CHEMICAL REACTION EFFECT ON MHD FLOW PAST A MOVING PLATE WITH VARIABLE MASS DIFFUSION AND HALL CURRENT

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ABSTRACT

Chemical reaction effect on unsteady MHD flow past a moving vertical plate with variable mass diffusion and Hall current is studied here. The wall temperature is kept constant. The fluid considered is viscous, incompressible and electrically conducting. The model under consideration is solved by Laplace transform method. The velocity profile for different parameters has been studied. This study is usefull in magnetic field controlled materials processing systems, planetary systems and MHD induction machine energy generators, etc.

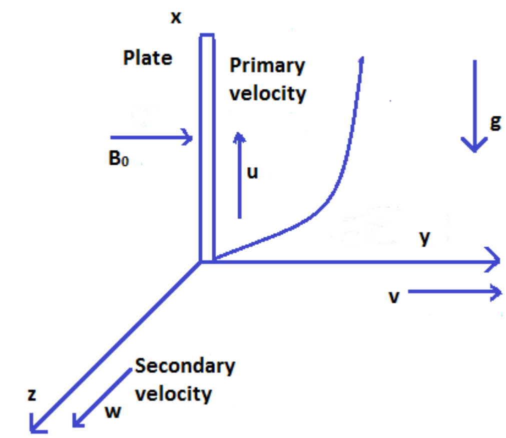
Keywords: Chemical reaction; Hall current; MHD flow; and Skin fraction.

1. INTRODUCTION

The magneto hydrodynamic flow with Hall effect plays important role in engineering and astrophysics. Some applications are in the field of MHD generators, MHD bearings, ion propulsion, stellar and solar structure, inter planetary and inter stellar matter, solar storms and the three-dimensional channel flow MHD pumps etc. Some articles are worth mentioning. Deka [5] has discussed Hall effects on MHD flow over an accelerated plate. Watanabe et al. [7] have studied Hall effects on MHD boundary layer flow on a continuous moving plate. Katagiri [9] has discussed the effect of Hall current on the MHD boundary layer flow past a semi-infinite plate. Unsteady Hartmann flow with heat transfer has been studied by Attia [6]. Further, mass diffusion effects on infinite vertical plate with heat flux have been studied by Deka et al [8]. Raptis et al. [4] have analysed viscous flow on stretching sheet with chemical reaction. Ibrahim et al. [3] have studied MHD boundary layer flow on moving vertical plate. Rajput and Sahu [2] have studied effects of chemical reaction on MHD flow between two vertical parallel plates. H. Kumar [1] has studied MHD free convection flow on a stretching porous sheet with heat source. We are considering Chemical reaction effect on unsteady MHD flow on a plate with Hall current. The results are shown with tables and some graphs.

2. MATHEMATICAL MODELING

The Geometric model of the flow problem is shown in Figure - A.



A vertical moving electrically non-conducting plate with constant wall temperature is considered here. A uniform magnetic field B is taken on the flow. Using the relation $\nabla \cdot B = 0,$ for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Due to Hall effect, there will be two components of the momentum equation. With usual assumptions the fluid model is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(u + mw), \tag{1}$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(w - mu), \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_C(C - C_\infty), \tag{3}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

The boundary conditions considered for the model are as follows:

$$\left.\begin{aligned}
&t \le 0: u = 0, w = 0, C = C_\infty, T = T_\infty, \text{ for all the values of } y,\\
&t > 0: u = u_0, w = 0, T = T_W, C = C_\infty + \left(C_W - C_\infty\right)\frac{u_0^{\,2}t}{\upsilon} \text{ at } y = 0,\\
&u \to 0, w \to 0, C \to C_\infty, T \to T_\infty \quad \text{as } y \to \infty.
\end{aligned}\right\} \quad (5)$$

Definition of symbols:

u – fluid velocity along x, *w* – fluid velocity along z, *m* - Hall parameter, *g* - gravity, β - coefficient of thermal expansion, β^* - coefficient of concentration expansion, *t* - time, C_∞ - the species concentration away from the plate, *C* - species concentration in the fluid, C_w - species concentration at the plate, *D* - mass diffusion, T_∞ - fluid temperature near the plate, T_w - plate temperature, *T* - fluid temperature, k - the thermal conductivity, υ - the kinematic viscosity, ρ - the fluid density, K_c -chemical reaction parameter, σ - electrical conductivity, μ - the magnetic permeability, C_P - specific heat at constant pressure.

Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To transform the equations (1) - (4) in dimensionless from, the following non - dimensional quantities are used:

$$\begin{aligned}
&\bar{u} = \frac{u}{u_0}, \bar{w} = \frac{w}{u_0}, \bar{y} = \frac{yu_0}{\upsilon}, Sc = \frac{\upsilon}{D}, Pr = \frac{\mu C_P}{k}, M = \frac{\sigma B_0^2 \upsilon}{\rho u_0^2},\\
&\bar{t} = \frac{tu_0^{\,2}}{\upsilon}, Gm = \frac{g\beta\upsilon(C_W - C_\infty)}{u_0^{\,3}}, Gr = \frac{g\beta\upsilon(T_W - T_\infty)}{u_0^{\,3}},\\
&\bar{C} = \frac{C - C_\infty}{C_W - C_\infty}, \theta = \frac{(T - T_\infty)}{(T_W - T_\infty)}, K_0 = \frac{\upsilon K_c}{u_0^2}.
\end{aligned} \quad (6)$$

The symbols in dimentionless form are: $\bar{u}$ - velocity, $\bar{w}$ - velocity, ϑ - temperature, $\bar{C}$ - concentration, *Gr* - thermal Grashof number, *Gm* - mass Grashof number, μ - the coefficient of viscosity, *Pr* - the Prandtl number, *Sc* - the Schmidt number, *M* - the magnetic parameter K_0 - the chemical reaction parameter. The dimensionless forms of equations (1), (2), (3) and (4) are as follows

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta + Gm\bar{C} - \frac{M(\bar{u} + m\bar{w})}{(1+m^2)}, \quad (7)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{M(\bar{w} - m\bar{u})}{(1+m^2)}, \quad (8)$$

$$\frac{\partial \overline{C}}{\partial \bar{t}} = \frac{1}{Sc}\frac{\partial^2 \overline{C}}{\partial \bar{y}^2} - K_0 \overline{C}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \tag{10}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &\bar{t} \le 0, \bar{u} = 0, w = 0, \overline{C} = 0, \theta = 0, \text{for all the values of y}, \\ &\bar{t} > 0, \bar{u} = 1, \overline{w} = 0, \theta = 1, \overline{C} = \bar{t} \text{ at } \bar{y} = 0, \\ &u \to 0, w \to 0, \overline{C} \to 0, \theta \to 0 \quad \text{as } \bar{y} \to \infty. \end{aligned}\right\} \tag{11}$$

Dropping the bars and combining the equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta + GmC - \left(\frac{M}{1+m^2}(1-mi)\right)q, \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_0 C, \tag{13}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{14}$$

Here $q = u + iw$. The corresponding boundary conditions become

$$\left.\begin{aligned} &t \le 0, q = 0, C = 0, \theta = 0, \text{for all the values of y}, \\ &t > 0, q = 1, \theta = 1, C = t \text{ at y} = 0, \\ &q \to 0, C \to 0, \theta \to 0 \quad \text{as y} \to \infty. \end{aligned}\right\} \tag{15}$$

