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MHD BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY SHRINKING VERTICAL SHEET WITH SUCTION

A. K. JHANKAL
Department of Mathematics
Army Cadet College, Indian Military Academy, Dehradun – 248 007, INDIA

ABSTRACT

An analysis is made to study the problem of steady laminar two-dimension MHD boundary layer flow and heat transfer of an incompressible viscous fluid in the presence of buoyancy force over an exponentially shrinking vertical sheet with suction. The shrinking velocity and wall temperature are assumed to have specific exponential function forms. The governing partial differential equations are transformed into self-similar ordinary differential equations using similarity transformations, which are then solved numerically using Runge-Kutta-Fehlberg fourth-fifth order method. The numerical results are plotted in some figures and the variations in physical characteristics of the flow dynamics and heat transfer for several parameters involved in the equations are discussed. It is found that these parameters have significantly effects on the flow and heat transfer.

Keywords: MHD boundary layer flow, heat transformation, shrinking sheet, similarity transformation, numerical study.

NOMENCLATURE

B_0	Constant applied magnetic field
C_p	Specific heat at constant pressure
Gr	Grashof number
f	Dimensionless stream function
g	Acceleration due to gravity
L	Characteristic length of the sheet
M	Magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
Re	Reynold number
S	Wall mass transformation parameter, S>0
T	Temperature of the fluid
U	Characteristic velocity
u, v	Velocity component of the fluid along the x and y directions, respectively
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	Thermal diffusivity
β	Thermal expansion coefficient
η	Similarity variable
λ	Mixed convection/Buoyancy parameter
ρ	Density of the fluid

μ Viscosity of the fluid
Ψ Stream function
σ_e Electrical conductivity
υ Kinematic viscosity
θ Dimensionless temperature

Superscript
$'$ Derivative with respect to η

Subscripts
w Properties at the plate
∞ Free stream condition

1. INTRODUCTION

The study of boundary layer flow over a shrinking sheet has generated much interest in recent years in view of its significant applications in industrial manufacturing such as glass-fibber and paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning etc. In his pioneering work, Sakiadis [29] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [7] extended the work of Sakiadis [29] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by number of researchers [5, 6, 14, 23, 25].

The boundary layer flow of an incompressible viscous fluid over a shrinking sheet has received considerable attention of modern day researchers because of its increasing application to many engineering systems. Wang [32] first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. Later, Miklavcic and Wang [19] obtained an analytical solution for steady viscous hydrodynamic flow over a permeable shrinking sheet. Then, Hayat et al. [12] derived both exact and series solution describing the magnetohydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. The problem of stagnation flow towards a shrinking sheet was studied by Wang [32]. Nadeem and Awais [21] studied thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity. Viscous flow over an unsteady shrinking sheet with mass transfer was studied by Fang and Zhang [9]. The problem ofboundary layer flow over a exponentially stretching sheet was studied by researcher [1, 4, 8, 15, 16, 27, 30]. However, for the case of exponentially shrinking sheet there is still only a few studies.Rohni et al. [26] solved the problem where shrinking velocity and wall temperature are assumed to have specific exponential function forms. Fang and Zhang [10] solved the Full N-S equation analytically for two dimensional MHD viscous flow due to a shrinking sheet. Fang and Zhang [11] investigated the heat transfer characteristics of the shrinking sheet problem with a linear velocity. Later on, Noor et al. [22] studied the MHD viscous flow due to shrinking sheet using Adomian Decomposition Method (ADM) and they obtained a series solution. Sajid and Hayat [28] applied homotopyanalysis method for the MHD viscous flow due to a shrinking sheet. Midya [17] studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction was studied numerically by Muhaimin et al. [20].

Midya [18] obtained a closed form analytical solution for the distribution of reactant solute in a MHD boundary layer flow over a shrinking sheet.

Motivated by works mentioned above and practical applications, the main concern of the present paper is to study the problem of MHD boundary layer flow over a vertical shrinking sheet with suction involving boundary conditions of exponential velocity and temperature distribution. The numerical results are plotted in some figures and the variations in physical characteristics of the flow dynamics and heat transfer for several parameters involved in the equations are discussed. It is found that these parameters have significantly effects on the flow and heat transfer.

