

Mathematical modeling on segregation of Biological species in relation to temperature variation

V.K Katiyar¹, K.S. Basavarajappa², Sathisha A.B³,

1. Department of Mathematics, Indian Institute of Technology, Roorkee, Uttarakhand
2. Department of Mathematics, Bapuji Institute of Engineering and Technology, Davangere
3. Department of Mathematics, Govt. Science College, Chitradurga

Abstract:

The study concerns the quantification of Biological species with respect to varying temperature. It is to identify the three varieties of populations of the same type of species with reference to three nest sites: wet marsh (I), dry marsh (II), dry levees (III) regions. The difference in the temperature varying from 27⁰C to 37⁰C is considered in the model for the determination of three aged groups. The model predicts male and female populations in these regions using mathematical modeling. Numerical method with three stages of approximations has been used to identify the species with a reference to sex determination depends on varying temperature and on genetic characteristics. The maximum output form in order to conclude the population for three different nest sites is employed. The reference of mean temperature is employed in the calculation for various Quantifications of the nest sites using numerical method with equal step size.

Keywords: Temperature, Wet marsh, Dry marsh, Hatchlings.

Introduction:

Study from Biological species predicts that, some species become extinct and other species do not become extinct. The reference of three families of crocodilian (alligators, crocodiles and gavials) not becomes extinct. The crocodile is clearly the prototype dragon. This is more stronger than alligator. Pooley and Gans [2] studied on Nile crocodile and describe among other things a unique social behavior the long survival. Ferguson [6], [7] explained the theoretical study of reproductive biology on such species. Generally in mammals and birds, sex is determined at conception. This is to the reference of genetic sex determination (GSD). Where as in environmental sex determination (ESD), sex is determined by environmental factors and occurs in other vertebrates and also some invertebrates. But in the case of reptiles, sex is determined depending on temperature variations (TSD). With turtles, it is the high temperature that gives only females except for the snapping turtle which is like the crocodile. The temperature that produce all male and female hatchlings very little between the different species of crocodilian. At 30⁰C, 90% of species are females, at 33⁰C, 99% of species are males but at 35⁰C, 90% of species are female hatchlings. It can be noted that, between 33.5⁰C-34.5⁰C, species produced will be both male and female.

The present study concerns, the new mathematical formulation to explain the variations in the temperatures related to segregation of male and female crocodilia. Further the different regions of temperatures are related to give the hypothesis for the expression of gene for the male determining factor. We assume three different types of nest sites (I) wet marsh, (II) dry marsh and (III) levee (Elevated from ground) to describe the segregation of male and female species. It is reported that in the mathematical model levee nests are hot 34⁰C and hatch approximately 100% males while in the wet marsh, nests are cool 30⁰C and hatch approximately 100% females. But in between 31.8⁰C and 33.4⁰C, the species produced will have very low relative fitness. The female alligator does not choose the sex of her offspring.

The temperature controls more than just the sex of embryos. It affects the growth and development from embryo to adulthood influence pigmentation pattern and the adults to regulate its own body temperature. Temperature in different nest regions protects the changing habitats of the species and to adjust the metabolic requirements in order to adjust with the privilege conditions. The survival rate is closely related to climate changes. The present study also focuses on new formulation between the segregation of species depending on

temperature, the sex ratio and the survivorship of the species. The formulation includes the density dependent age-structured model for population dynamics. This needs the incubation of egg at various time periods. Woodward and Murray [10] Lotka [3], McKendrick [4] studied theoretically the population in reference to age-structured model dynamics to segregate male and female populations by temperature of egg incubation. In his model the stability criteria shows no possibility of ESD over GSD, only the history data has been used in the theoretical analysis. Their analysis was restricted to linear models but not with reference to density dependent one. So the population might stabilize to a nonzero equilibrium when we use Maltusian model. In his model the population either grows or decays exponentially.

