COMBINED EFFECTS OF SORET AND CHEMICAL REACTION ON UNSTEADY FREE CONVECTION MHD FLOW PAST A MOVING VERTICAL PLATE THROUGH POROUS MEDIUM WITH VARIABLE TEMPERATURE AND CONSTANT MASS DIFFUSION IN AN INCLINED MAGNETIC FIELD

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ABSTRACT

Combined effects of Soret and chemical reaction on unsteady free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion in an inclined magnetic field is studied here. Fluid considered is electrically conducting. The Laplace transform technique has been used to find the solutions for velocity and temperature profiles. The results obtained are discussed with the help of graphs drawn for different parameters like thermal and mass Grashof numbers, the Prandtl number, permeability parameter, the Hartmann number, the Soret number, the Schmidt number, chemical reaction parameter, time and inclination of the magnetic field.

KEYWORDS: MHD, Soret effect, chemical reaction, free convection, Heat transfer, Mass transfer, Porous medium.

INTRODUCTION

The effect of magnetic field on viscous, incompressible and electrically conducting fluid plays very important role in many applications such as glass manufacturing, control processing, paper industry, textile industry, magnetic materials processing and purification of crude oil etc. Bejan and Khair [1] have studied heat and mass transfer by natural convection in a porous medium. Nelson and Wood [2] have studied combined heat and mass transfer natural convection between vertical parallel plates with uniform heat flux. MHD flow between two parallel plates with heat transfer have been studied by Attia and Katb [4]. Rajput and Kumar [12] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Hossain and Shayo [3] have discussed skin-friction in unsteady free convection flow past an exponentially accelerated plate. Das and Jana [10] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [11] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Soret effect on unsteady MHD flow through porous medium past an oscillating inclined plate with variable wall temperature and mass diffusion have been discussed by Rajput and Kumar [15]. Postelnicu [5] has studied influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al. [6] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [9] has discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak et al. [13] have studied Soret and

Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects has studied by Postelnicu [7]. Ibrahim [8] has discussed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. Kesavaiah and Satyanarayana [14] have studied radiation absorption and Dufour effects to MHD flow in vertical surface.

In this paper we are analyzing combined effects of Soret and chemical reaction on unsteady free convection MHD flow past a moving vertical plate through porous medium with variable temperature and constant mass diffusion in an inclined magnetic field.

MATHEMATICAL ANALYSIS

Consider an unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate with velocity u_0. The *x*- axis is taken along the plate in the upward direction and *y*- axis is taken normal to it. The plate is electrically non-conducting. A uniform inclined magnetic field B_0 is applied on the plate with angle α from vertical. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t > 0$, temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w.

The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho}\sin^2(\alpha)u - \frac{\upsilon}{K}u, \quad (1)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - K_C(C - C_\infty), \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}, \quad (3)$$

The initial and boundary conditions are given as:

$$\left.\begin{aligned} &t \le 0; u = 0, T = T_\infty, C = C_\infty \text{ for each value of } y, \\ &t > 0; u = u_0, T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, C = C_w \text{ at } y = 0, \\ &u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty. \end{aligned}\right\} \quad (4)$$

Here u is the velocity of the fluid, g – the acceleration due to gravity, β – volumetric coefficient of thermal expansion, β^* – volumetric coefficient of concentration expansion, t – time, T – the temperature of the fluid,

