jaimala et al. [9] analysed the effect of magnetic field and Hall current on an electrically conducting couple-stress fluid layer heated from below; and they concluded that in the presence or absence of Hall current, magnetic field has a stabilizing effect on the thermal convection. In 2014, Reddy [10] worked on heat and mass transfer effects on unsteady MHD radiative flow of a chemically reacting fluid past an impulsively started vertical plate. He [10] observed that the presence of chemical reaction retards the fluid velocity, decreases the concentration and increases the thermal boundary layer thickness. Recently Mahmoudpour Molaei et al. [11] analysed the MHD free convection flow of a non-newtonian power-law fluid over a vertical plate with suction effects.

The model under consideration analyzes the combined effect of Hall current and rotation on an unsteady free convective flow past an exponentially accelerated infinite non-conducting vertical flat plate through a porous medium with heat source. The problem is solved analytically using the Laplace transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction have been tabulated.

2. MATHEMATICAL ANALYSIS

Consider an unsteady flow of a viscous, incompressible, electrically conducting fluid past an exponentially accelerated vertical infinite non-conducting flat plate in porous medium. Let $x'-$ axis be chosen vertically upward along the motion of the plate and the $z'-$axis normal to the plate. The fluid and the plate rotate as a rigid body with a constant angular velocity Ω' about $z'-$axis. A uniform magnetic field B_o is applied normal to the plate and the fluid is assumed to be electrically conducting whose magnetic Reynolds number is very small, so the induced magnetic field produced by fluid motion is negligible in comparison to the applied one, so $\vec{\mathbf{B}} = (0, 0, B_o)$. Also it is assumed that no applied and polarization voltage exist, so induced electric field $\vec{\mathbf{E}} = (0,0,0)$. As the plate is of infinite extent, therefore all physical variables depends only on z' and t'. Initially, at time $t' \le 0$, the fluid and the plate are at rest and at a uniform temperature T_o. At time $t' > 0$, the plate starts moving with a velocity $u_o e^{-c't'}$ in vertically upward direction and the temperature of the plate is raised to T_p. The fluid motion is induced due to the impulsive movement of the plate as well as the free convection. From the equation of conservation of electric charge $\nabla . \vec{\mathbf{J}}$, we have $J_{z'} =$ constant, where $\vec{\mathbf{J}} = (J_{x'}, J_{y'}, J_{z'})$ is the current density vector. Since the plate is assumed to be non-conducting therefore at the plate $J_{z'} = 0$. Thus $J_{z'} = 0$ everywhere in the fluid. So, under the above assumptions, the governing equations with Boussinesq's approximations are as follows:

$$\frac{\partial u'}{\partial t'} - 2\Omega'\ v' = \upsilon\frac{\partial^2 u'}{\partial z'^2} + g\beta(T - T_0) + \frac{B_o}{\rho}J_{y'} - \frac{\upsilon}{K'}u', \tag{1}$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \upsilon\frac{\partial^2 v'}{\partial z'^2} - \frac{B_o}{\rho}J_{x'} - \frac{\upsilon}{K'}v', \tag{2}$$

$$\frac{\partial T}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z'^2}, \tag{3}$$

The boundary conditions taken are as under:

$$\left.\begin{aligned} &t' \le 0 : u' = 0, v' = 0, T = T_o \quad \forall\ z', \\ &t' > 0 : u' = u_o\, \mathrm{e}^{-c't'} (c' > 0), v' = 0, T = T_o + (T_p - T_o)\frac{u_o^2}{\upsilon}t',\ at\ z' = 0, \\ &u' \to 0, v' \to 0, T \to T_o,\ as \quad z' \to \infty. \end{aligned}\right\} \tag{4}$$

Taking Hall current into account and neglecting the electron pressure gradient, the ion slip and the thermo-electric effects, the generalised Ohm's law can be written as-

$$\vec{\mathbf{J}} + \frac{\omega_e \tau_e}{B_o}(\vec{\mathbf{J}} \times \vec{\mathbf{B}}) = \sigma(\vec{\mathbf{E}} + \vec{\mathbf{q}} \times \vec{\mathbf{B}}) \tag{5}$$

