ANALYTIC STUDY OF A SYSTEM WITH CONCURRENT OPERATION OF STANDBY UNITS AND MAINTENANCE FACILITY

Upasana Sharma[*1], Gunjan Sharma[*2]

[*]**Department of Statistics, Punjabi University, Patiala, India**

ABSTRACT:

a*The present paper deals with the stochastic model under which there is one main unit and two cold standby units. The maintenance facility is provided in it in order to bring out smooth functioning of the system. Initially, there is one main unit and two cold standby units. The main unit goes under maintenance solely but both the cold standby units go under maintenance together. When both main and standby unit stops functioning, the system goes to failed state. There is one repairman for repair as well as maintenance purposes. The reliability and profit analysis has also been done for the model. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique.*

KEYWORDS: *Standby systems; semi Markov process; Regenerative point technique.*

INTRODUCTION

Reliability can be defined as the ability of a component or machine to perform its intended function under given circumstances and within specified time interval. Maintenance is intended to improve or maintain the functioning of the system or component. The standby systems play a vital role in the field of reliability engineering. Reliability models have been extensively used by various researchers under different situations [1-7]. Most of the studies deal with the systems having standby units, so that the system may function efficiently with the operation of standby unit under the situation of failure in the main unit. The literature still lacks behind considering the situation where maintenance is provided to the system in order to keep the system efficient with its smooth functioning. Our motive is to study this situation and thus filling the gap.The system comprises of one main unit and two cold standby units. Initially, the main unit is in operative state and other two units in the cold standby state. The main or standby units may go under maintenance as per requirement of the system. There is only one repairman available to do the job of repair as well as maintenance. Various measures of system effectiveness such as MTSF and Profit are obtained using semi Markov process and Regenerative point technique. The graphical interpretation has also been done for the present study.

NOTATIONS

λ	Constant failure rate of main unit (Unit 1)
λ_1/ λ_2	Constant failure rate of cold standby units (Unit 2/3)
α_1	Constant rate of Unit 1 (main unit) to go under maintenance
α_2	Constant rate of Unit 2 and 3 (bothl of the standby units) to go under maintenance
g(t)/ G(t)	pdf/ cdf of repair time of the main unit at failed state (Unit 1)
$g_1(t)$/ $G_1(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 2)
$g_2(t)$/ $G_2(t)$	pdf/ cdf of repair time of the standby unit at failed state (Unit 3)
$h_1(t)$/ $H_1(t)$	pdf/ cdf of maintenance time of the main unit (Unit 1)
$h_2(t)$/ $H_2(t)$	pdf/ cdf of maintenance time of standby units together (Unit 2,3)
O_I/ O_{II}/ O_{III}	Unit 1/2/3 is in operative state
CS_{II}/ CS_{III}	Unit 2/3 is in cold standby state
$U_{mI}/U_{mII}/U_{mIII}$	Unit 1/2/3 is under maintenance respectively
$U_{MI}/U_{MII}/U_{MIII}$	Unit 1/2/3 is under maintenance respectively from the previous state, i.e., maintenance is continuing from previous state
$F_{rI}/F_{rII}/F_{rIII}$	Unit 1/2/3 is under repair respectively
$F_{wrI}/F_{wrII}/F_{wrIII}$	Unit 1/2/3 is waiting for repair respectively
$F_{RI}/F_{RII}/F_{RIII}$	Unit 1/2/3 is under repair respectively from the previous state, i.e., repair is continuing from previous state

TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram in fig. 1 shows various transitions of the system. The epochs of entry into states 0,1,2,3,9 and 10 are regenerative points and thus these are regenerative states. The states 4,5,6,7 and 8 are failed states.