T_∞ – the temperature of the plate at $y \to \infty$, C – species concentration in the fluid, C_∞ – species concentration at $y \to \infty$, υ – the kinematic viscosity, ρ – the density, C_P – the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T – thermal diffusion ratio, D – the mass diffusion constant, D_m – the effective mass diffusivity rate, T_w – the temperature of the plate at $y=0$, C_w – species concentration at the plate at $y=0$, T_m – mean fluid temperature, B_0 – the uniform magnetic field, σ – electrical conductivity, K – permeability of the porous medium and α – angle of inclination from vertical, K_C – chemical reaction parameter.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{aligned}
&\bar{y} = \frac{yu_0}{\upsilon}, \bar{t} = \frac{tu_0^2}{\upsilon}, \bar{u} = \frac{u}{u_0}, \theta = \frac{T-T_\infty}{T_w-T_\infty}, \bar{C} = \frac{C-C_\infty}{C_w-C_\infty}, Sc = \frac{\upsilon}{D}, Ha^2 = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2}, \bar{K} = \frac{Ku_0^2}{\upsilon^2},\\
&Gr = \frac{g\beta\nu(T_w-T_\infty)}{u_0^3}, \Pr = \frac{\mu C_p}{k}, M = Ha^2 * \sin^2(\alpha), \mu = \upsilon\rho, Gm = \frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},\\
&Sr = \frac{D_m K_T (T_w-T_\infty)}{\upsilon T_m (C_w-C_\infty)}, K_O = \frac{K_C \upsilon}{u_0^2}.
\end{aligned}\right\} \quad (5)$$

Here $\bar{u}$ is dimensionless velocity, $\bar{t}$ – dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr*- thermal Grashof number, *Gm*- mass Grashof number, θ - dimensionless temperature, $\bar{C}$ - dimensionless concentration, *Ha*- the Hartmann number, μ - the coefficient of viscosity, $\bar{K}$ – permeability parameter K_O – dimensionless chemical reaction parameter and *Sr* - Soret number. Then model is transformed into the following non dimensional form of equations:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta + Gm\bar{C} - A\bar{u}, \quad (6)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Sc}\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + Sr\frac{\partial^2 \theta}{\partial \bar{y}^2} - \mathrm{K_O}\bar{\mathrm{C}}. \quad (7)$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2}, \quad (8)$$

Here $A = M + \frac{1}{\bar{K}}$.

The initial and boundary condition become:

$$\left.\begin{array}{l}\bar{t} \le 0; \bar{u} = 0, \theta = 0, \overline{C} = 0 \text{ for each value of } \bar{y}, \\ \bar{t} > 0; \bar{u} = 1, \theta = \bar{t}, \overline{C} = 1 \text{ at } \bar{y} = 0, \\ \bar{u} \to 0, \theta \to 0, \overline{C} \to 0 \text{ as } \bar{y} \to \infty.\end{array}\right\} \quad (9)$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - Au, \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + Sr\frac{\partial^2 \theta}{\partial y^2} - \mathrm{K_O C}. \quad (11)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}, \quad (12)$$

Here $A = M + \frac{1}{K}$.

The initial and boundary condition become:

$$\left.\begin{array}{l}t \le 0; u = 0, \theta = 0, C = 0 \text{ for each value of } y, \\ t > 0; u = 1, \theta = t, C = 1 \text{ at } y = 0, \\ u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty.\end{array}\right\} \quad (13)$$

Now the solution of equations (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for species concentration *C*, fluid temperature θ and fluid velocity *u* are respectively:

$$\theta = -\frac{A_{41}\sqrt{\Pr t}\, y}{\sqrt{\pi}} - \left(t + \frac{\Pr y^2}{2}\right)A_7$$

$$C = \frac{A_{10}(1 + A_{11} + A_{12}A_{10}^{-2})}{2} + \frac{\Pr Sr(A_{10}B_{26} + A_{14}A_{13}^{-1}B_7 - 2(-1 + A_{17}) + A_{14}A_{13}^{-1}(-1 - A_{13}^2 - A_{18} + A_{19}A_{13}^2))}{2K_O}$$

$$u = B_8 + C_{10} + \frac{A_{28}A_{20}A_{29}B_{13}Gm}{2(K_O Sc - A)} - C_{11} - \frac{A\sqrt{\pi}(K_O Sc - A)\Pr ScGm}{A_{36}} C_{12} - C_{13} + \frac{A_{28}A_{30}A_{29}B_{21}Gm}{2(K_O Sc - A)}$$
$$+ \frac{Gm \Pr Sr\sqrt{Sc}}{A_{36}}(C_{14} - C_{15}) + Gm\frac{A_{10}(1 + A_{11} + A_{12}A_{10}^{-2})}{2(K_O Sc - A)} - \frac{GmSr}{A_{42}}(2\sqrt{\pi}(1 - A_{17})B_{27} - K_O ScB_{20}$$
$$+ A_{14}A_{13}^{-1}\sqrt{\pi}\Pr B_{27}(-\Pr A + Sc))$$

The expressions for the constants involved in the above equations are given in the appendix.