Eric L. Charnov et al [5] proposed that labile sex determination which is favored by natural selection when an individual's fitness is strongly influenced by environmental conditions and where the individual has little control over which environment it will experience and their argument also applies to sex expression in hermaphrodites. D. C. Deeming et al [6] presented the data on the effects of incubation at 30⁰C to 33⁰C on the development of embryos of alligator mississippiensis using a sample of eggs different to that described in his previous paper. D. C. Deeming et al [7] summarizes the pattern of TSD in crocodylians and suggests that a hypothesis for a mechanism of temperature dependent sex determination in reptiles also finally the phylogenetic advantages of TSD in crocodylians, other reptiles and vertebrates are described in relation to the new hypothesis. Jeffrey E. Lovich et al [8] discussed the male biased in a population of the diamond back terrapin and evaluate the applicability of the biased sample hypothesis. Each explanation was tested as an alternative hypothesis to determine its utility. Fredric J. Janzen [9] investigated the response of hatchling sex ratio to incubation temperature among families of the common snapping turtle (*Chelydras serpentina*), a species with TSD and used an experimental design in which families were crossed with three incubation temperatures to determine if genotype X environment (G X E) interactions might be the mechanism by which genetic variation in sex ratio is maintained in this system. Thane Wibbels et al [11] addresses several of the hypothesis reviewed estradiol-17 β , tamoxifen, testosterone showed similar type of effect utilizing male producing temperatures if sex determination is sensitive to DTH at constant temperatures which produce mixed sex ratios in control groups and also includes a double treatment experiment which examines if masculinizing effects of DHT and feminizing effects of estradiol-17 β can be simultaneously stimulated at a mixed sex ratio incubation temperature. Adam Markham [12] identified Species adaptation abilities depend not only on genetic variability but also on dispersal and migration capacity. Genetic variability within populations and ecosystem resilience is being reduced through habitat fragmentation. It will be further reduced by pressures resulting from human-induced climate forcing at any level. Tackling this question presents empirical challenges. We studied on TSD which is sensitive to climatic variation in terms of sex-ratio outcomes. Second, given that TSD is sensitive to climatic variation, we identify the heritability of TSD and quantify the evolutionary potential of specific traits involved in TSD.

Therefore the study focuses on the remodeling of basic assumptions and then the formulation for segregation of male and female populations of species depending on the variation of temperature. These assumptions help to provide key ideas for mathematical model to quantify the populations in three nest regions.

Proposed assumptions:

Assumptions remodeled include male and female populations quantification with respect to only time .

- I) In "wet marsh" we can notice female hatchlings because of low incubation temperature
- II) In "dry marsh" we can notice approximately 50% male and 50% female hatchlings.
- III) In "dry levees" we can notice male hatchlings because of higher incubation temperature.
- IV) Temperature range is considered to be between 25⁰C- 35⁰C in present model.
- V) Some crucial spatial elements are under consideration in the model based on the above observations.
- VI) Limited numbers of nest sites are taken into consideration near the water.
- VII) These limited number of nest sites may prevent female population.
- VIII) Present model ratio segregation of male and female species is proposed as follows.
1: 2: 12 \longleftrightarrow 6.7% : 13.6% : 79.7%

Wet Marsh : $\{f_1(t)\}$: Female hatchlings
 Dry Marsh : $\{f_2(t)\}\{m_2(t)\}$: 50% Female : 50% Male hatchlings
 Dry Levees : $\{m_3(t)\}$: Male population.
 Total Female population: $f(t) = f_1(t) + f_2(t)$
 Total Male population : $m(t) = m_3(t) + m_2(t)$

Further assuming that there is a limited number of nest sites near the water which prevents a totally female population: typical figures for percentages of the total nest sites in each of these regions are given by Joanen [1] as 79.7% for region I, 13.6% for region II and 6.7% for region III.

The population at time t is divided into four classes, $f_1(t)$ and $f_2(t)$ denoting females themselves incubated in regions I and II respectively and $m_2(t)$ and $m_3(t)$ denoting males incubated in II and III.

