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CRITICAL PATH WITH COMPLEMENT OF TYPE-2 FUZZY NUMBERS

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Abstract

This paper presents the critical path problem with the complement of trapezoidal type-2 fuzzy numbers. New ranking methods are introduced to identify the fuzzy critical path and its length in a acyclic project network. An illustrative example is also included to demonstrate our proposed approach.

Keywords: acyclic project network, fuzzy Critical path, ranking function, trapezoidal type-2 fuzzy numbers.

Introduction

The critical path method(CPM) , worked out at the beginning of the 1960s[11], has become one of the tools that are most useful in practice and are applied in the planning and control of realization of complex projects. First of all it consists in the identification of the so-called critical paths, critical activities and critical events in the network, which is the project model, assuming the earliest possible completion time of the whole project.

Consequently, the fuzzy set theory can play a significant role in this kind of decision making environment to tackle the unknown or the vagueness about the time duration of activities in a project network. To effectively deal with the ambiguities involved in the process of linguistic estimate times, the trapezoidal fuzzy numbers are used to characterize fuzzy measures of linguistic variable.

Dubois et al[12] extended the fuzzy arithmetic operations to compute the latest starting time of each activity in a project network. To find critical path in a project network yao et al[13] used signed distance ranking of fuzzy number. In this paper , we propose the concept of complement for finding critical path in a project network.

The organization of the paper is as follows: In section 2, we have some basic concepts, section 3, gives some properties of total slack fuzzy time, Section 4, gives the network terminology. section 5, gives an algorithm to find the critical path and its length combined with trapezoidal type-2 fuzzy number using ranking method. To illustrate the proposed algorithm the numerical example is solved in section 6.

2. Basic concepts

2.1 Type-2 Fuzzy number

Let $\tilde{A}$ be a type-2 fuzzy set defined in the universe of discourse R, if the following conditions are satisfied, then $\tilde{A}$ is called a type-2 fuzzy number.

1. $\tilde{A}$ is normal.
2. $\tilde{A}$ is a convex set.
3. The support of $\tilde{A}$ is closed and bounded.

2.2 Normal type-2 fuzzy number

A type-2 fuzzy number(T2fs), $\tilde{A}$ is said to be normal if its Foot of Uncertainty (FOU) is normal interval type-2 fuzzy numbers(IT2FS) and it has a primary membership function.

2.3 Addition on type-2 fuzzy numbers

Let $\tilde{A} = \bigcup_{for all \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{A}_{\tilde{\alpha}}) = (A^L, A^M, A^N, A^U)$

$= ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U)$

Let $\tilde{B} = \bigcup_{for all \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{B}_{\tilde{\alpha}}) = (B^L, B^M, B^N, B^U)$

$= ((b_1^L, b_2^L, b_3^L, b_4^L), (b_1^M, b_2^M, b_3^M, b_4^M)(b_1^N, b_2^N, b_3^N, b_4^N)(b_1^U, b_2^U, b_3^U, b_4^U)$

be two normal type-2 fuzzy numbers. By using extension principle, we have $\tilde{A} + \tilde{B}$ =

$$\left[\bigcup_{for all \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{A}_{\tilde{\alpha}})\right] + \left[\bigcup_{for all \tilde{\alpha}} \tilde{\alpha} FOU(\tilde{B}_{\tilde{\alpha}})\right] = (A^L + B^L, A^M + B^M, A^N + B^N, A^U + B^U)$$

$$= \begin{matrix} ((a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L), (a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M, a_4^M + b_4^M) \\ (a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N, a_4^N + b_4^N)(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U)) \end{matrix}$$

2.4 Ranking Function on type-2 Fuzzy number

Let $\tilde{A} = ((a_1^L, a_2^L, a_3^L, a_4^L), (a_1^M, a_2^M, a_3^M, a_4^M)(a_1^N, a_2^N, a_3^N, a_4^N)(a_1^U, a_2^U, a_3^U, a_4^U)$

be a type-2 normal trapezoidal fuzzy number, then the ranking function is defined as, $R(\tilde{A})$ = $(a_1^L + 2a_2^L + a_3^L + a_4^L + 2a_1^M + 4a_2^M + 4a_3^M + 2a_4^M + 2a_1^N + 4a_2^N + 4a_3^N + 2a_4^N + a_1^U + 2a_2^U + 2a_3^U + a_4^U)/36$

2.5 Complement(Negation)

The complement of the membership function is defined as:

$$\mu_{\tilde{A}}(X) = 1 - \mu_{\tilde{A}}(X), \qquad \forall x \in X.$$

2.6 Fuzzy critical path

In a project network a path p_c such that

FCPM(p_c)=min{FCPM(p_k)/$p_k \in$ P,k=1tom} is defined as a fuzzy critical path.