RESULTS AND DISCUSSION

The numerical values of velocity, concentration and temperature are computed for different parameters like thermal Grashof number *Gr,* mass Grashof number *Gm*, Hartmann number *Ha*, Prandtl number *Pr*, Schmidt number *Sc*, inclination α, permeability parameter *K,* Soret number *Sr,* chemical reaction parameter *Ko* and time *t*. The values of the parameters considered are *Ha* = 2, 4, 6, *Gm* = 30, 40, 50, $\alpha = 30°, 45°, 60°$, *Pr* = 0.025, 0.71, 7.00, *Gr* = 5, 50, 200, *Sc* = 0.66, 2.01, 4.00, *K* = 0.2, 0.3, 0.4, *Sr* = 2, 3 , 4, *Ko* = 2, 3, 4 and *t* = 0.2, 0.3, 0.4. Figures 5, 6, 7, 8, 9 and 10 show that velocity increases when one of the following parameters, *Sr, K, Gm, Gr, Pr and t,* are increased keeping others constant. Figures 1, 2, 3 and 4 show that velocity decreases when α, *Ha, Ko* and *Sc* are increased keeping others constant. Figure 12 shows that temperature increases when *t* is increased; and Figure 11 shows that temperature decreases when *Pr* is increased.

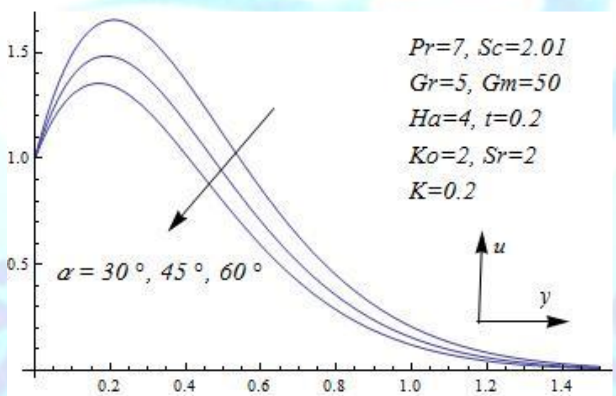

Figure-1: Velocity profile for different values of α

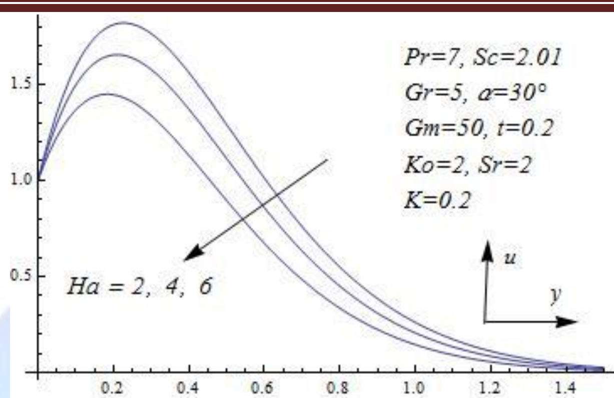

Figure-2: Velocity profile for different values of *Ha*

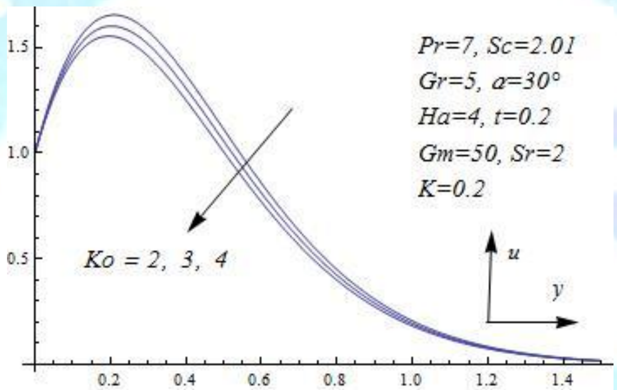

Figure-3: Velocity profile for different values of *Ko*

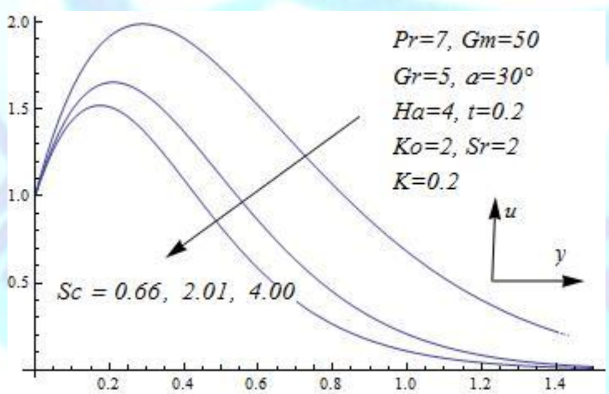

Figure-4: Velocity profile for different values of *Sc*

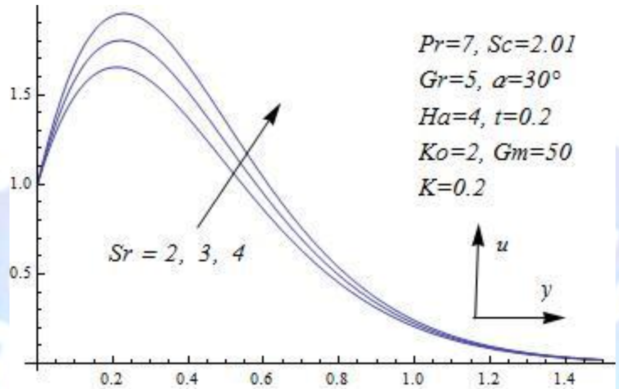

Figure-5: Velocity profile for different values of *Sr*

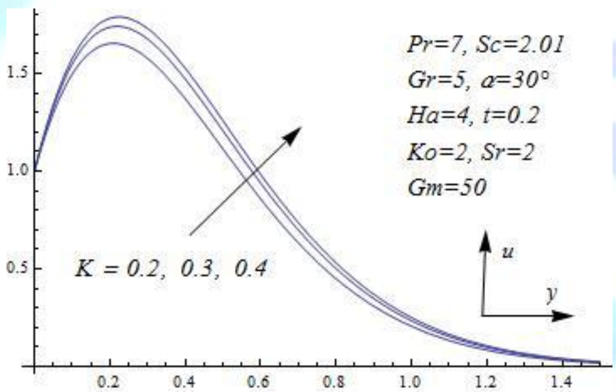

Figure-6: Velocity profile for different values of *K*

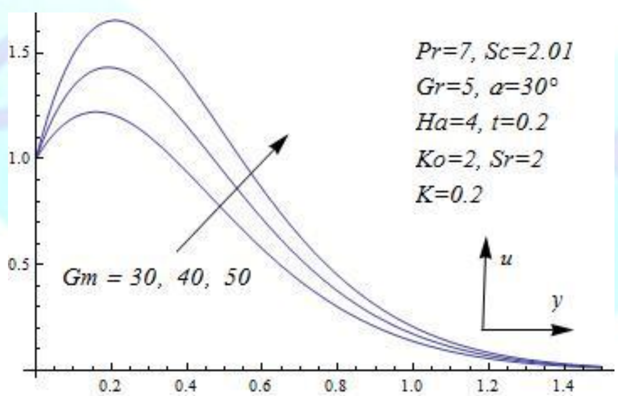

Figure-7: Velocity profile for different values of *Gm*