- I. Wet marsh - all female hatchlings: $f_1(t)$
- II. Dry marsh - 50% female, 50% male hatchlings: $f_2(t), m_2(t)$
- III. Dry levees - all male hatchlings: $m_3(t)$ (1)

Then the total female population and male populations are modeled as

Total female population = $f(t) = f_1(t) + f_2(t)$
 Total male population = $m(t) = m_2(t) + m_3(t)$

Formulation and analysis

Only a fraction of females can incubate their eggs [1] in the wet marsh region (I). Taking k_1 denote the carrying capacity of region I. This fraction (F) must be a function of k_1 and the female population f_1 and it must satisfy certain criteria. If there are only a few females f_1 , $F \approx 1$ since essentially all of them can nest in region I while for a very large number of females f_1 , $F = 0$ since in this situation most of them have to move away from the wet marsh region I. As an approximation to this function, the fraction

$$F(k_1, f_1) = \frac{k_1}{k_1 + f_1} \tag{2}$$

satisfies the following conditions.

$$F(k_1, f_1) = \frac{k_1}{k_1 + f_1} \rightarrow 0 \text{ as } f_1 \rightarrow \infty,$$

and $F(k_1, f_1) = \frac{k_1}{k_1 + f_1} \rightarrow 1 \text{ as } f_1 \rightarrow 0,$

It is an approximation to the actual fraction function. Strictly $F(f_1, k_1)$ is zero until f_1 reaches the carrying capacity k_1 of region I after which the extra females have to move away from the wet marsh region. For best approximation we could choose ($f_1 = k_1$, the carrying capacity, $F = 0.5$ whereas it should still be zero). We use this form, (2), for algebraic simplicity;

If b is the effective birth rate, that is, it includes clutch size, C , and the high mortality of hatchlings and egg predation, in other words survival, S , then, in a simple population model a dynamic equation for the population in region I (all females)

$$\frac{df_1(t)}{dt} = b \left[\frac{k_1}{k_1 + f_1} \right] f_1 - df_1 \tag{3}$$

The birth rate, b , is a function of the total male population, m , and is reasonably taken as

$$b = b(m) = \frac{b_0 m}{(c+m)} \rightarrow b_0 \text{ for small } c, \tag{4}$$

Where c is a constant which, very small. Then equation (3) is uncoupled from the other equations in the model system below.

If we now consider region II where both females and males are produced, the fraction of females which have to move from the wet marsh region I to the dry marsh region II is simply. So, the total number of females who want to nest in region II is the number who like this temperature plus those that had to move from region I:

$$f(t) = f_1(t) + f_2(t) = \frac{f_1^2}{k_1 + f_1} + f_2 \tag{5}$$

k_2 relates to the maximum number of hatchlings possible in the dry marsh region II. For algebraic simplicity we approximate this fraction by

$$F_{\max} = \frac{k_2}{k_2 + f_1 + f_2}$$

Which has roughly the same qualitative behavior. Compared with other assumptions and approximations this approximation is minor. It can, of course, easily be included in a numerical simulation of the equations: this was done and the resulting solutions were in general qualitative agreement. Thus the equations for the females and males in region II are:

$$\frac{df_2}{dt} = \frac{1}{2} \left\{ b_0 \left[\frac{f_1^2}{k_1 + f_1} + f_2 \right] \left[\frac{k_2}{k_2 + f_1 + f_2} \right] \right\} - df_2 \quad (6.1)$$

$$\frac{dm_2}{dt} = \frac{1}{2} \left\{ b_0 \left[\frac{f_1^2}{k_1 + f_1} + f_2 \right] \left[\frac{k_2}{k_2 + f_1 + f_2} \right] \right\} - dm_2 \quad (6.2)$$

Finally in region III, the dry levees, the number of females forced to move from region II to III to nest is

$$F(k_1, f_1, f_2) = \left[\frac{f_1^2}{k_1 + f_1} + f_2 \right] \left[\frac{\frac{f_1^2}{k_1 + f_1} + f_2}{k_2 + \frac{f_1^2}{k_1 + f_1} + f_2} \right]$$

The remaining females cannot nest in any suitable site. So, with these expressions the equation for males in region III (in our model there are only males here) is

$$\frac{dm_3}{dt} = b_0 \left[\frac{k_3}{k_3 + f_1 + f_2} \right] \left[\frac{f_1^2}{k_1 + f_1} + f_2 \right] \left[\frac{f_1 + f_2}{k_2 + f_1 + f_2} \right] \quad (7)$$

The system of equations (3), (6.1), (6.2) and (7) constitute the model for the populations in the various regions and we can obtain the sex ratio of the total population.