On solving (5), we get $J_{x'} = \dfrac{\sigma B_o (v' + m\,u')}{1 + m^2}, J_{y'} = \dfrac{\sigma B_o (m\,v' - u')}{1 + m^2}$

Here the symbols used are: $T -$ temperature of the fluid, $T_o -$ temperature of the fluid far away from the plate, $T_p -$ temperature at the plate, $B_o -$ external magnetic field, , $u' -$ primary velocity of the fluid, $v' -$ secondary velocity of the fluid, $u_o -$ velocity of the Plate, $c' -$ exponential parameter, $K' -$ permeability parameter, $t' -$ time, $\beta -$ volumetric coefficient of thermal expansion, $\alpha -$ thermal diffusivity, $\omega_e -$ cyclotron frequency of electron, $\tau_e -$ electron collision time, $g -$ acceleration due to gravity, $\rho -$ density of fluid, $\upsilon -$ kinematic viscosity, $m(= \omega_e \tau_e) -$ hall parameter, $J_{x'} -$ current density along $x' -$ axis and $J_{y'} -$ current density along $y' -$ axis.

To obtain the equations in dimensionless form, the following non-dimensional quantities are introduced:

$$\left.\begin{aligned} &u = \frac{u'}{u_o},\ v = \frac{v'}{u_o},\ t = \frac{u_o^2}{\upsilon}t',\ \theta = \frac{(T - T_o)}{(T_p - T_o)}, \mathrm{P}_r = \frac{\upsilon}{\alpha},\ M^2 = \frac{\sigma B_o^2\ \upsilon}{\rho u_o^2}, \\ &\mathrm{c} = \frac{\upsilon}{u_o^2}\mathrm{c}', z = \frac{u_o}{\upsilon}z', \Omega = \frac{\upsilon}{u_o^2}\Omega',\ K = \frac{u_o^2}{\upsilon^2}K'\ , G_r = \frac{g\beta\upsilon(T_p - T_\infty)}{u_o^3}. \end{aligned}\right\} \tag{6}$$

Here u is dimensionless primary velocity of the fluid, $v -$ dimensionless secondary velocity of the fluid, $z -$ dimensionless spatial coordinate normal to the plate, $\theta -$ dimensionless temperature, $P_r -$ Prandlt number, $G_r -$ Thermal Grashof number, $t -$ dimensionless time, $\Omega -$ dimensionless rotation parameter and $M -$ magnetic field parameter.

Using equation (6), equations (1), (2), (3) and (4) respectively, become:

$$\frac{\partial u}{\partial t}-2\Omega\, v=\frac{\partial^2 u}{\partial z^2}+\frac{M^2}{(1+m^2)}(m\, v-u)+G_r\theta-\frac{u}{K}, \tag{7}$$

$$\frac{\partial v}{\partial t}+2\Omega\, u=\frac{\partial^2 v}{\partial z^2}-\frac{M^2}{(1+m^2)}(v+m\, u)-\frac{v}{K}, \tag{8}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial z^2}. \tag{9}$$

$$\left.\begin{array}{l} t\le 0: u=0,\ v=0,\ \theta=0,\quad \forall\, z,\\ t>0: u=e^{-ct},\ v=0,\ \theta=t,\quad at\ \ z=0,\\ u\to 0,\ v\to 0,\ \theta\to 0,\ \ as\ \ z\to\infty. \end{array}\right\} \tag{10}$$

To solve above system, assume $\mathbf{V}=u+iv$. Then using equations (7) and (8), we get,

$$\frac{\partial \mathbf{V}}{\partial t}=\frac{\partial^2 \mathbf{V}}{\partial z^2}-b\mathbf{V}+G_r\theta, \tag{11}$$

The boundary conditions (10) are transformed:

$$\left.\begin{array}{l} t\le 0: \mathbf{V}=0,\ \theta=0,\qquad \forall\, z,\\ t>0: \mathbf{V}=e^{-ct},\ \theta=t,\ at\ z=0,\\ \mathbf{V}\to 0,\ \theta\to 0,\qquad as\ z\to\infty. \end{array}\right\} \tag{12}$$