2.7 Fuzzy critical path length

The sum of the Fuzzy activity time of the corresponding path P_c is said to be the fuzzy critical path length.

2.8 Complement of fuzzy critical path

In a project network a path p_c such that

CFCPM(p_c)=max{FCPM(p_k)/$p_k \in$ P,k=1tom} is defined as a complement of a fuzzy critical path

2.9.Notations:

A_{ij}= The activity between node i and j.

$F\tilde{E}S_j$ =The earliest starting fuzzy time of node j.

$F\tilde{L}F_i$ = The latest finishing fuzzy time of node i.

$F\tilde{T}S_{ij}$ = The total slack fuzzy time of A_{ij}.

p_k= the k^{th} fuzzy path.

P = the set of all fuzzy paths in a project network.

FCPM(p_k)= The total slack fuzzy time of path p_k in a project network.

CFCPM(p_k)= The total slack complement fuzzy time of path p_k in a project network.

3.Properties of total slack fuzzy time

Property :3.1(Forward pass calculation)

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie) $\tilde{FES}_1 = (0.0, 0.0, 0.0, 0.0)$

$\tilde{FES}_j = \max_i \{\tilde{FES}_i + \tilde{FET}_{ij}\}, j \neq i, j \in N$, i=number of preceding nodes.($\tilde{FES}_j$ =The earliest starting fuzzy time of node j).

Ranking value is utilized to identify the maximum value. Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

Property :3.2 (Backward pass calculation)

To calculate the latest finishing time in the project network set $\tilde{FLF}_n = \tilde{FES}n$. $\tilde{FLF}_i = \min_j \{\tilde{FLF}_{ij} (-) \tilde{FET}_{ij}\}, i \neq n, i \in N$, j = number of succeeding nodes. Ranking value is utilized to identify the minimum value.

Latest starting fuzzy time= Latest finishing fuzzytime(-)Fuzzy activity time.

Property :3.3

For the activity A_{ij}, i<j

Total fuzzy slack:

$$\tilde{FTS}_{ij} = \tilde{FLF}_j (-)(\tilde{FES}_i (+) \tilde{FET}_{ij}) \text{ (or) } (FLF_j (-) \tilde{FET}_{ij})(-)\tilde{FES}_i,\ 1 \leq i \leq j \leq n;\ i, j \in N,$$

Property :3.4

$FCPM(p_k) = \sum_{\substack{1 \leq i \leq j \leq n \\ i, j \in p_k}} \tilde{FTS}_{ij}$, $p_k \in P$, p_k is the possible paths in a network from source node to the destination node, k=1 to m.

4.Network Terminology

Consider a directed acyclic project network G(V,E) consisting of a finite set of nodes V={1,2,…..n} and a set of m directed edges $E \subseteq V \times V$. Each edge is denoted by an ordered pair (i,j) where $i.j \in V$ and $i. \neq j$. In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path p_{ij} as a sequence $p_{ij}=[i=i_1,(i_1,i_2),i_2,\ldots.i_{l-1},i_l=j]$ of alternating nodes and edges. The existence of at least one path p_{si} in G(V,E) is assumed for every node $i \in V - \{S\}$. $\tilde{d}_{ij}$ denotes a trapezoidal type – 2 fuzzy number associated with the edge (i,j), corresponding to the length necessary to transverse (i,j) from i to j. The fuzzy length along the path P is denoted by $\tilde{d}(P)$ and is defined as $\tilde{d}(P) = \sum_{(i,j \in p)} \tilde{d}_{ij}$

5.1.Algorithm:1 (for finding critical path)

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $\tilde{FES}_1 = (0.0, 0.0, 0.0, 0.0)$ and calculate $\tilde{FES}_j$, j =2,3,…. n by using property 1.

Step 3: Let $\tilde{FLF}_n = \tilde{FES}n$ and calculate $\tilde{FLF}_i$,i=n-1, n-2,….. 2,1. By using property 2.

Step 4 : Calculate $\tilde{FTS}_{ij}$ with respect to each activity in a project network by using property 3.