The solution of the above equations is obtained by the Laplace transform method, which is an under

$$\theta = Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$C = \frac{1}{4\sqrt{ScK_0}} e^{-y\sqrt{ScK_0}} [B_{11}](-Scy + 2t\sqrt{ScK_0}) + e^{2y\sqrt{ScK_0}}$$
$$[B_{12}](Scy + 2t\sqrt{ScK_0}),$$

$$q=\frac{1}{2}e^{\sqrt{a}y}(A_0)+\frac{1}{2a}(-e^{-\sqrt{a}y}(A_0)+e^{\frac{at}{-1+Pr}-y\sqrt{\frac{at}{-1+Pr}}}(A_{13}+e^{2y\sqrt{\frac{at}{-1+Pr}}}A_{14}))Gr+\frac{1}{4(a-K_0Sc)^2}yGm(\frac{2e^{-\sqrt{a}y}A_{01}}{y}(1-a_0)$$

$$+\sqrt{a}e^{-\sqrt{a}y}(A_0-e^{2\sqrt{a}y}A_{12})+\frac{1}{y}2e^{\frac{at}{-1+Sc}-\frac{tK_0Sc}{-1+Sc}-yA_{17}}(-1-e^{2yA_{17}}+A_{15}+e^{2yA_{17}}A_{16})[1-Sc]+\frac{1}{y}2e^{-\sqrt{ay}}(-1-e^{\sqrt{ay}}-A_{11}$$

$$+e^{2\sqrt{ay}}A_{12})Sc[1-tK_0]-\frac{1}{\sqrt{a}}e^{-\sqrt{ay}}(A_0-e^{2\sqrt{ay}})K_0Sc)-\frac{1}{2a}(e^{\frac{at}{-1+Pr}-y\sqrt{\frac{at}{-1+Pr}}}(1+A_{18}+e^{2y\sqrt{\frac{aPr}{-1+Pr}}}A_{19})-2Erfc[\frac{y\sqrt{Pr}}{2\sqrt{t}}])Gr$$

$$+\frac{1}{4(a-K_0Sc)^2}yGm\sqrt{Sc}(-\frac{1}{\sqrt{K_0}}ae^{-y\sqrt{K_0Sc}}(-1-e^{2y\sqrt{K_0Sc}}+A_{20}+e^{2y\sqrt{K_0Sc}}A_{21})+\frac{1}{y\sqrt{Sc}}2e^{-y\sqrt{K_0Sc}}(-1-e^{2y\sqrt{K_0Sc}}+A_{20}$$

$$+e^{2y\sqrt{K_0Sc}}A_{21})[1-t-Sc]-\frac{1}{y\sqrt{Sc}}2e^{\frac{at}{-1+Sc}-yA_{17}\frac{tK_0Sc}{-1+Sc}}(-1-e^{2yA_{17}}-A_{22}+e^{2y\sqrt{\frac{a-K_0}{-1+Sc}}\sqrt{Sc}}A_{23})[1-Sc]$$

$$+e^{-y\sqrt{K_0Sc}}[\frac{1}{y}2t((-1-e^{2y\sqrt{K_0Sc}}-A_{20}+e^{2y\sqrt{K_0Sc}}A_{21})K_0Sc)+(1-e^{2y\sqrt{K_0Sc}}+A_{20}+e^{2y\sqrt{K_0Sc}}\sqrt{K_0}Sc)].$$

2. 1 SKIN FRACTION

The dimensionless skin friction at the plate $y = 0$ is computed by

$$\left(\frac{dq}{dy}\right)_{y=0}=\tau_x+i\tau_z$$

2.2 SHERWOOD NUMBER

The dimensionless Sherwood number at the plate $y = 0$ is computed by

$$Sh=\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

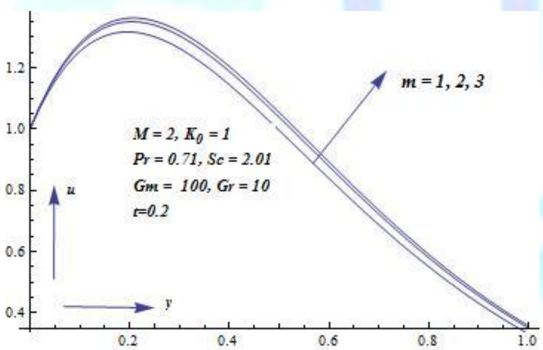
$$Sh = -\frac{1}{4}e^{-y\sqrt{ScK_0}}(B_{11}(-Scy + 2t\sqrt{ScK_0}) + e^{2y\sqrt{ScK_0}}(B_{12}(Scy + 2t\sqrt{ScK_0})) + \frac{1}{4\sqrt{ScK_0}}e^{-y\sqrt{ScK_0}}(-ScB_{11}e^{2y\sqrt{ScK_0}}ScB_{12})$$