The governing non-dimensional partial differential equations (9) and (11) subject to the above boundary conditions prescribed in equation (12) are solved using the Laplace Transform technique. The solution is as under:

$$\begin{aligned} \mathrm{V}(z,t)=&\ a_1e^{-B_1t}\{-2cosh(a_3z)+e^{-a_3z}Erf\,(\eta-a_3\sqrt{t})+e^{a_3z}Erf\,(\eta+a_3\sqrt{t})\}-2(a_1-a_{11}t)cosh(a_7z)\\ &+(a_1-a_1B_1t+a_4z)e^{-a_2z}Erf\,(a_2\sqrt{t}-\eta)+(-a_1+a_1B_1t+a_4z)e^{a_2z}Erf\,(a_2\sqrt{t}+\eta)\\ &-(a_1-a_{11}t+a_6z)e^{-a_7z}Erf\,(-a_9\eta)+(a_1-a_{11}t-a_6z)e^{a_7z}Erf\,(a_8\eta)\\ &+a_1e^{-B_1t}\{2cosh(a_9z)+e^{-a_9z}Erf\,(a_{10}\sqrt{t}-a_8\eta)-e^{a_9z}Erf\,(a_{10}\sqrt{t}+a_8\eta)\}\\ &+2a_1(1-B_1t)\{cosh(a_2z)\}-\tfrac{1}{2}e^{a_5z-ct}Erf\,(a_5\sqrt{t}+\eta)+2a_6z\ sinh(a_7z) \end{aligned}$$

$$\theta(z,t)=a_{12}z\ sinh(a_7z)-(a_{13}z+a_{15}t\ )\mathrm{e}^{a_7z}\ Erf\,(a_8\eta)+a_{14}t\ cosh(a_7z)+(a_{15}t-a_{13}z\ )\mathrm{e}^{-a_7z}\ Erf\,(-a_8\eta)$$

The dimensionless skin-friction components τ_x, τ_y are obtained as: $\tau_x+i\tau_y=-\left.\frac{\partial \mathbf{V}}{\partial z}\right|_{z=0}$

$$\begin{aligned} &=(b_1-b_3-b_5t)Erf\,(a_2\sqrt{t})+b_6\sqrt{t}\\ &\qquad +e^{-B_1t}\left(b_6Erf\,(a_{10}\sqrt{t}-b_2Erf\,(a_3\sqrt{t}\right)+e^{-bt}\left(\tfrac{1}{\sqrt{\pi t}}-b_4\sqrt{t}\right)+a_5e^{-ct}Erf\,(a_5\sqrt{t}) \end{aligned}$$

3. RESULTS AND DISCUSSION

In order to explain the significance of the study a representative set of numerical results for different parameters involved is shown graphically in Figures 1 to 6. It is noticed from figures 1 to 4 that magnitude of secondary velocity *v* attains a distinctive maximum value near the surface of plate and then decreases properly while primary velocity *u* decreases continuously on increasing boundary layer coordinate *z* to approach free stream value. Figures 1 and 2 shows the effect of acceleration parameter c on the fluid velocity at different time (t = 0.3 and t = 0.6). And it is observed that both the components of velocity decrease with c but as the time increases the rate of decrease becomes comparatively slow. Figure 3 show the effect of rotation parameter Ω on the velocity and it is analysed that primary velocity decreases and secondary velocity increases with Ω. Also the rate of increase becomes high as time increases. The effect of Hall parameter can be seen from Figures 4, which shows that the primary component of velocity increases and the secondary component of velocity decreases with increase in *m*. Figures 5 and 6 depict the temperature variation near the plate and it can be seen that from Figures 5 that at a particular time the thermal boundary layer thickness decreases with P_r and it increases with time (Figure 6).

The effects of various parameters on the skin-friction are shown in tables 1 and 2. It is found from table -1, that the value of τ_x increases when the value of M is increased (keeping other parameters fixed) but if values of *m* and *K* is increased, it gets decreased. Also, it is observed that τ_y increases with *m* and it decreases when *M* and *K* are increased. From the table 2 it can be seen that τ_x increases when the values of Ω is increased while it gets decreased with c. On the other hand τ_y increases with c. And it decreases when Ω is increased.