Step 5 : Find all the possible paths and calculate CFCPM(p_k) by using property 4.

Step 6 : Identify the critical path by using Definition 2.5.

6.Numerical example

The problem is to find the critical path and critical path length between source node to destination node in the acyclic fuzzy project network having 6 vertices and 7 edges with trapezoidal type-2 fuzzy number.

$\tilde{R}$

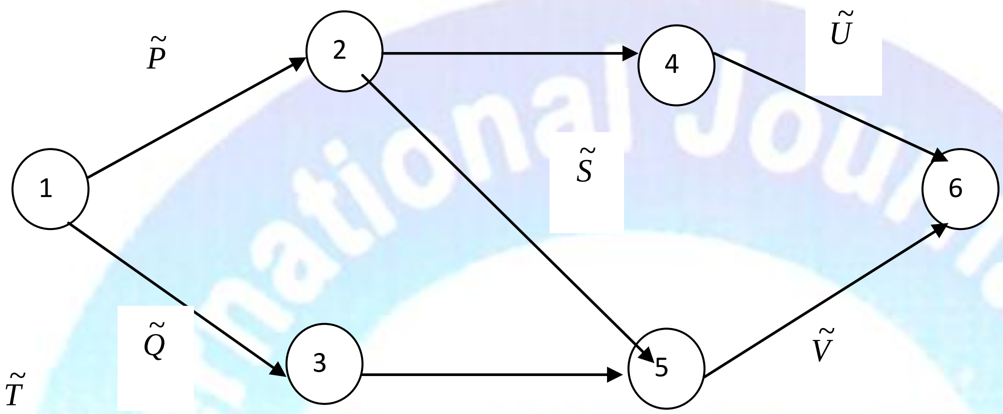

Fig 6.1

The edge lengths are

$\tilde{P} = ((1.5,1.8,2.0,2.5;0.3),(1.2,1.5,2.0,2.2),(1.1,1.3,1.5,2.0),(1.0,1.3,1.5,2.0))$

$\tilde{Q} = ((1.6,1.7,1.8,2.0;0.4),(1.5,1.6,2.0,2.5),(1.3,1.5,2.0,2.3),(1.2,1.4,1.5,1.8))$

$\tilde{R} = ((1.8,1.9,2.2,2.6;0.7),(1.6,1.9,2.2,2.6),(1.5,1.7,2.1,2.3),(1.2,1.8,2.3,2.6))$

$\tilde{S} = ((1.3,1.5,1.9,2.2;0.9),(1.3,1.4,2.3,2.4,),(1.2,1.6,2.3,2.8),(1.0,1.5,1.9,2.3))$

$\tilde{T} = ((1.8,1.9,2.3,2.5;0.5),(1.6,1.8,2.3,2.4,),(1.5,1.6,1.9,2.3),(1.3,1.6,1.9,2.5))$

$\tilde{U} = ((1.5,1.7,2.0,2.5;0.7),(1.3,1.6,2.2,2.5),(1.2,1.6,2.3,2.6),(1.1,1.5,1.8,2.0))$

$\tilde{V} = ((1.9,2.0,2.3,2.4;0.6),(1.8,2.1,2.3,2.5),(1.5,1.7,2.0,2.3),(1.4,1.7,2.0,2.5))$

Complement of the edges are

$\overline{\tilde{P}} = ((-0.5,-0.8,-1.0,-1.5;0.3),(-0.2,-0.5,-1.0,-1.2),(-0.1,-0.3,-0.5,-1.0),(0.0,-0.3,-0.5,-1.0))$

$\overline{\tilde{Q}} = ((-0.6,-0.7,-0.8,1.0;0.4),(-0.5,-0.6,-1.0,-1.5),(-0.3,-0.5,-1.0,-1.3),(0.2,-0.4,-0.5,-0.8))$

$\overline{\tilde{R}} = ((-0.8,-0.9,-1.2,-1.6;0.7),(-0.6,-0.9,-1.2,-1.6),(-0.5,-0.7,-1.1,-1.3),(0.2,-0.8,-1.3,-1.6))$

$\overline{\tilde{S}} = ((-0.3,-0.5,-0.9,-1.2;0.9),(-0.3,-0.4,-1.3,-1.4),(-0.2,-0.6,-1.3,-1.8),(0.0,-0.5,-0.9,-1.3))$

$\overline{\tilde{T}} = ((-0.8,-0.9,-1.3,-1.5;0.5),(-0.6,-0.8,-1.3,-1.4),(-0.5,-0.6,-0.9,-1.3),(0.3,-0.6,-0.9,-1.5))$