$$-\sqrt{\frac{Sc}{\pi t}}(e^{B_{11}^{\ 2}}(-Sc + 2t\sqrt{ScK_0}) + e^{-B_{12}^{\ 2}+2y\sqrt{ScK_0}}(Sc + 2t\sqrt{ScK_0})) + 2e^{2y\sqrt{ScK_0}}B_{12}(Sc + 2t\sqrt{ScK_0}).$$

3. INTERPRETATION OF RESULTS

The velocity, skin friction and Sherwood number computed for different parameters.

Figures 1 and 5 show that u increases when *m,* and *t* are increased. Figures 2, 3, 4, and 6 show that u decreases when *M, Sc*, *Pr* and, K_0 are increased. Further, it is deduced from figures 8 and 11 that w increases when *M,* and *t* are increased. Figures 7, 9, 10 and 12 show that w decreases when *m, Pr, Sc* and K_0 are increased. Further, Figures 13 and 14 show that concentration decreases when K_0, and *Sc* are increased. These results are in agreement with the actual flow of the fluid. From table – 1 it is deduced that Sherwood number decreases with increase in *Sc,* K_0, and *t.* From table - 2 it is deduced that τ_x decreases with increase in *Pr, M, Sc* and K_0, and it increases with increase in *m, Gm, Gr,* and *t*. The value of τ_z increases with increase in *Gr, t, Gm,* and *M.* Further, it decreases when *Sc, Pr,* K_0 and *m* are increased. Effectively it is as per the expectations of the boundary layer theory.



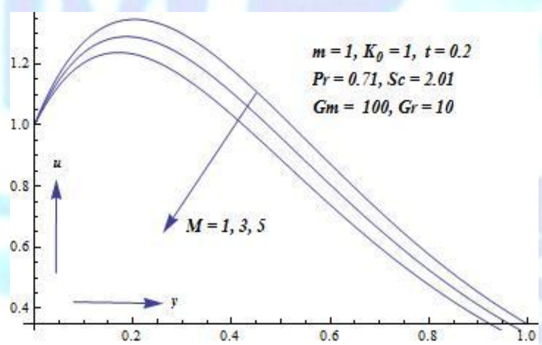
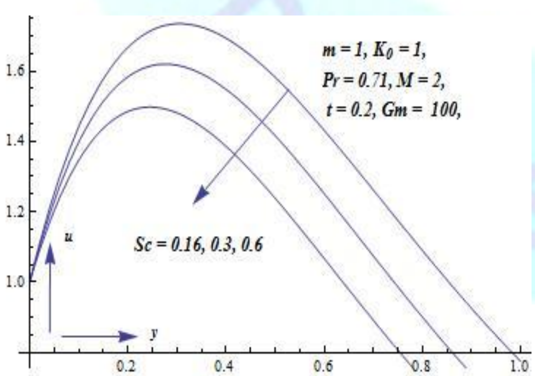