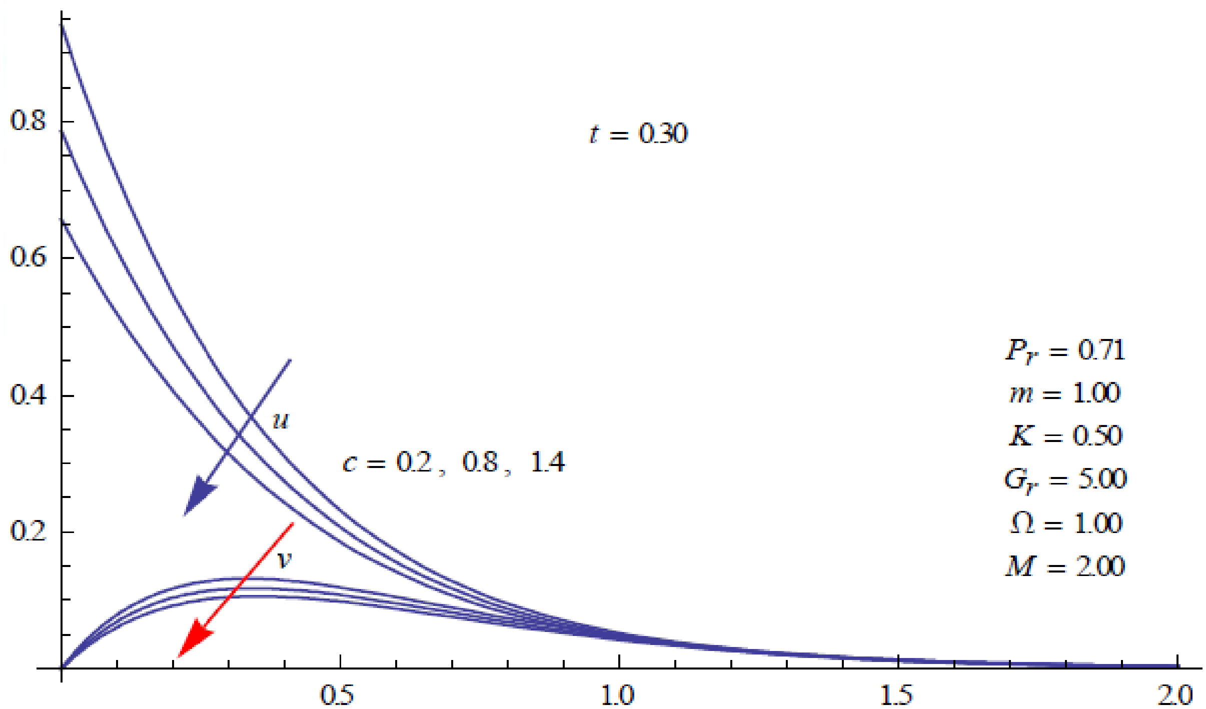

Fig.1.velocity for different values of c at t = 0.3

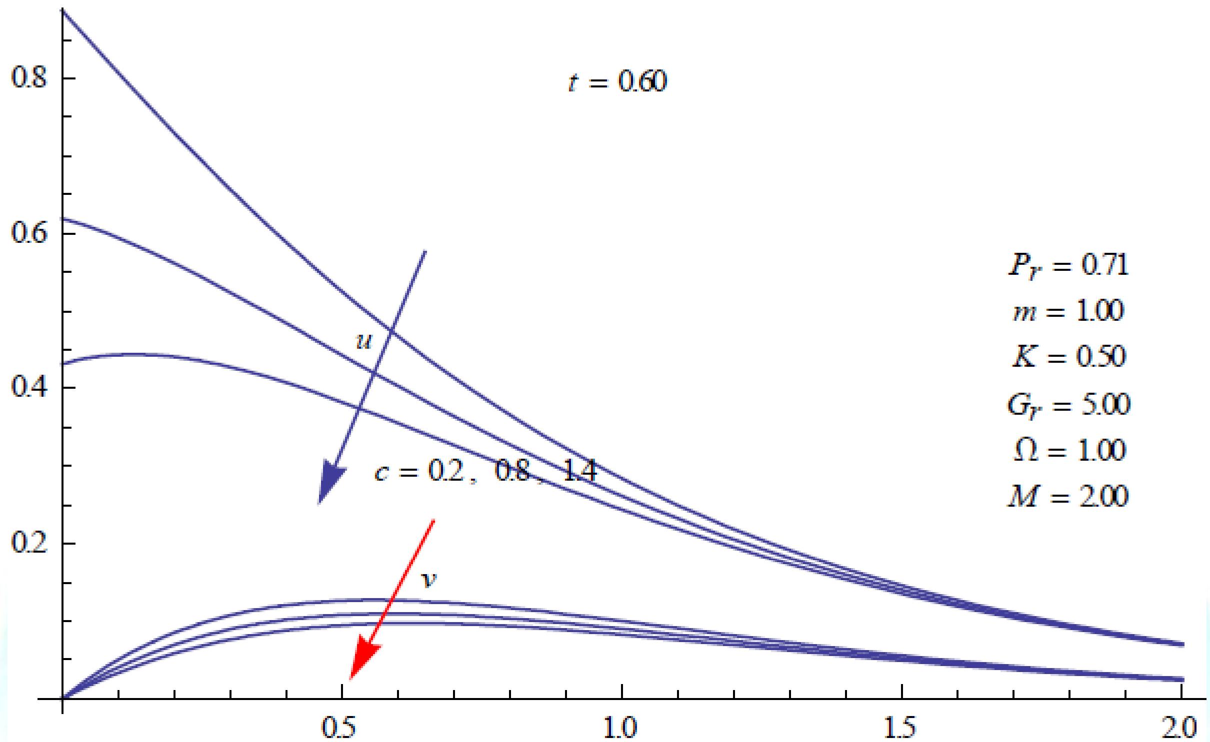

Fig.2.velocity for different values of c at t = 0.6

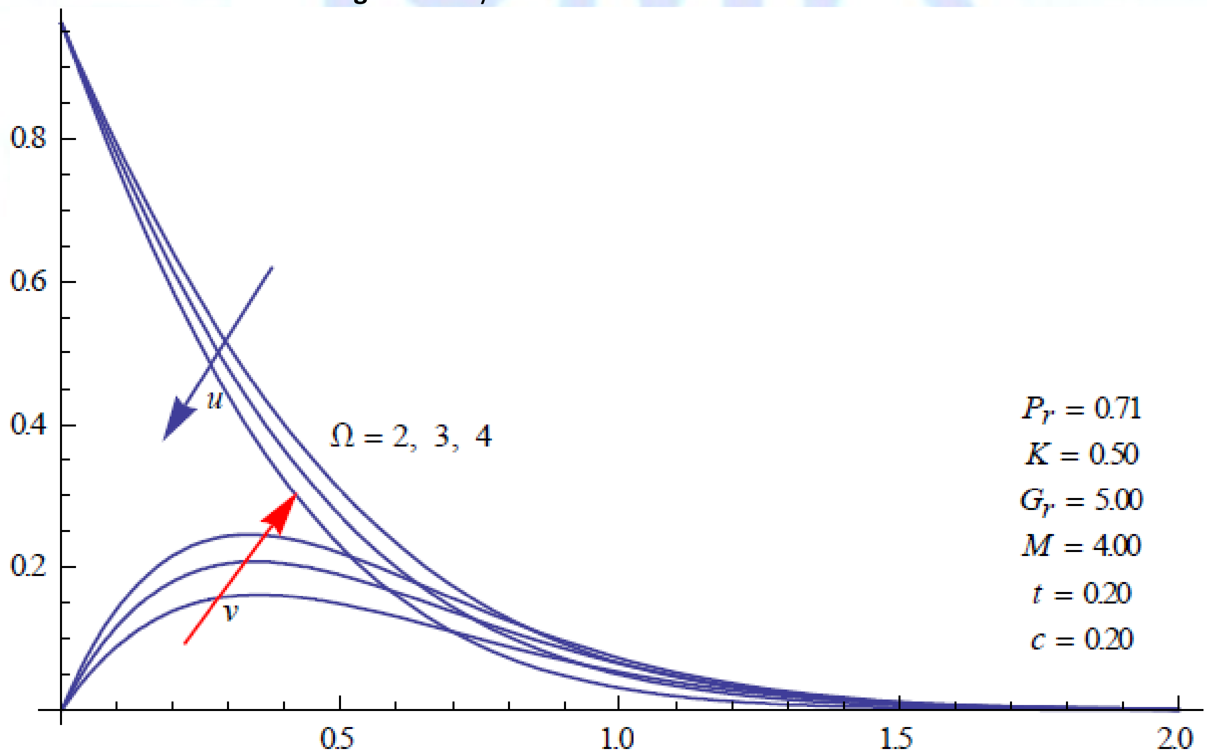

Fig.3.velocity for different values of Ω

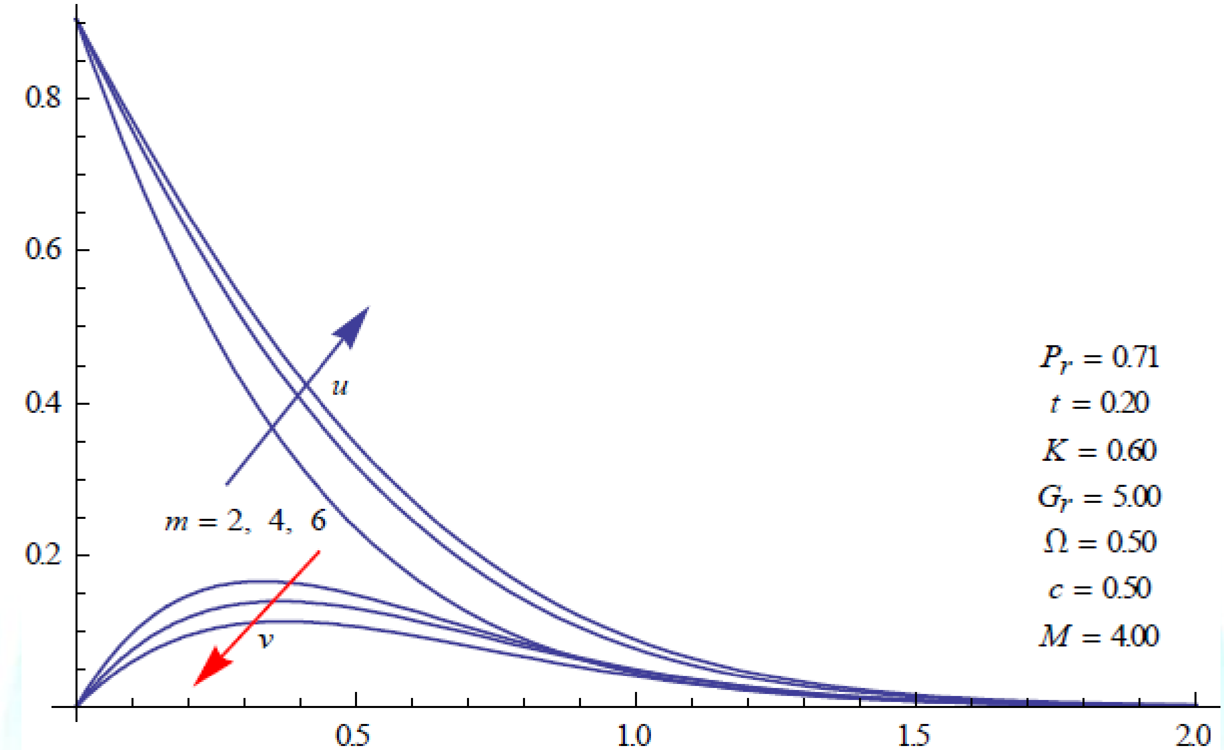

Fig.4.velocity for different values of *m*

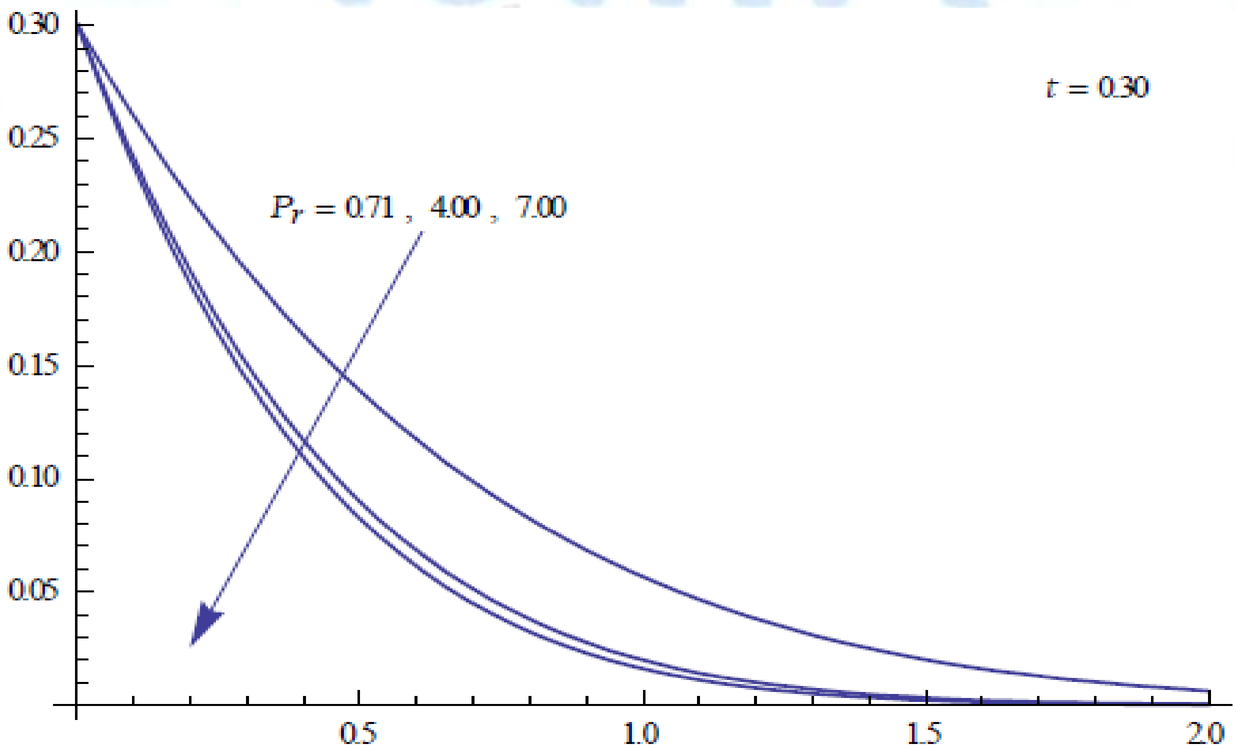

Fig.5. θ for different values of P_r