2. FORMULATION OF THE PROBLEM

Let us consider the problem of steady two-dimensional laminar MHD flow past a permeable vertically moving sheet with exponential velocity towards the origin and an exponential surface temperature. The fluid is an electrically conducting incompressible viscous fluid. The x and y are the coordinate along and normal to the shrinking sheet respectively. The velocity of the shrinking sheet is $u_w(x)$, the free stream temperature $T_\infty$is constant, the surface temperature is $T_w(x)$, where $T_w(x) > T_\infty$ corresponds to a heated sheet (assisting flow) and $T_w(x) < T_\infty$ corresponds to a cooled sheet (opposing flow). $B_0$is the externally imposed magnetic field in the y-direction. The induced magnetic field effect is neglected for the small magnetic Reynolds number flow. It is also assumed that the external electric field is zero, the electric field owing to polarization of charges and the Hall effect are neglected. Incorporating the Boussineq's approximation within the boundary layer, the governing equations of continuity, momentum and energy(Pai [24], Schlichting [31], Bansal [2]) under the influence of externally imposed transverse magnetic field (Jeffery [13], Bansal [3]) are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{...(1)}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma_e B_0^2}{\rho}u \qquad \text{...(2)}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \qquad \text{...(3)}$$

Along with the boundary conditions are:

$$y = 0: u = u_w(x) = -U_w \exp\left(\frac{x}{L}\right), v = v_w(x), \qquad T = T_w(x) = T_\infty + T_0 \exp\left(\frac{2x}{L}\right)$$

$$y \to \infty: u \to 0, T \to T_\infty \qquad \text{...(4)}$$

The stream function Ψ is defined by $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$, therefore, the continuity equation is automatically satisfied.Invoking the following similarity variables:

$$\eta = y\sqrt{\frac{U_w}{2\nu L}}\exp\left(\frac{x}{2L}\right), f(\eta) = \frac{\Psi}{\sqrt{2\nu L U_w}\exp(x/2L)}, \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \qquad \text{...(5)}$$

Where η is the independent similarity variable. The transformed nonlinear ordinary differential equations are:

$$f''' + ff'' - 2f'^2 + 2\lambda\theta - 2Mf' = 0 \qquad \text{...(6)}$$

$$\frac{1}{Pr}\theta'' + f\theta' - 4f'\theta = 0 \qquad \text{...(7)}$$

The transformed boundary conditions are:

$$f(0) = S, \ f'(0) = -1, \theta(0) = 1 \ \text{and} f'(\infty) \to 0, \theta(\infty) \to 0. \qquad \text{...(8)}$$

Where prime denotes differentiation with respect to, $M=\frac{\sigma_e B_0^2}{\rho U_w(x)\exp(x/L)}$ is the dimensionless magnetic parameter, $Pr=\frac{\mu c_p}{\kappa}$ is the Prandtl number, $\lambda=\frac{Gr}{Re^2}$ is the buoyancy parameter, $Gr=\frac{g\beta T_0 L^3}{\nu^2}$ is the Grashof number, and $Re=\frac{U_w L}{\nu}$ is the Reynolds number. It is worth mentioning that λ>0 corresponds to an assisting flow where the plate is heated, λ<0 corresponds to the opposing flow where the plate is cooled and finally, λ=0 corresponds to forced convection flow $(T_w=T_\infty)$i.e. non-buoyant case.

3. RESULTS AND DISCUSSION

The system of coupled ordinary nonlinear differential equations (6) and (7) along with boundary conditions (8) has been solved numerically using Runge-Kutta-Fehlberg forth-fifth order method. To solve these equations we adopted symbolic algebra software Maple. Maple uses the well knownRunge-Kutta-Fehlberg forth-fifth order (RKF45) method to generate the numerical solution of a boundary value problem. The boundary conditions η = ∞ were replaced by those at η = 2 in accordance with standard practice in the boundary layer analysis. Numericalcomputations of these solutions have been carried out to study the effect of various physical parameters when Pr=1.