$\overline{\tilde{U}} = ((-0.5,-0.7,-1.0,-1.5;0.7),(-0.3,-0.6,-1.2,-1.5),(-0.2,-0.6,-1.3,-1.6),(0.1,-0.5,-0.8,-1.0))$

$\overline{\tilde{V}} = ((-0.9,-1.0,-1.3,-1.4;0.6),(-0.8,-1.1,-1.3,-1.5),(-0.5,-0.7,-1.0,-1.3),(0.4,-0.7,-1.0,-1.5))$

Table:1 Activities, fuzzy durations and total slack fuzzy time for each activity

Activity (i-j)i<j	Duration $F\tilde{E}T_{ij}$	$F\tilde{E}S_i$	$F\tilde{L}F_j$	$F\tilde{T}S_{ij}$
1-2	((-0.5,-0.8,-1.0,-1.5;0.3), (-0.2,-0.5,-1.0,-1.2), (-0.1,-0.3,-0.5,-1.0), (0.0,-0.3,-0.5,-1.0))	(0.0,0.0,0.0,0.0)	((1.4,-0.1,-1.6,-2.8;0.0), (1.8,0.4,-2.1,-3.2), (2.1,0.8,-1.5,-3.4), (2.2,0.6,-1.1,-3.5))	((2.9,0.9,-0.8,-2.3;0.0), (3.0,1.4,-1.6,-3.0), (3.1,1.3,-1.2,-3.3), (3.2,1.1,-0.8,-3.5))
1-3	((-0.6,-0.7,-0.8,-1.0;0.4), (-0.5,-0.6,-1.0,-1.5), (-0.3,-0.5,-1.0,-1.3), (-0.2,-0.4,-0.5,-0.8))	(0.0,0.0,0.0,0.0)	((1.2,0.3,-1.3,-2.4;0.0), (1.6,0.6,-1.7,-2.7), (1.8,0.3,-1.5,-3.1), (2.6,0.4,-1.1,-3.1))	((2.2,1.1,-0.6,-1.8;0.0), (3.1,1.6,-1.1,-2.2), (3.1,1.3,-1.0,-2.8), (3.4,0.9,-0.7,-2.9))
2-4	((-0.8,-0.9,-1.2,-1.6;0.7), (-0.6,-0.9,-1.2,-1.6), (-0.5,-0.7,-1.1,-1.3), (-0.2,-0.8,-1.3,-1.6))	((-0.5,-0.8,-1.0,-1.5;0.3), (-0.2,-0.5,-1.0,-1.2), (-0.1,-0.3,-0.5,-1.0), (0.0,-0.3,-0.5,-1.0))	((-0.2,-1.3,-2.5,-3.6;0.0), (0.2,-0.8,-3.0,-3.8), (0.8,-0.3,-2.2,-3.9), (0.6,-0.7,-1.9,-3.7))	((2.9,0.9,-0.8,-2.3;0.0), (3.0,1.4,-1.6,-3.0), (3.1,1.3,-1.2,-3.3), (3.2,1.1,-0.8,-3.5))
2-5	((-0.3,-0.5,-0.9,-1.2;0.9), (-0.3,-0.4,-1.3,-1.4), (-0.2,-0.6,-1.3,-1.8) ,(0.0,-0.5,-0.9,-1.3))	((-0.5,-0.8,-1.0,-1.5;0.3), (-0.2,-0.5,-1.0,-1.2), (-0.1,-0.3,-0.5,-1.0), (0.0,-0.3,-0.5,-1.0))	((-0.3,-1.0,-2.2,-3.2;0.0), (0.2,-0.7,-2.5,-3.3), (0.5,-0.6,-2.1,-3.6), (1.1,-0.5,-1.7,-3.4))	((2.4,0.9,-0.9,-2.4;0.0), (2.8,1.6,-1.6,-2.8), (3.3,1.2,-1.2,-3.3), (3.4,0.9,-0.9,-3.4))
3-5	((-0.8,-0.9,-1.3,-1.5;0.5), (-0.6,-0.8,-1.3,-1.4), (-0.5,-0.6,-0.9,-1.3), (-0.3,-0.6,-0.9,-1.5))	((-0.6,-0.7,-0.8,-1.0;0.4), (-0.5,-0.6,-1.0,-1.5), (-0.3,-0.5,-1.0,-1.3), (-0.2,-0.4,-0.5,-0.8))	((-0.3,-1.0,-2.2,-3.2;0.0), (0.2,-0.7,-2.5,-3.3), (0.5,-0.6,-2.1,-3.6), (1.1,-0.5,-1.7,-3.4))	((2.2,1.1,-0.6,-1.8;0.0), (3.1,1.6,-1.1,-2.2), (3.1,1.3,-1.0,-2.8), (3.4,0.9,-0.7,-2.9))
4-6	((-0.5,-0.7,-1.0,-1.5;0.7), (-0.3,-0.6,-1.2,-1.5), (-0.2,-0.6,-1.3,-1.6), (-0.1,-0.5,-0.8,-1.0))	((-1.3,-1.7,-2.2,-3.1;0.0), (-0.8,-1.4,-2.2,-2.8), (-0.6,-1.0,-1.6,-2.3), (-0.2,-1.1,-1.8,-2.6))	((-1.7,-2.3,-3.2,-4.1;0.0), (-1.3,-2.0,-3.6,-4.1), (-0.8,-1.6,-2.8,-4.1), (-0.4,-1.5,-2.4,-3.8))	((2.9,0.9,-0.8,-2.3;0.0), (3.0,1.4,-1.6,-3.0), (3.1,1.3,-1.2,-3.3), (3.2,1.1,-0.8,-3.5))
5-6	((-0.9,-1.0,-1.3,-1.4;0.6), (-0.8,-1.1,-1.3,-1.5), (-0.5,-0.7,-1.0,-1.3), (-0.4,-0.7,-1.0,-1.5))	((-0.8,-1.3,-1.9,-2.7;0.0),(-0.5,-0.9,-2.3,-2.6),(-0.3,-0.9,-1.8,-2.8),(0.0,-0.8,-1.4,-2.3))	((-1.7,-2.3,-3.2,-4.1;0.0), (-1.3,-2.0,-3.6,-4.1), (-0.8,-1.6,-2.8,-4.1), (-0.4,-1.5,-2.4,-3.8))	((2.4,0.9,-0.9,-2.4;0.0), (2.8,1.6,-1.6,-2.8), (3.3,1.2,-1.2,-3.3), (3.4,0.9,-0.9,-3.4))