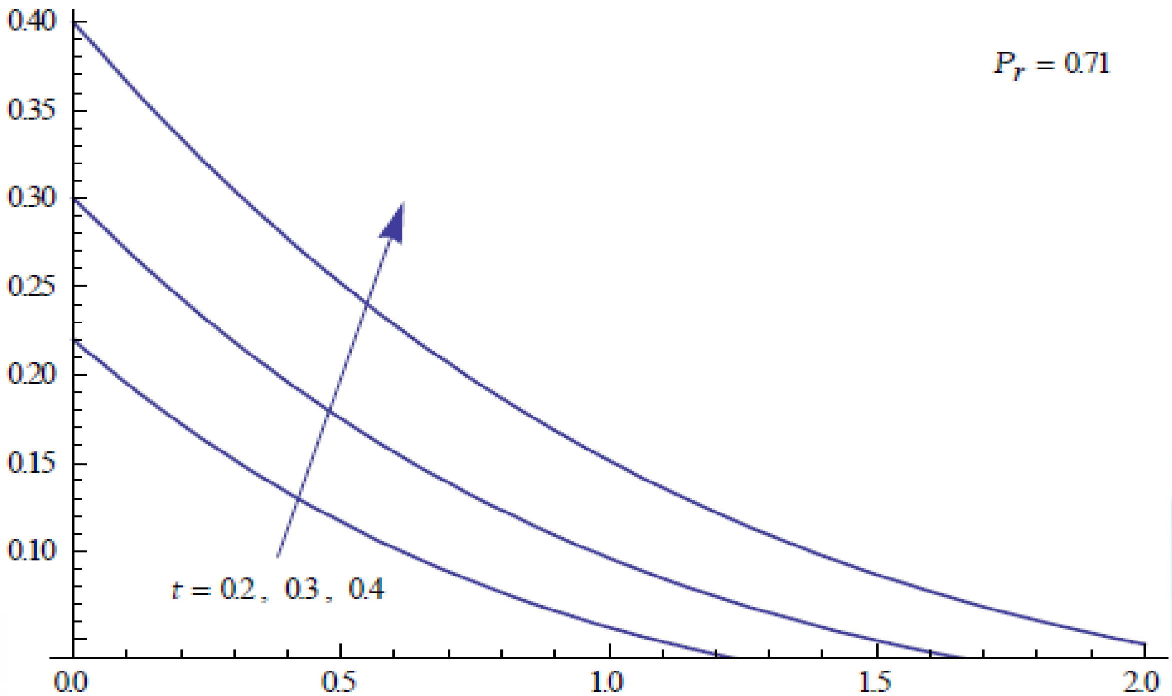

Fig.6. θ for different values of t

Table.1. Skin Friction for different values of m

t = 0.2	m	K = 0.3		K = 0.5	
		τ_x	$-\tau_y$	τ_x	$-\tau_y$
M = 2	1	1.951	0.523	1.713	0.561
	2	1.726	0.484	1.473	0.521
	3	1.642	0.419	1.381	0.453
M = 3	1	2.425	0.839	2.226	0.891
	2	1.971	0.801	1.739	0.859
	3	1.779	0.681	1.532	0.732

Table.2. Skin Friction for Ω

t = 0.2	Ω	τ_x	$-\tau_y$
c = 0.2	0.2	1.871	0.469
	0.4	1.888	0.585
	0.6	1.901	0.662