For λ = 0:The velocity profiles for various values of magnetic parameter M are depicted in figures 1 to 3. From the figures it is noted that for a fixed values of η, velocity profiles increase as magnetic parameter M increases and it makes the momentum boundary layer thickness thinner. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. To reduce momentum boundary layer thickness the generated Lorentz force enhances the fluid motion in the boundary layer region. On the other hand, from figures 4 to 6, it is noticed that temperature θ(η) at a point decreases with magnetic parameter.

Now we see the effects of mass suction parameter S on the velocity and temperature profiles. The velocity profiles (figures 1 to 3) increase as applied suction increases and it makes the momentum boundary layer thickness thinner. This is because suction implies an increase in skin friction coefficient which is caused by the reduction momentum boundary layer thickness. The dimensionless temperature profiles for several values of suction parameter are demonstrated in figures 4 to 6. It is seen that the wall mass suction affects temperature distribution in addition to the velocity field, with increasing suction the temperature θ(η) for fixed η decreases and consequently, the thermal boundary layer thickness reduces.

For λ = 1:The velocity profiles for various values of magnetic parameter M are depicted in figures 7 to 9. Fluid velocity increases due to increase in magnetic parameter M for η<1, while reverse effect is observed when η>1. Further, from figures 10 to 12, it is noticed that temperature θ(η) at a point decreases with magnetic parameter. It is also observed from figures 7 to 9 velocity profile increases as mass suction parameter S increases for η<1, while reverse effect is observed when η>1. On the other hand, temperature decreases with increasing values of S.

For λ = -1: Figures 13 to 18 which illustrate the effect of magnetic parameter M on velocity and temperature profiles. We infer from these figures that for a given S the velocity increases with the increasing values of M but reversed phenomenon is observed for temperature. Similar results occur for increasing values of mass suction parameter S.

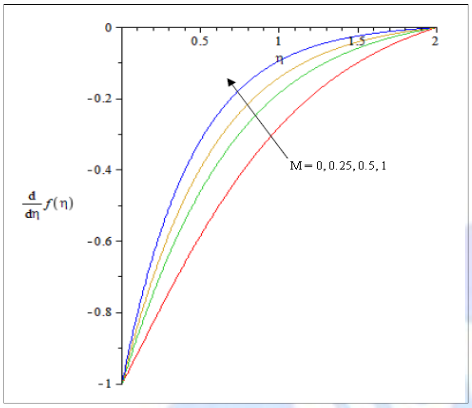

Figure 1: Effect of M on the velocity profiles S=2, λ=0 andPr=1.

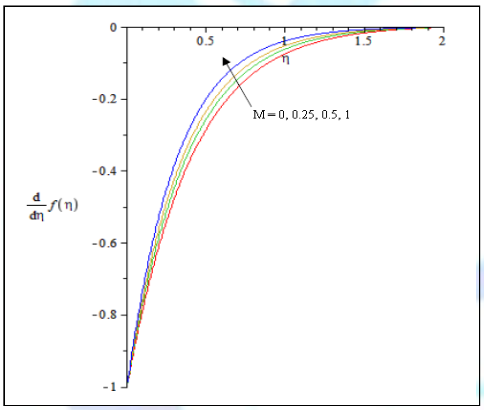

Figure 2: Effect of M on the velocity profiles for S=3, λ=0 andPr=1.

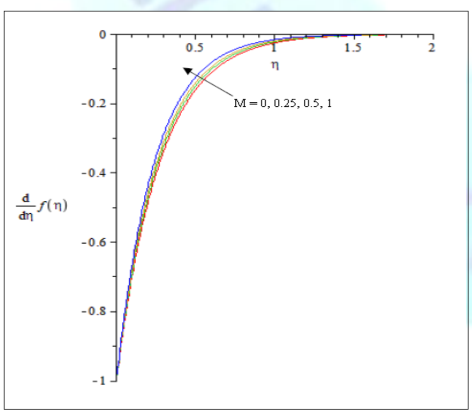

Figure 3: Effect of M on the velocity profiles S=2, λ=0 andPr=1.