Fig 1: *u vs y* for different values of *m*

Fig 2: *u vs y* for different values of *M*



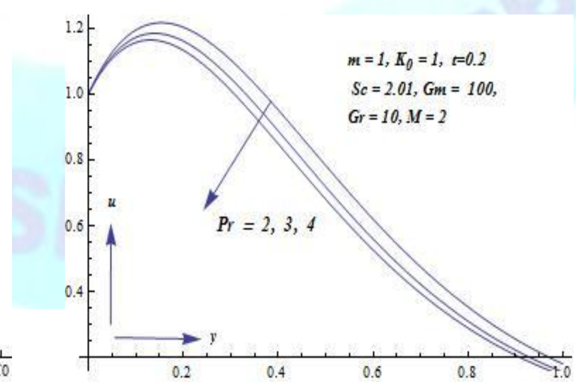


Fig 3: *u vs y* for different values of *Sc*

Fig 4: *u vs y* for different values of Pr

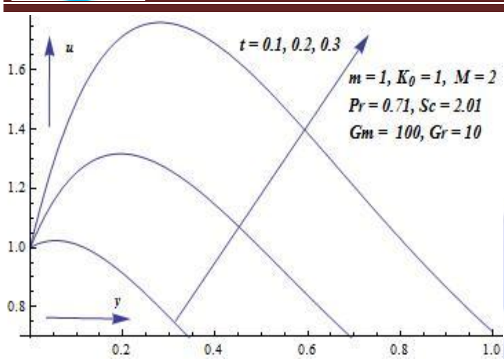


Fig 5: ***u vs y*** **for different values of t**

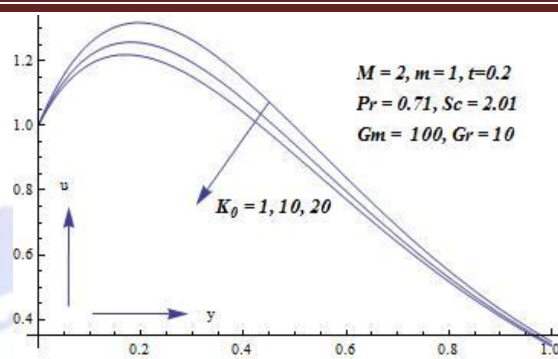


Fig 6 ***u vs y*** **for different values of K_0**

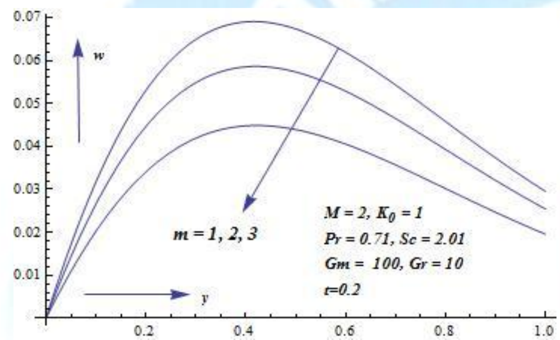


Fig 7: ***w vs y*** **for different values of m**

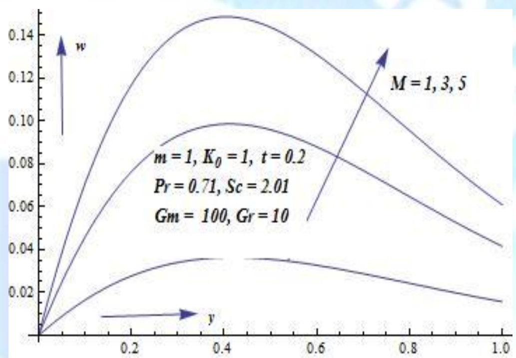


Fig 8: ***w vs y*** **for different values of M**

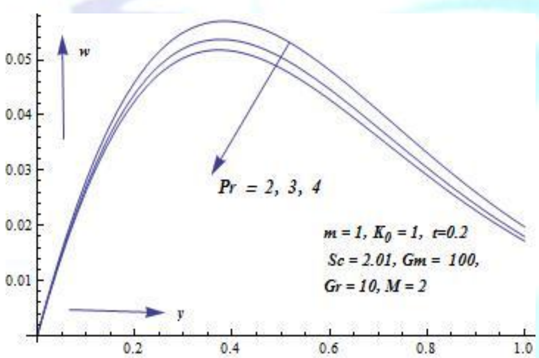


Fig 9 ***w vs y*** **for different values of Pr**

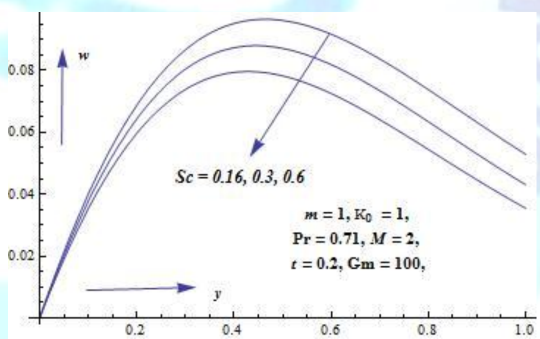


Fig 10: ***w vs y*** **for different values of Sc**

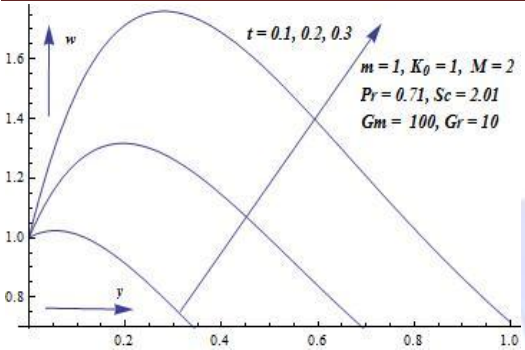


Fig 11: ***w vs y*** **for different values of t**

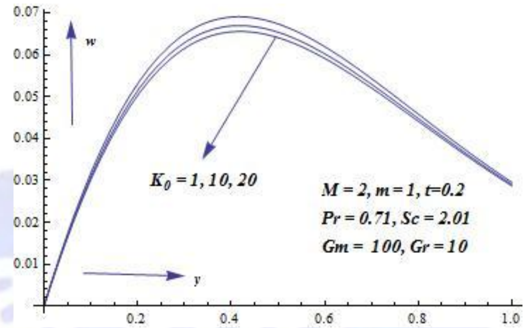


Fig 12: ***w vs y*** **for different values of K_0**

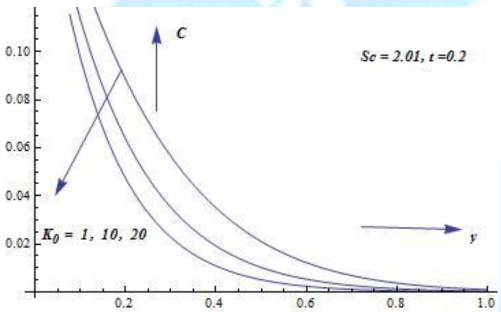


Fig 13 :*C vs y* for different values of K_0

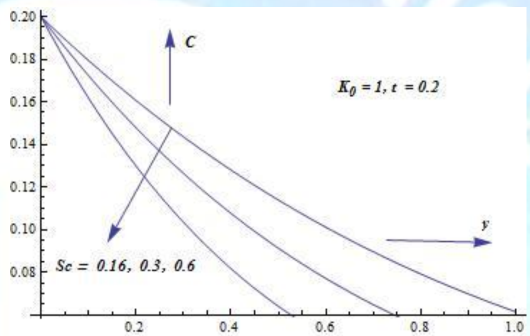