The quantification of f_1, f_2, m_2, m_3 are estimated numerically by solving simultaneously with repeated iterations using Runge – Kutta fourth order method at 6 (six) decimal places in each stage. Results are compared with findings of Ferguson [6] [7]. In their analysis computations of f_1, f_2, m_2, m_3 are taken using finite difference scheme.

Where $\frac{df_1}{dt} = \frac{f_1(t+h) - f_1(t)}{h} + O(h)$

$$\frac{df_1}{dt} = \left(\frac{b_0 m}{c + m} \right) \left(\frac{k_1}{k_1 + f_1} \right) f_1 - df_1$$

$$e_1: -57.849 \text{ to } -57.849$$

$$e_2: 116.04225 \text{ to } 116.04225$$

$$e_3: -405.796 \text{ to } -405.79759$$

$$e_4: 2377.145 \text{ to } 2377.154926$$

$$f_{1,1}: 299.9646 \text{ to } 299.96588$$

$$\frac{df_2}{dt} = \frac{f_2(t+h) - f_2(t)}{h} + O(h)$$

$$\frac{df_2}{dt} = \frac{b_0}{2} \left(\frac{f_1^2}{k_1 + f_1} + f_2 \right) \left(\frac{k_2}{k_2 + f_1 + f_2} \right) - df_2$$

The values of g_1, g_2, g_3, g_4 for $t_0=1, 3, 6, 9, 12$, are computed as

$$g_1: -29.318 \text{ to } -29.318.$$

$$g_2: 45 \text{ to } 58.7435.$$

$$g_3: -205.510 \text{ to } -205.49132$$

$$g_4: 1203.81 \text{ to } 1203.69807.$$

$$f_{2,1}: 151.827 \text{ to } 151.81436.$$

$$\frac{dm_2}{dt} = \frac{m_2(t+h) - m_2(t)}{h} + O(h)$$

$$\frac{dm_2}{dt} = \frac{b_0}{2} \left(\frac{f_1^2}{k_1 + f_1} + f_2 \right) \left(\frac{k_2}{k_2 + f_1 + f_2} \right) - dm_2$$

The values of h_1, h_2, h_3, h_4 for $t_0=1, 3, 6, 9, 12$, are computed as

$$h_1: -29.318 \text{ to } -29.318.$$

$$h_2: 58.745 \text{ to } 58.7435.$$

$$h_3: -205.4987 \text{ to } -205.49132$$

$$h_4: 1203.81 \text{ to } 1203.699807.$$

$$m_{2,1}: 151.832 \text{ to } 151.81436$$

$$\frac{dm_3}{dt} = \frac{m_3(t+h) - m_3(t)}{h} + O(h)$$

$$\frac{dm_3}{dt} = b_0 \left(\frac{k_3}{k_3 + f_1 + f_2} \right) \left(\frac{f_1^2}{k_1 + f_1} + f_2 \right) \left(\frac{f_1 + f_2}{k_2 + f_1 + f_2} \right) - dm_3$$

The values of l_1, l_2, l_3, l_4 for $t_0 = 1, 3, 6, 9, 12$, are computed as

$$l_1: -57.4432 \text{ to } -57.4432.$$

$$l_2: 115.61476 \text{ to } 115.61476.$$

$$l_3: -405.9163 \text{ to } -403.9161927.$$

$$l_4: 2366.5088 \text{ to } 2366.509774.$$

$$m_{3,1}: 298.74375 \text{ to } 298.7439514.$$

Subaverage species of region-I and region-II at $t = 1, 3, 6, 9, 12$ days are

Subaverage of region-I and region-II: 301.81 to 301.79.

Subaverage species of region-II and region-III at $t = 1, 3, 6, 9, 12$ days are

Subaverage of region-II and region-III: 301.20 to 301.19.

Results and Discussion.

The study is carried out to compute the segregated population of species depending on variation of temperature in three different regions. The average temperature is observed near the bifurcation lines between I, II, III regions. When temperature is varied in regions I, II and III, we can notice the computation of male and female populations species (Crocodiles). Three species choose their survivorship in the nested regions. $f_{1,1}, f_{2,1}, m_{2,1}$ and $m_{3,1}$ denote the populations in their nested regions as the temperature (Table-1) varies between 30°C to 37°C in three regions.