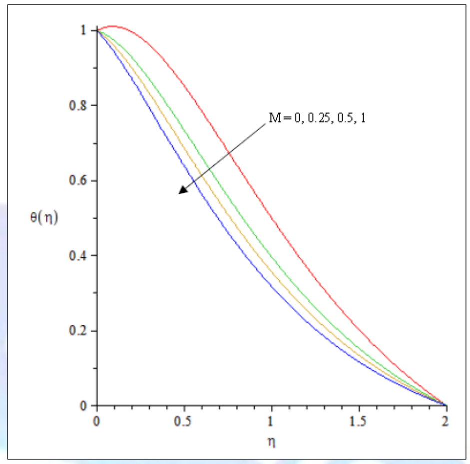

Figure 4: Effect of M on the temperature profiles for S=2, λ=0 and Pr=1.

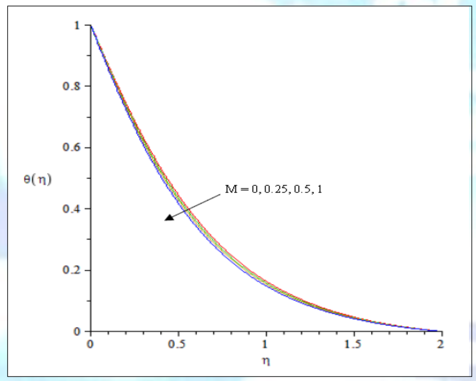

Figure 5: Effect of M on the temperature profiles for S=3, λ=0 and Pr=1.

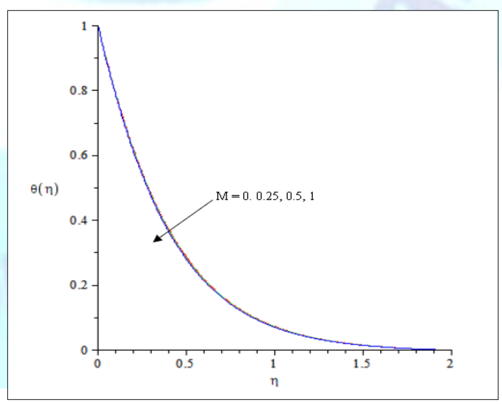

Figure 6: Effect of M on the temperature profiles for S=4, λ=0 and Pr=1.

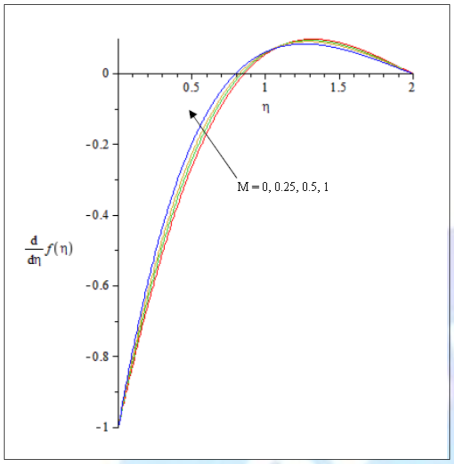

Figure 7: Effect of M on the velocity profiles for S=0.5, λ=1 andPr=1.

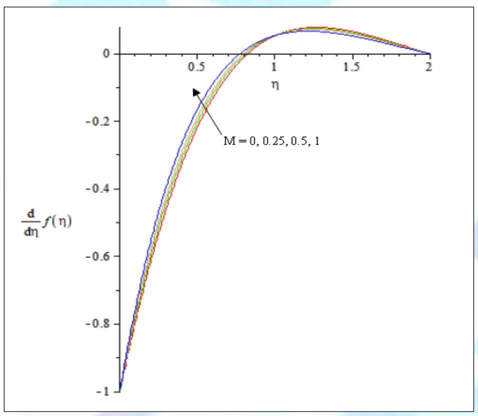

Figure 8: Effect of M on the velocity profiles for S=1, λ=1 andPr=1.

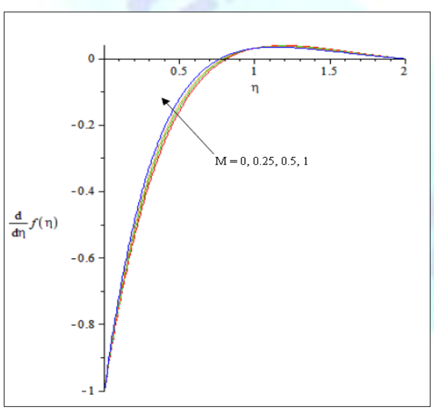

Figure 9: Effect of M on the velocity profiles for S=2, λ=1 andPr=1.