Fig 14 : *C vs y* for different values of Sc

Table - 1 Sherwood number

Sc	***K_0***	***t***	***Sh***
0.16	1.0	0.2	-0.2150
0.3	1.0	0.2	-0.2944
0.6	1.0	0.2	-0.4164
2.01	**10**	0.2	-1.1182
2.01	**20**	0.2	-1.4264
2.01	1.0	**0.1**	-0.5225
2.01	1.0	**0.2**	-0.7621

Table – 2 Skin fraction

m	*Gr*	*Gm*	*M*	*Sc*	K_0	*Pr*	*t*	τ_x	τ_z
1.0	10	**10**	2	2.01	1.0	0.71	0.2	1.3939	0.3315
1.0	10	**20**	2	2.01	1.0	0.71	0.2	1.6614	0.3357
1.0	**5**	100	2	2.01	1.0	0.71	0.2	2.4815	0.3239
1.0	**10**	100	2	2.01	1.0	0.71	0.2	3.8014	0.3695
2.0	10	100	2	2.01	1.0	0.71	0.2	4.0317	0.3079
3.0	10	100	1	2.01	1.0	0.71	0.2	4.1129	0.2341
1.0	10	100	**3**	2.01	1.0	0.71	0.2	3.6056	0.5357
1.0	10	100	**5**	2.01	1.0	0.71	0.2	3.2202	0.8345
1.0	10	100	2	2.01	1.0	**2.00**	0.2	3.1942	0.3320
1.0	10	100	2	2.01	1.0	**3.00**	0.2	2.9636	0.3204
1.0	10	100	2	**0.16**	1.0	0.71	0.2	5.7437	0.4533
1.0	10	100	2	**0.30**	1.0	0.71	0.2	5.3035	0.4304
1.0	10	100	2	2.01	1.0	0.71	**0.1**	0.9034	0.2139
1.0	10	100	2	2.01	1.0	0.71	**0.2**	3.8014	0.3695
1.0	10	100	2	2.01	**10**	0.71	0.2	3.3781	0.3598
1.0	10	100	2	2.01	**20**	0.71	0.2	3.0806	0.3531