Figure (1) denotes the survivor species in three nested regions

Figure (2) denotes the quantification of female populations $f_{1,1}$ in Region I.

Figure (3) denotes the quantification of female populations $f_{2,1}$ and male $m_{2,1}$ populations in Region II.

Figure (4) denotes the quantification of male $m_{3,1}$ populations in Region III.

Table (1) Quantification of female and male populations between 30°C to 37°C

Sl no.	Female populations
1	30°C
2	to
3	33.5°C
	Male populations
1	32.5°C
2	to
3	37°C

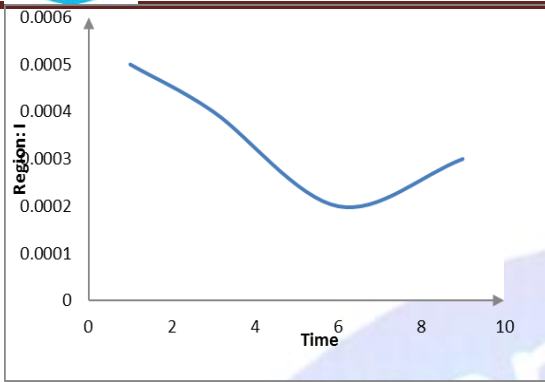


Fig 2: Wet marsh (Region-I): Female population

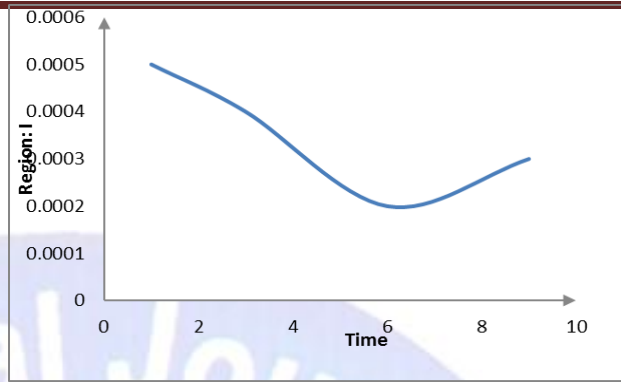


Fig 3: Dry marsh (Region-II): Female and male populations

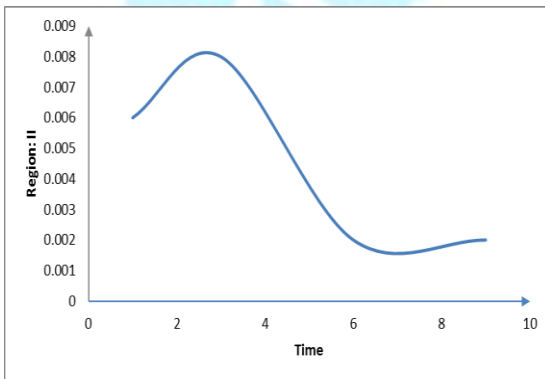


Fig 4: Dry levees (Region-III): Male populations

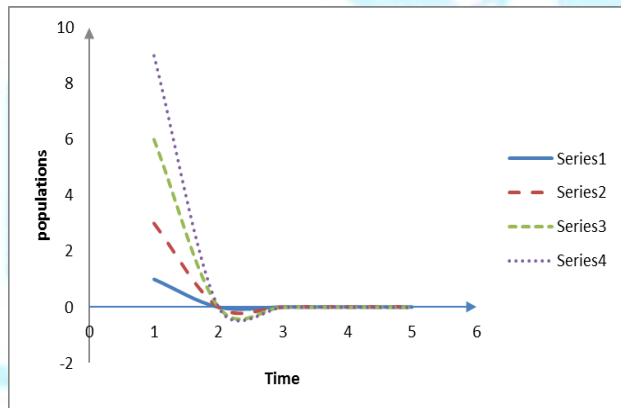


Fig 5: Uneven temperature variation gives the populations in one region.