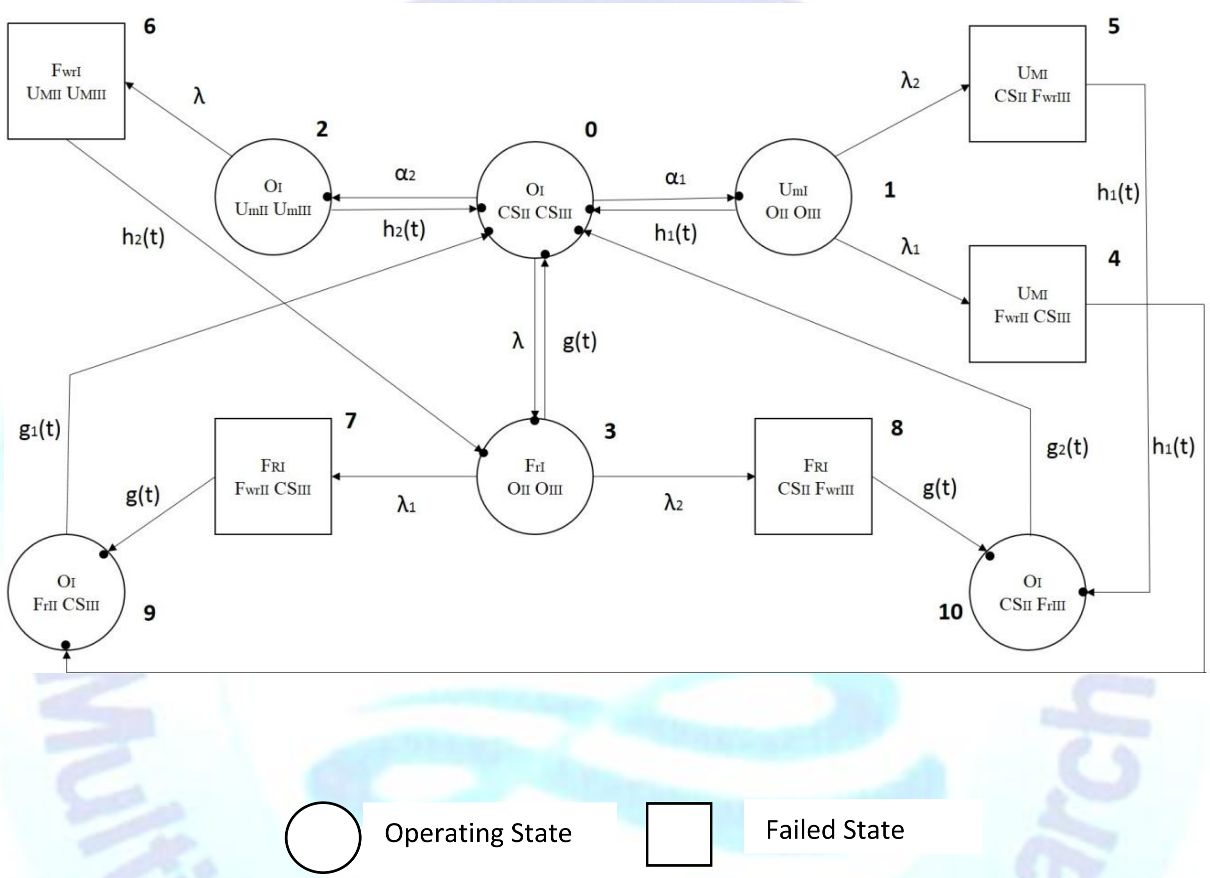

Fig. 1

The non-zero elements p_{ij} , are obtained as under :

$$p_{01} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \lambda} \qquad p_{02} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \lambda}$$

$$p_{03} = \frac{\lambda}{\alpha_1 + \alpha_2 + \lambda} \qquad p_{10} = h_1^*(\lambda_1 + \lambda_2)$$

$$p_{14} = \frac{\lambda_1[1 - h_1^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{19}^{(4)} \qquad p_{15} = \frac{\lambda_2[1 - h_1^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{1,10}^{(5)}$$