4. CONCLUSION

Velocity *u* increases with the increase in *m*, and *t*. But it decreases with the increase in *M, Pr,* K_0 and *Sc.* Further, velocity *w* increases with increase in *t* and *M*. However, it decreases with the increase in *m, Pr,* K_0 and *Sc.* The value of τ_x decreases with the increase in *Pr, M, Sc,* K_0, and it increases with the increase in *m, Gm, Gr,* and *t*. The value of τ_z increases with the increase in *Gr, t, Gm*, and *M*. But it decreases when *Sc, Pr,* K_0 and *m* are increased. Concentration decreases with increase in *Sc* and K_0. Sherwood number decreases with increase in *Sc,* K_0 and *t*.

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APPENDIX $A_0 = 1 + A_{11} + e^{2\sqrt{a}y} A_{12},\ A_{01} = 1 + e^{2\sqrt{a}y} + A_{11} - e^{2\sqrt{a}y} A_{12},$

$$A_{11} = Erf\{\frac{2a\sqrt{t} - y}{2\sqrt{t}}\},\ A_{12} = Erf\{\frac{2a\sqrt{t} - y}{2\sqrt{t}}\},\ A_{13} = Erf\{\frac{y - 2t\sqrt{a(1 + \frac{1}{-1+\Pr})}}{2\sqrt{t}}\},\ A_{14} = Erf\{\frac{y + 2t\sqrt{a(1 + \frac{1}{-1+\Pr})}}{2\sqrt{t}}\},$$

$$A_{15} = Erf\{\frac{y - 2t\sqrt{y - \frac{(a-K_0)Sc}{-1+Sc})}}{2\sqrt{t}}\},\ A_{16} = Erf\{\frac{y + 2t\sqrt{y - \frac{(a-K_0)Sc}{-1+Sc})}}{2\sqrt{t}}\},\ A_{17} = \sqrt{\frac{(a-K_0)Sc}{-1+Sc}}),\ A_{18} = Erf\{\frac{2t\sqrt{\frac{a}{-1+\Pr}} - y\sqrt{\Pr}}{2\sqrt{t}}\},$$

$$A_{19} = Erf\{\frac{2t\sqrt{\frac{a}{-1+\Pr}} + y\sqrt{\Pr}}{2\sqrt{t}}\},\ A_{20} = Erf\{\frac{2t\sqrt{K_0} - y\sqrt{Sc}}{2\sqrt{t}}\},\ A_{21} = Erf\{\frac{2t\sqrt{K_0} + y\sqrt{Sc}}{2\sqrt{t}}\},\ A_{22} = Erf\{\frac{2t\sqrt{\frac{a-K_0}{-1+Sc}} - y\sqrt{Sc}}{2\sqrt{t}}\},$$

$$A_{22} = Erf\{\frac{2t\sqrt{\frac{a-K_0}{-1+Sc}} - y\sqrt{Sc}}{2\sqrt{t}}\},\ B_{11} = Erfc\{\frac{\sqrt{Sc}y - 2t\sqrt{K_0}}{2\sqrt{t}}\},\ B_{12} = Erfc\{\frac{\sqrt{Sc}y + 2t\sqrt{K_0}}{2\sqrt{t}}\},\ a = \frac{M}{1+m^2}(1 - im),$$