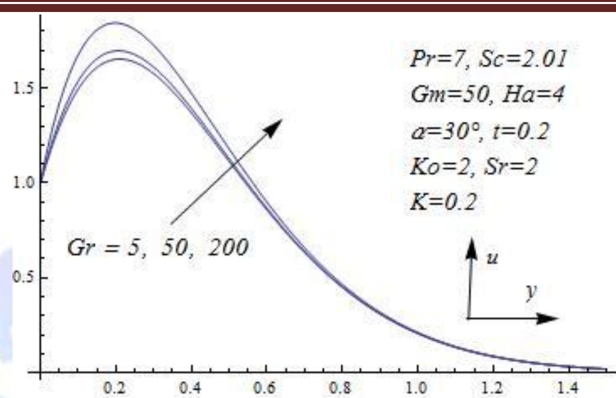

Figure-8: Velocity profile for different values of *Gr*

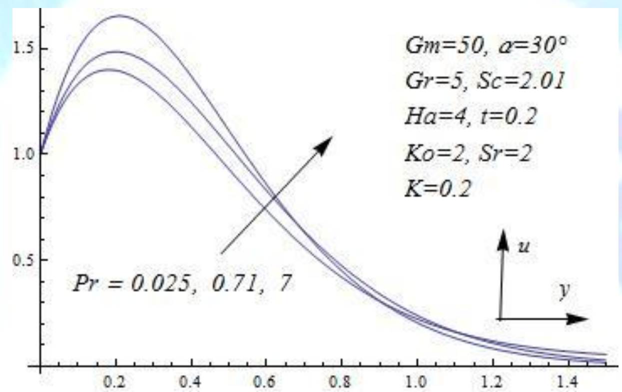

Figure-9: Velocity profile for different values of *Pr*

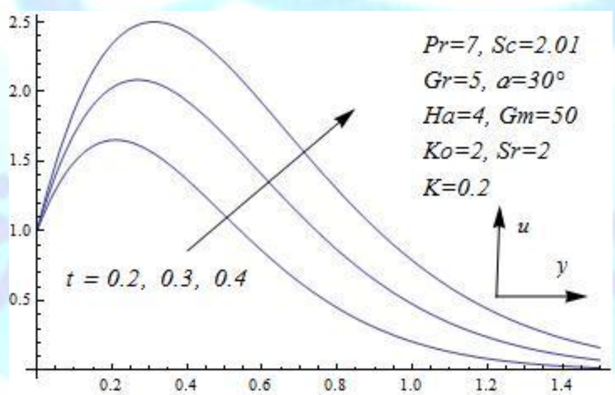

Figure-10: Velocity profile for different values of *t*

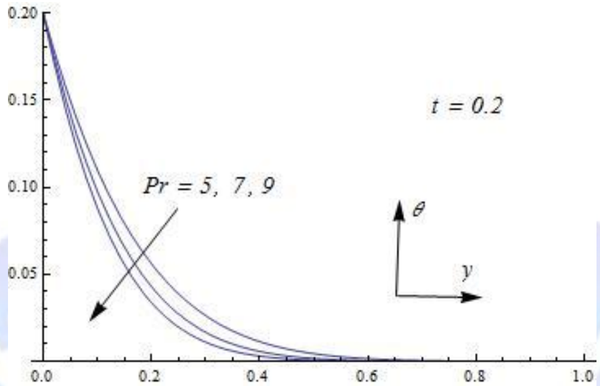

Figure-11: Temperature profile for different values of *Pr*

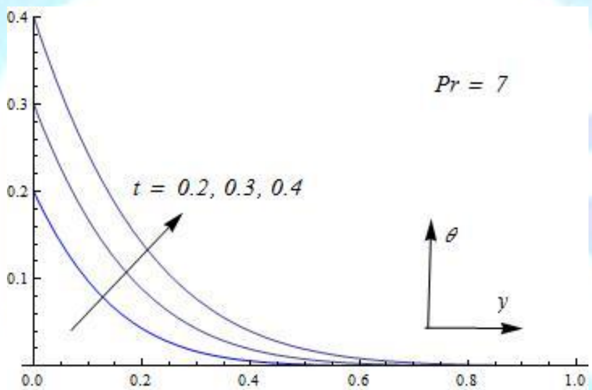

Figure-12: Temperature profile for different values of *t*

CONCLUSION

Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of *Sr, K, t, Gm, Gr* and *Pr.*
- The velocity of the fluid decreases with increasing the values of *Ha, Ko,* α and *Sc.*
- The temperature of the fluid increases with increasing the values of *t.*
- The temperature of the fluid decreases with increasing the values of *Pr*.