$$p_{20} = h_2^*(\lambda) \qquad p_{26} = 1 - h_2^*(\lambda) = p_{23}^{(6)}$$

$$p_{30} = g^*(\lambda_1 + \lambda_2) \qquad p_{37} = \frac{\lambda_1[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{39}^{(7)}$$

$$p_{38} = \frac{\lambda_2[1 - g^*(\lambda_1 + \lambda_2)]}{\lambda_1 + \lambda_2} = p_{3,10}^{(8)} \qquad p_{49} = h_1^*(0) = p_{5,10}$$

$$p_{63} = h_2^*(0) \qquad p_{79} = g^*(0) = p_{8,10}$$

$$p_{90} = g_1^*(0) \qquad p_{10,0} = g_2^*(0)$$

By these transition probabilities, it can be verified that

$$p_{01} + p_{02} + p_{03} = 1$$

$$p_{10} + p_{14} + p_{15} = 1 \qquad p_{10} + p_{19}^{(4)} + p_{1,10}^{(5)} = 1$$

$$p_{20} + p_{26} = 1 \qquad p_{20} + p_{23}^{(6)} = 1$$

$$p_{30} + p_{37} + p_{38} = 1 \qquad p_{30} + p_{39}^{(7)} + p_{3,10}^{(8)} = 1$$

$$p_{49} = p_{5,10} = p_{63} = 1 \qquad p_{79} = 1$$

$$p_{8,10} = 1 = p_{90} = p_{10,0}$$

The unconditional mean time taken by the system to transit for any regenerative

state j, when it is counted from epoch of entrance into that state i, is mathematically stated as -

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0), Thus -$$

$$m_{01} + m_{02} + m_{03} = \mu_0$$

$$m_{10} + m_{14} + m_{15} = \mu_1 \qquad m_{10} + m_{19}^{(4)} + m_{1,10}^{(5)} = m_1$$

$$m_{20} + m_{26} = \mu_2 \qquad m_{20} + m_{23}^{(6)} = m_2$$

$$m_{30} + m_{37} + m_{38} = \mu_3 \qquad m_{30} + m_{39}^{(7)} + m_{3,10}^{(8)} = k$$

where,

$$k = \int_0^\infty \overline{G}(t)dt \qquad m_1 = \int_0^\infty \overline{H}_1(t)dt$$

$$m_2 = \int_0^\infty \overline{H}_2(t)dt$$

The mean sojourn time in the regenerative state i (μ_i) is defined as the time of stay in that state before transition to any other state, then we have -

$$\mu_0 = \frac{1}{\lambda + \alpha_1 + \alpha_2} \qquad \mu_1 = \frac{1 - h_1^*(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2}$$

$$\mu_2 = \frac{1 - h_2^*(\lambda)}{\lambda} \qquad \mu_3 = \frac{1 - g^*(\lambda_1 + \lambda_2)}{\lambda_1 + \lambda_2}$$

$$\mu_4 = -h_1^*(0) = \mu_5 \qquad \mu_6 = -h_2^*(0)$$

$$\mu_7 = -g^*(0) = \mu_8 \qquad \mu_9 = -g_1^*(0)$$

$$\mu_{10} = -g_2^*(0)$$

MEAN TIME TO SYSTEM FAILURE

The mean time to system failure when the system starts from the state 0, is

$$T_0 = \frac{N}{D}$$

where

$$N = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03}$$
$$D = 1 - p_{10} p_{10} - p_{02} p_{20} - p_{03} p_{30}$$