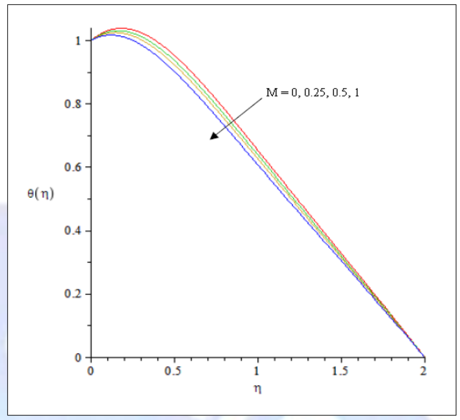

Figure 10: Effect of M on the temperature profiles for S=0.1, λ=1 and Pr=1.

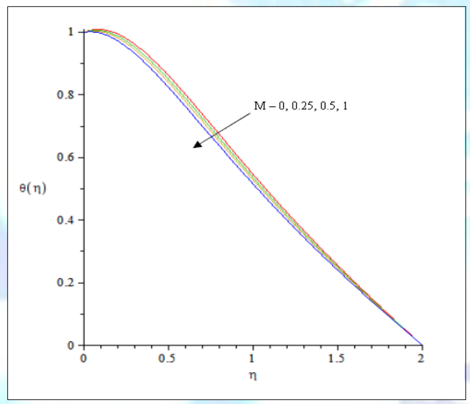

Figure 11: Effect of M on the temperature profiles for S=0.5, λ=1 and Pr=1.

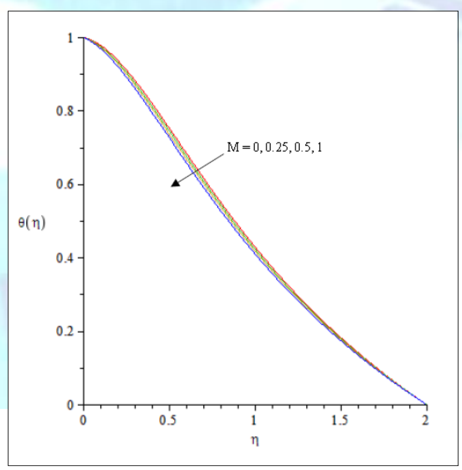

Figure 12: Effect of M on the temperature profiles for S=1, λ=1 and Pr=1.

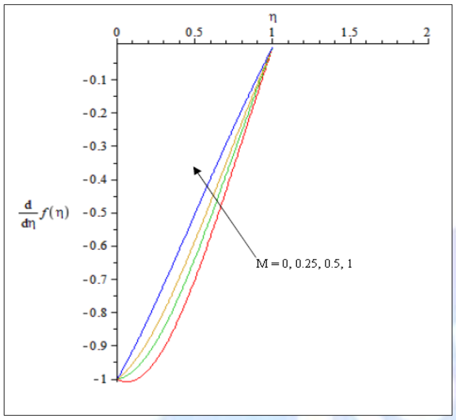

Figure 13:Effect of M on the velocity profiles for S=1, λ=-1 andPr=1.

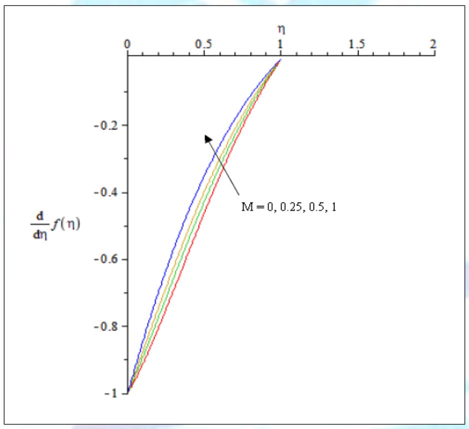

Figure 14: Effect of M on the velocity profiles for S=2, λ=-1 andPr=1.