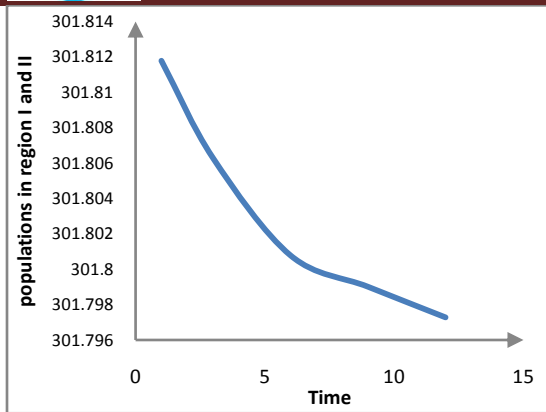


Fig 6: Subaverage populations in region-I and region-II

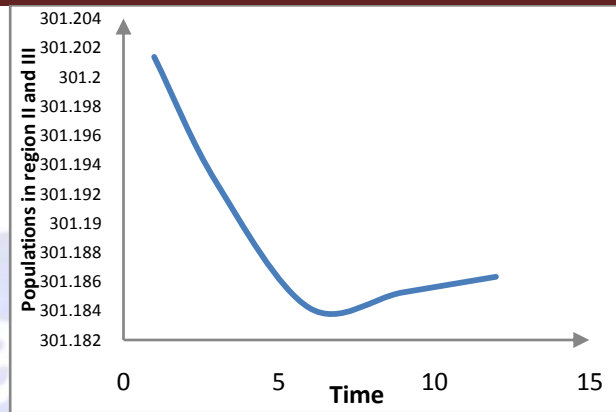


Fig 7: Subaverage populations in region-II and region-III

Nomenclature

t - time

$f_1(t)$ – female population in region-I

$f_2(t)$ - female population in region-II

$m_2(t)$ -male population in region-II

$m_3(t)$ -male population in region-III

F- Fraction

k_1 – carrying capacity in region-I

k_2 -carrying capacity in region-II

k_3 -carrying capacity in region-III

b– Birth rate

C– Clutch size

$f(t) = f_1(t) + f_2(t)$ – total female population in region-I and region-II

$m(t) = m_3(t) + m_2(t)$ - total male population in region-I and region-II

References:

1. Joanen. T., "Nesting ecology of alligators in Louisiana", Proc. Southeastern Association, Game and Fish commissioners, Conf. 24: 175-193 (1969).
2. A. C. Pooley and Carl Gans, "The Nile Crocodile". Scientific American, Volume 57, Issue 5: pp 839-840 (1776).
3. Alfred J. Lotka, "The Relation Between Birth Rate and Death Rate in a Normal Population and the Rational Basis of an Empirical Formula for the Mean Length of Life Given by William Farr". Vol. 16, No. 123, pp. 121-130, (1918).
4. McKendrick, 1926, "Applications of mathematics to medical problems". Proceedings of the Edinburgh Mathematical Society (1926).
5. Eric L. Charnov & James Bull. "When is sex environmentally determined?". Nature 266, 828 - 830 (28 April 1977)
6. D. C. Deeming, M. W. J. Ferguson. "Effects of incubation temperature on growth and development of embryos of Alligator mississippiensis". Vol:159, pp 183-193, (1989)
7. D. C. Deeming, M. W. J. Ferguson. "The Mechanism of Temperature Dependent Sex Determination in Crocodylians: A Hypothesis". Amer. Zool., 29:973-985 (1989).
8. Jeffrey E. Lovich et al. "Age at maturity influences adult sex ratio in the turtle malaclemys terrapin". OIKOS 59: 126-134, (1990).
9. Fredric J. Janzen, "Heritable Variation for Sex Ratio Under Environmental Sex Determination in the Common Snapping Turtle". Genetics 131: 155-161, (1992).
10. Woodward and Murray. "On the Effect of Temperature-Dependent Sex Determination on Sex Ratio and Survivorship in Crocodylians". rspb, vol 252, issue 1334, (1993).
11. Thane Wibbels et al, "Steroid induced sex determination at incubation temperatures producing mixed sex ratios in a turtle with TSD". General and comparative endocrinology-100, 53-60 (1995).
12. Adam Markham, "Potential impacts of climate change on ecosystems: a review of implications for policymakers and conservation biologists". Climate Research, Vol. 6: 179-191, (1996).