EXPECTED UP-TIME OF THE SYSTEM

The steady state availability of the system is given by

$$A_0 = \frac{N_1}{D_1}$$

where

$$N_1 = \mu_0 + \mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03} + k_1[p_{01} p_{19}^{(4)} + p_{39}^{(7)}(p_{03} + p_{02} p_{23}^{(6)})] + k_2[p_{01} p_{1,10}^{(5)} + p_{3,10}^{(8)}(p_{03} + p_{02} p_{23}^{(6)})]$$

$$D_1 = \mu_0 + m_1 p_{01} + m_2 p_{02} + k[p_{03} + p_{02} p_{23}^{(6)}] + k_1[p_{01} p_{19}^{(4)} + p_{39}^{(7)}(p_{03} + p_{02} p_{23}^{(6)})] + k_2[p_{01} p_{1,10}^{(5)} + p_{3,10}^{(8)}(p_{03} + p_{02} p_{23}^{(6)})]$$

BUSY PERIOD OF A REPAIRMAN (REPAIR ONLY)

The steady state busy period of the system is given by:

$$B_R = \frac{N_2}{D_1}$$

Where

$$N_2 = W_3[p_{03} + p_{02}p_{23}^{(6)}] + W_9[p_{01}p_{19}^{(4)} + p_{39}^{(7)}(p_{03} + p_{02}p_{23}^{(6)})]$$
$$+W_{10}[p_{01}p_{1,10}^{(5)} + p_{3,10}^{(8)}(p_{03} + p_{02}p_{23}^{(6)})]$$

and D_1 is already specified.

BUSY PERIOD OF A REPAIRMAN (MAINTENANCE ONLY)

The steady state busy period of the system is given by:

$$B_M = \frac{N_3}{D_1}$$

where

$$N_3 = W_1 p_{01} + W_2 p_{02}$$

and D_1 is already specified.

EXPECTED NO. OF VISITS OF REPAIRMAN

The steady state expected no. of visits of the repairman is given by:

$$V_R = \frac{N_4}{D_1}$$

where

$N_4 = p_{01} + p_{02} + p_{03} = 1$

and D_1 is already specified.

PROFIT ANALYSIS

The expected profit incurred of the system is -

$$P = C_0 A_0 - C_1 B_R - C_2 B_M - C_3 V_R$$

C_0 = Revenue per unit up time of the system

C_1 = Cost per unit up time for which the repairman is busy in repair

C_2 = Cost per unit up time for which the repairman is busy doing maintenance

C_3 = Cost per visit of the repairman

GRAPHICAL INTERPRETATION AND CONCLUSION

For graphical analysis following particular cases are considered:

$g(t)=\beta e^{-\beta t}$ $g_1(t)=\beta_1 e^{-\beta_1 t}$

$g_2(t)=\beta_2 e^{-\beta_2 t}$ $h_1(t)=\gamma_1 e^{-\gamma_1 t}$

$h_2(t)=\gamma_2 e^{-\gamma_2 t}$

Graphical study has been made for the profit with respect to failure rate of main unit (λ), w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure rate of main unit (λ) and w.r.t. cost of repairman for busy in doing maintenance (C_2) for different values of rate of failure rate of main unit (λ).