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APPENDIX

$A_7 = Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_8 = Erf\left[\sqrt{\frac{\Pr tK_O}{\Pr - Sc}} + \frac{\sqrt{Sc}y}{2\sqrt{t}}\right],\ A_9 = Erf\left[\sqrt{tK_O} + \frac{\sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{10} = e^{-\sqrt{K_O Sc}y},\ A_{11} = Erf\left[\sqrt{tK_O} - \frac{\sqrt{Sc}y}{2\sqrt{t}}\right],$

$A_{12} = Erfc\left[\sqrt{tK_O} + \frac{\sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{13} = e^{\sqrt{\frac{\Pr ScK_O}{\Pr - Sc}}y},\ A_{14} = e^{\frac{tScK_O}{\Pr - Sc}}\ A_{15} = Erf\left[\frac{2\sqrt{\frac{K_O \Pr}{\Pr - Sc}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{16} = Erfc\left[\frac{2\sqrt{\frac{K_O \Pr}{\Pr - Sc}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],$

$A_{17} = Erf\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{18} = Erf\left[\frac{2\sqrt{\frac{K_O Sc}{\Pr - Sc}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{19} = Erf\left[\frac{2\sqrt{\frac{K_O Sc}{\Pr - Sc}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{20} = e^{-\sqrt{A}y},\ A_{21} = Erf\left[\frac{2\sqrt{A}t - y}{2\sqrt{t}}\right],$

$A_{22} = Erfc\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right],\ A_{23} = Erf\left[\frac{2\sqrt{A}t + y}{2\sqrt{t}}\right],\ A_{24} = e^{\sqrt{\frac{\Pr A}{\Pr - 1}}y},\ A_{25} = e^{\frac{At}{\Pr - 1}},\ A_{26} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr - 1}}t - y}{2\sqrt{t}}\right],\ A_{27} = Erf\left[\frac{2\sqrt{\frac{A\Pr}{\Pr - 1}}t + y}{2\sqrt{t}}\right],$

$A_{28} = e^{\frac{-K_O Sct}{Sc-1}},\ A_{29} = e^{\sqrt{\frac{(A-Ko)Sc}{Sc-1}}y},\ A_{30} = e^{\frac{At}{Sc-1}},\ A_{31} = Erf\left[\frac{2\sqrt{\frac{(A-Ko)Sc}{Sc-1}}t - y}{2\sqrt{t}}\right],\ A_{32} = Erf\left[\frac{2\sqrt{\frac{(A-Ko)Sc}{Sc-1}}t + y}{2\sqrt{t}}\right],$

$A_{33} = e^{\sqrt{\frac{\Pr A - ASc + KoSc}{\Pr - Sc}}y},\ A_{34} = Erf\left[\sqrt{\frac{A\Pr - ASc + KoSc}{\Pr - Sc}}\sqrt{t} - \frac{y}{2\sqrt{t}}\right],\ A_{35} = Erf\left[\sqrt{\frac{A\Pr - ASc + KoSc}{\Pr - Sc}}\sqrt{t} + \frac{y}{2\sqrt{t}}\right],$

$A_{36} = 2Ko\sqrt{\pi}(-A + KoSc)(KoSc(\Pr - 1) + A(Sc - \Pr)),\ A_{37} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr - 1}}t - \sqrt{\Pr}y}{2\sqrt{t}}\right],\ A_{38} = Erf\left[\frac{2\sqrt{\frac{A}{\Pr - 1}}t + \sqrt{\Pr}y}{2\sqrt{t}}\right],$

$A_{39} = Erf\left[\frac{2\sqrt{\frac{A-Ko}{Sc-1}}t - \sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{40} = Erf\left[\frac{2\sqrt{\frac{A-Ko}{Sc-1}}t + \sqrt{Sc}y}{2\sqrt{t}}\right],\ A_{41} = e^{\frac{-\Pr y^2}{4t}},\ A_{42} = 2AKo\sqrt{\pi}(A(\Pr - Sc) - KoSc(\Pr - 1)),$