Find all the possible paths and calculate CFCPM(p_k) by using property3.4. The possible paths are (1-2-4-6),(1-2-5-6),(1-3-5-6). Which are denoted by p_1, p_2, p_3.

For path p_1, CFCPM(p_1)= $F\tilde{T}S_{12} + F\tilde{T}S_{24} + F\tilde{T}S_{46}$

=((8.7,2.7,-2.4,-6.9),(9.0,4.2,-4.8,-9.0),(9.3,3.9,-3.6,-9.9),(9.6,3.3,-2.4,-10.5)

For path p_2,C FCPM(p_2)= $F\tilde{T}S_{12} + F\tilde{T}S_{25} + F\tilde{T}S_{56}$

= ((7.7,2.7,-2.6,-7.1),(8.6,4.6,-4.8,-8.6),(9.7,3.7,-3.6,-9.9),(10.0,2.9,-2.6,-10.3))

For path p_3, CFCPM(p_3)= $F\tilde{T}S_{13} + F\tilde{T}S_{35} + F\tilde{T}S_{56}$

=((6.8,3.1,-2.1,-6.0),(9.0,4.8,-3.8,-7.2),(9.5,3.8,-3.2,-8.9),(10.2,2.7,-2.3,-9.2))

Now we have to find the fuzzy critical path by algorithm1(using step5), the ranking value of CFCPM(p_i), i=1,2,3, can be obtained.

R(CFCPM(p_1))=0.025,R(CFCPM(p_2))=0.00833,R(CFCPM(p_3))=0.4389

Since R(CFCPM(p_2))<R(CFCPM(p_1))<R(CFCPM(p_3)), the fuzzy complement of critical path is p_3 and its path length is

((6.8,3.1,-2.1,-6.0),(9.0,4.8,-3.8,-7.2),(9.5,3.8,-3.2,-8.9),(10.2,2.7,-2.3,-9.2))

Conclusion

In this work , to find the complement of fuzzy critical path and its length in a acyclic project network, complement of trapezoidal type-2 fuzzy numbers are used. The results are obtained with the help of ranking function which helps the decision makers to get the required fuzzy critical path in a complement nature.

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