c = 2.0	0.2	0.997	0.365
	0.4	1.012	0.455
	0.6	1.023	0.514

4. CONCLUSION

It is found that Hall current has tendency to accelerate the flow in the primary direction whereas it retard the secondary flow. At a particular time as the parameter c increases the momentum boundary layer thickness decreases. The rotation parameter retards the primary flow whereas it accelerates the secondary flow. The skin-friction increases with the increase in rotation parameter and it decreases with increase in *m* and *c*. The results obtained will have applications in the research related to the solar physics dealing with the sunspot development, the solar cycle, the structure of rotating magnetic stars, and geophysics.

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APPENDIX

$$b=\frac{M^2 i}{m+i}+2i\Omega+\frac{1}{K},A_1=\frac{G_r}{1-P_r},B_1=\frac{b+P_r}{1-P_r},a_1=\frac{A_1}{2B_1^2},a_2=\sqrt{b},a_3=\sqrt{b-B_1},a_4=\frac{A_1}{4B_1\sqrt{b}},$$

$$a_5=\sqrt{b-c},a_6=\frac{A_1\sqrt{P_r}}{4B_1},a_7=\sqrt{P_r},a_8=\sqrt{P_r},a_9=\sqrt{-B_1P_r},a_{10}=\sqrt{B_1},a_{11}=\frac{A_1}{2B_1},a_{12}=\frac{P_r}{2a_9},$$

$$a_{13}=\frac{P_r}{4a_9},a_{14}=\frac{a_7}{a_9},a_{15}=\frac{a_7}{2a_9},b_1=2a_1a_2,b_2=2a_1a_3,b_3=\frac{a_{11}}{a_2},b_4=\frac{2a_{11}}{\sqrt{\pi}},b_5=2a_{11}a_2,$$

$$b_6=2a_1a_8a_{10},b_7=\frac{2a_{11}a_8}{\sqrt{\pi}},\eta=\frac{z}{2\sqrt{t}}.$$