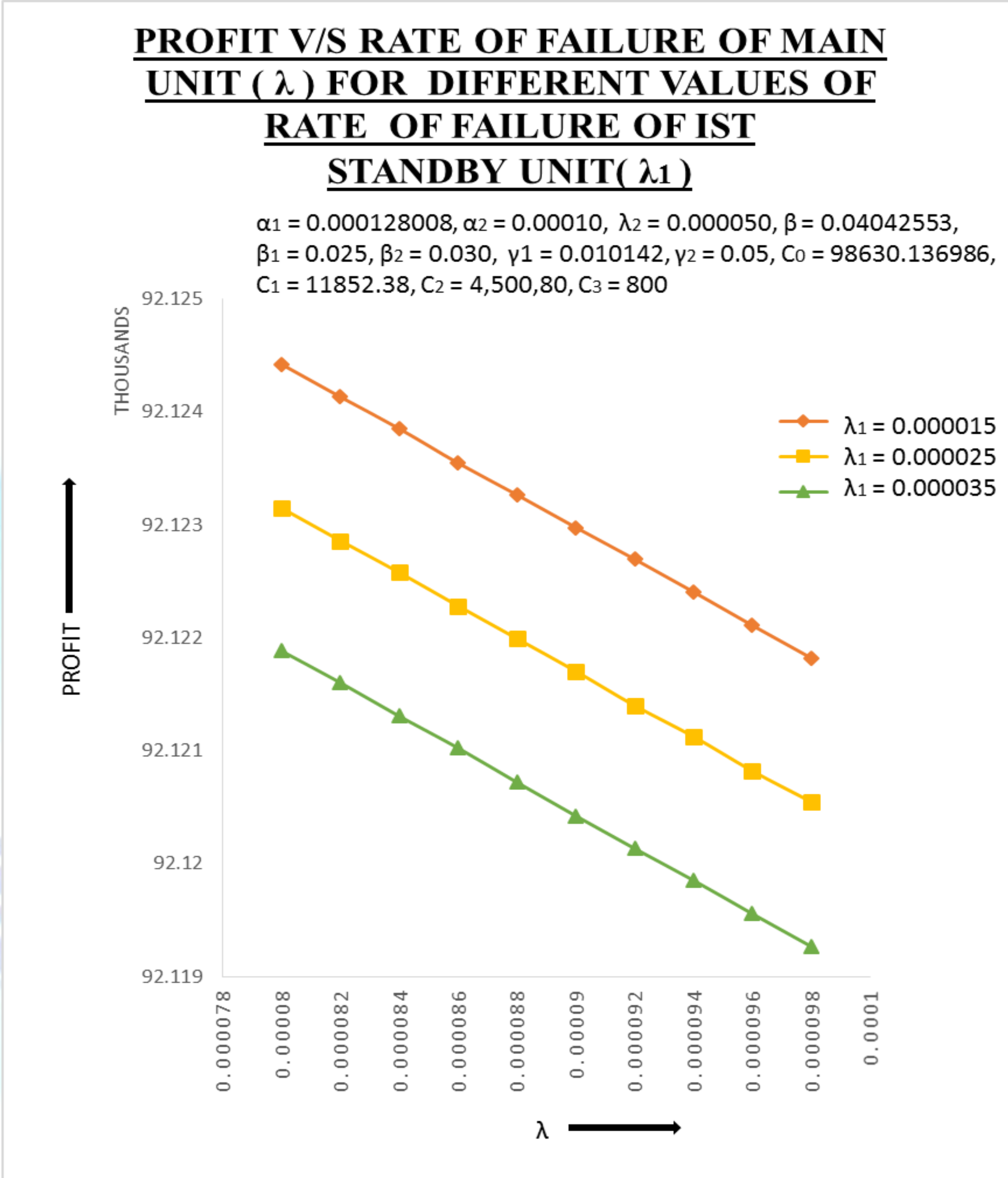

Fig. 3

Fig. 3 shows the behaviour of profit w.r.t. the failure rate of main unit (λ) for different values of failure rate of Ist standby unit (λ_1). As the values of failure rate of main unit (λ) increases, the profit decreases. Also, the profit decreases as failure rate of Ist standby unit (λ_1) increases.