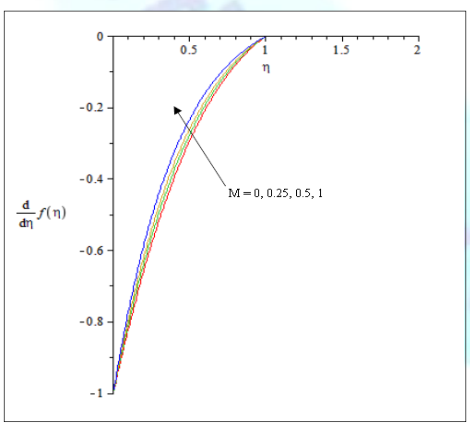

Figure 15: Effect of M on the velocity profiles for S=3, λ=-1 andPr=1.

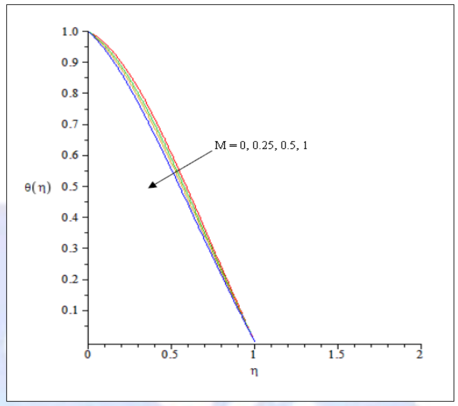

Figure 16: Effect of M on the temperature profiles for S=1, λ=-1 and Pr=1.

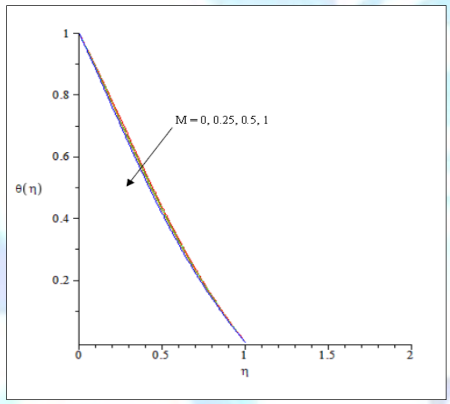

Figure 17: Effect of M on the temperature profiles for S=2, λ=-1 and Pr=1.

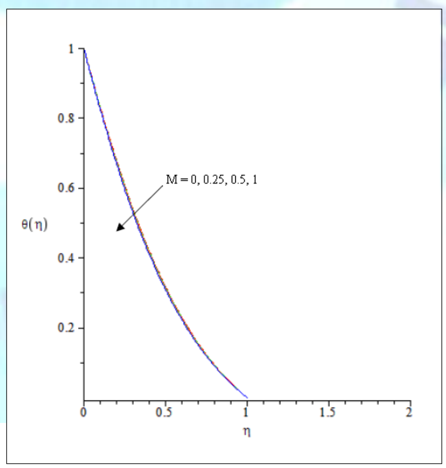

Figure 18: Effect of M on the temperature profiles for S=3, λ=-1 and Pr=1.

4. CONCLUSION

A mathematical model has been presented for the MHD boundary layer flow and heat transfer over an exponentially shrinking vertical sheet with suction. The governing partial differential equations are converted into ordinary differential equations by using similarity transformations. The effect of magnetic parameter M and mass suction parameter S controlling the velocity and temperature profiles for Pr=1 are shown graphically and discussed briefly. The influence of the physical parameters viz., the Magnetic parameter M, and the mass suction parameter S on dimensionless velocity and temperature profiles were examined. From the study, following conclusion can be drawn:

(i) As the magnetic parameter M increases, we can find the velocity profile increases in the flow region (but reversed phenomenon is observed when $\lambda=1$ and $\eta>1$) and to decrease the temperature profile. This happens due to the Lorentz force arising from the interaction of magnetic and electric fields during the motion of the electrically conducting fluid. Thus we conclude that we can control the velocity field and temperature by introducing magnetic field.

(ii) The velocity gradient increases with increasing values of mass suction parameter S. Since the effect of suction is to suck away the fluid near the wall, the velocity boundary layer is reduced due to suction. Consequently the velocity increases. Hence the velocity gradient increases with increasing suction.

(iii) The temperature in the boundary layer decreases due to suction. The effect of suction can be used as means of cooling.

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