$B_7 = 1 + A_{15} + A_{13}^2 A_{16},\ B_8 = \frac{1}{2}A_{20}(1 + A_{21} + A_{20}^{-2}A_{22}),\ B_9 = A_{30}A_{28}A_{29}^{-1}Ko\sqrt{\pi Sc}(1 - Sc),\ B_{10} = 1 + A_{20}^{-2} + A_{21} - A_{20}^{-2}A_{23},$

$B_{11} = 1 - A_{20}^{-2} + A_{21} + A_{20}^{-2}A_{23},\ B_{12} = (-1 - A_{24}^2 - A_{26} + A_{24}^2 A_{27})(1 - \Pr),$

$B_{13} = -A_{30}A_{20}^{-1} + A_{28}^{-1}A_{29} - A_{30}A_{20}^{-1}A_{29}^2 + A_{29}A_{28}^{-1}A_{20}^{-2} + A_{21}A_{29}A_{28}^{-1} - A_{31}A_{30}A_{20}^{-1} - A_{29}A_{23}A_{20}^{-2}A_{20}^{-2} + A_{30}A_{32}A_{20}^{-1}A_{29}^2$

$B_{14} = 1 - \Pr - At,\ B_{15} = A\Pr - ASc + KoSc - KoSc\Pr,\ B_{16} = A_{30}A_{28}A_{29}^{-1}Ko\sqrt{\pi}Sc(1 - Sc)(-1 - A_{29}^2 - A_{31} + A_{29}^2 A_{32})$

$B_{17} = A_{14}A_{33}^{-1}(-1 - A_{33}^2 - A_{34} + A_{33}^2 A_{35}),\ B_{18} = A\Pr - ASc - Sc\Pr Ko + Sc^2 Ko,$

$B_{19} = 2A_{41}\sqrt{t}\Pr y + \sqrt{\pi}\Pr y^2(A_{17} - 1),\ B_{20} = A_{25}A_{24}^{-1}\sqrt{\pi}\Pr(\Pr - 1)(1 + A_{24}^2 + A_{37} - A_{24}^2 A_{38}),$

$B_{21} = 1 + A_{29}^2 + A_{39} - A_{29}^2 A_{40},\ B_{22} = -1 - A_{10}^{-2} - A_{11} + A_{10}^{-2}A_9,\ B_{23} = A(\sqrt{Sc} - \frac{\Pr}{\sqrt{Sc}}) + \sqrt{Sc}Ko(\Pr - 1)$

$B_{24} = -1 - A_{13}^2 - A_{15} + A_{13}^2 A_8,\ B_{25} = \frac{A\Pr}{\sqrt{Sc}} - \frac{\sqrt{Sc}}{A} - \sqrt{Sc}Ko\Pr + Sc^{3/2}Ko,\ B_{26} = A\Pr^2 - ASc\Pr + KoSc\Pr - Ko\Pr^2 Sc$

$B_{27} = -1 - A_{13}^2 - A_{18} + A_{13}^2 A_{19},$

$C_{10} = \frac{Gr}{4A^2}(2A_{20}^{-1}B_{10}B_{14} + \sqrt{A}A_{20}^{-1}yB_{11} + 2A_{25}A_{24}^{-1}B_{12}),\ C_{11} = \frac{Gm\Pr Sr}{A_{36}}(-A_{20}\sqrt{\pi}B_{10}B_{15} - B_{16} - \sqrt{\pi}B_{17}B_{18}),$

$C_{12} = -A_{20}B_{10}B_{15} - A_{25}A_{24}^{-1}KoScB_{12} - A\Pr B_{17}(\Pr - Sc),\ C_{13} = \frac{-Gr}{2A^2\sqrt{\pi}}(-2\sqrt{\pi}(-1 + A_{17})B_{14} + AB_{19} + B_{20})$

$C_{14} = A_{10}\sqrt{\pi}B_{22}B_{23} + A_{14}A_{13}^{-1}\sqrt{\pi}B_{24}B_{25},\ C_{15} = A_{30}A_{28}A_{29}^{-1}Ko\sqrt{\pi Sc}B_{21}(1 - Sc)$