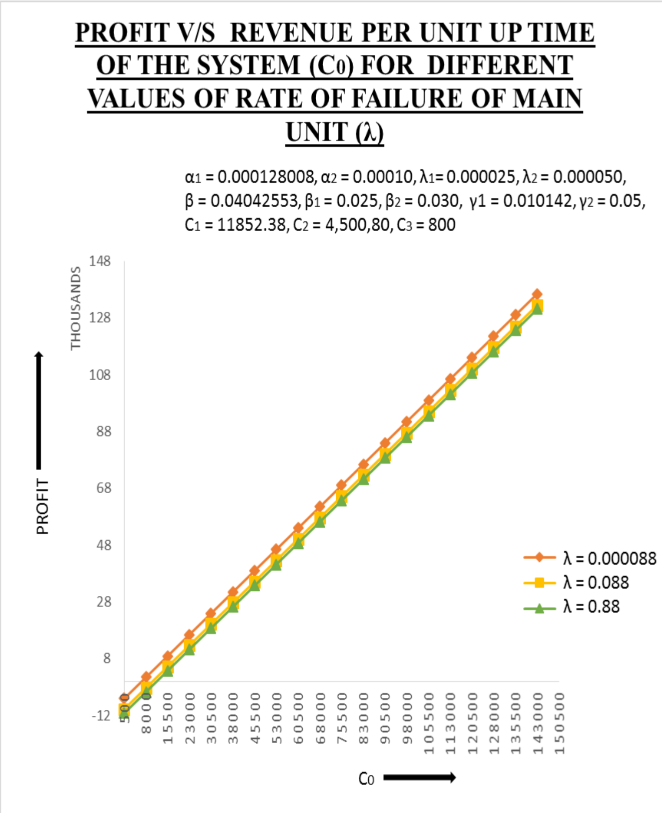

Fig. 4

Fig. 4 depicts the behaviour of the profit w.r.t. revenue per unit uptime of the system (C_0) for different values of rate of failure of main unit (λ). It can be interpreted that the profit increases with increase in the values of C_0. Following conclusions can be drawn from the graph:

1. For λ = 0.000088, profit is > or = or < according as C_0 > or = or < 6498, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 6498 to get positive profit.
2. For λ = 0.088, profit is > or = or < according as C_0 > or = or < 10172, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 10172 to get positive profit.
3. For λ = 0.88, profit is > or = or < according as C_0 > or = or < 11645, i.e. the revenue per unit uptime of the system in such a way so as to give C_0 not less than 11645 to get positive profit.

Fig. 5 shows the behaviour of profit w.r.t. to Cost per unit uptime for which the repairman is busy doing Maintenance (C_2) for different values of rate of failure of main unit (λ). As the value of Cost per unit uptime for which the repairman is busy in Maintenance (C_2) increases, the profit decreases. Also, the profit decreases as failure of main unit (λ) increases.

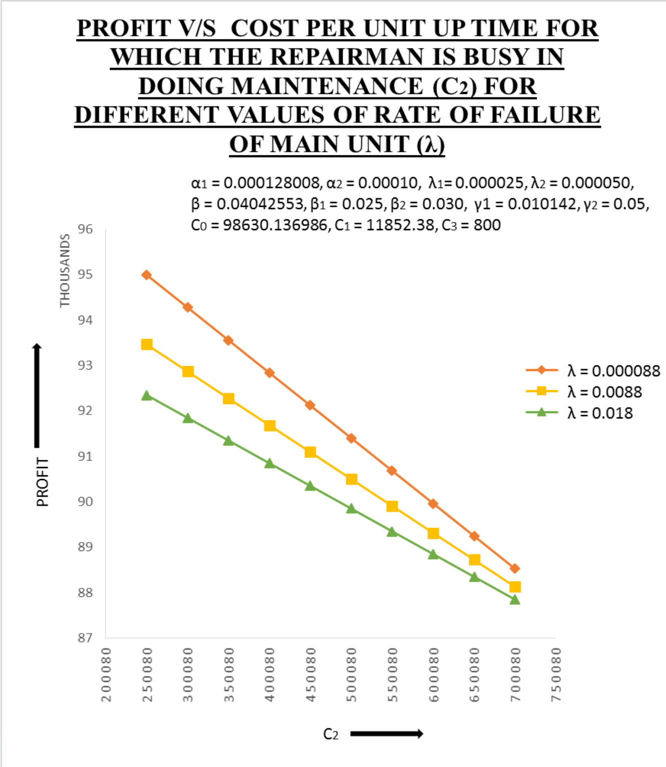

Fig. 5

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