Vol.09 Issue-01, (January - June, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

CHEMICAL REACTION AND SORET EFFECTS ON FREE CONVECTION MHD FLOW PAST A MOVING INCLINED PLATE WITH VARIABLE TEMPERATURE AND CONSTANT MASS DIFFUSION

U S Rajput and N K Gupta
Department of Mathematics & Astronomy, Lucknow University, Lucknow-226007 U.P, India

ABSTRACT
In this paper, we study the chemical reaction and Soret effects on unsteady free convective MHD flow past a moving inclined plate with variable temperature and constant mass diffusion. Fluid considered is electrically conducting. The Laplace transform method has been used to find the exact solutions for the momentum, energy and concentration equations. The flow model consists of a plate inclined from vertical by an angle ξ. The flow considered is unsteady, and the heat is transferred through free convection. The velocity and temperature profiles are discussed with the help of graphs drawn for different parameters like thermal and mass Grashof numbers, the Prandtl number, the magnetic field parameter, the Soret number, the Schmidt number, chemical reaction parameter, time and inclination of plate. The numerical values of skin frictions and Nusselt numbers have been discussed with the help of table for different parameters.

KEYWORDS: MHD, inclined plate, Soret effect, chemical reaction, free convection, mass transfer, heat transfer.

INTRODUCTION

The effect of magnetic field on viscous, incompressible and electrically conducting fluid plays very important role in many applications such as glass manufacturing, control processing, paper industry, textile industry, magnetic materials processing and purification of crude oil etc. Bejan and Khair [1] have studied heat and mass transfer by natural convection in a porous medium. MHD flow between two parallel plates with heat transfer have been studied by Attia and Katb [3]. Hossain and Shayo [2] have discussed unsteady free convection flow past an exponentially accelerated plate. Das and Jana [8] have studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Rajesh [9] has studied MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Soret effect on unsteady MHD flow through porous medium past an oscillating inclined plate with variable wall temperature and mass diffusion have been discussed by Rajput and Kumar [15]. Postelnicu [5] has studied Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Alam et al. [4] have studied Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. Reddy [7] has discussed Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Dipak et al. [10] have studied Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate. Ibrahim [6] has discussed analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and Hall effect. The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source have been discussed by Bhavana et al. [12]. Kesavaiah and Satyanarayana [13] have studied radiation absorption and Dufour effects to MHD flow in vertical surface. Rajput and Gupta [14] have discussed Dufour effect on unsteady free convection

MHD flow past an exponentially accelerated plate through porous media with variable temperature and constant mass diffusion in an inclined magnetic field. Dufour and Soret effect on steady MHD flow in presence of Heat generation and magnetic field past an inclined stretching sheet have been studied by Karim et al. [11].

In this paper we are analyzing the chemical reaction and Soret effects on unsteady free convection MHD flow past a moving inclined plate with variable temperature and constant mass diffusion.

MATHEMATICAL ANALYSIS

Consider an unsteady viscous incompressible electrically conducting fluid past an impulsively started vertical plate with velocity u_o. The x axis is taken along the vertical plane and *y* normal to it. The plate is inclined at angle ξ from vertical and plate is electrically non-conducting. The magnetic field B_o of uniform strength is applied perpendicular to the flow. We assume that the Magnetic Reynolds number is so small that the induced magnetic field can be neglected in comparison to the applied one. Initially the fluid and plate are at the same temperature T_∞ and the concentration of the fluid is C_∞. At time $t \geq 0$ temperature of the plate is raised to T_w and the concentration of the fluid is raised to C_w.

The governing equations under the usual Boussinesq's approximations are as follows:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta Cos(\xi)(T - T_\infty) + g\beta^* Cos(\xi)(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u, \tag{1}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} + \frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} - K_C(C - C_\infty), \tag{2}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

The initial and boundary conditions are given as:

$$\left.\begin{aligned} & t \le 0; u = 0, T = T_\infty, \mathrm{C} = \mathrm{C}_\infty \text{ for each value of } y, \\ & t > 0; u = u_0, T = T_\infty + (T_w - T_\infty)\frac{u_0^2 t}{\upsilon}, C = C_w \; at \; y = 0, \\ & u \to 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty. \end{aligned}\right\} \tag{4}$$

Here u is the velocity of the fluid, g – the acceleration due to gravity, β – volumetric coefficient of thermal expansion, β^* – volumetric coefficient of concentration expansion, t – time, T – the temperature of the fluid,

T_∞ - the temperature of the plate at $y \to \infty$, C - species concentration in the fluid, C_∞ - species concentration at

$y \to \infty$, υ - the kinematic viscosity, ρ - the density, C_P - the specific heat at constant pressure, k – thermal conductivity of the fluid, K_T - thermal diffusion ratio, D – the mass diffusion constant, D_m - the effective mass diffusivity rate, T_w - the temperature of the plate at $y=0$, C_w - species concentration at the plate at $y = 0$, T_m - mean fluid temperature, B_o - the uniform magnetic field, σ - electrical conductivity, ξ - angle of inclination from vertical and K_c - chemical reaction parameter.

By using the following dimensionless quantities, the above equations (1), (2), and (3) can be transformed into dimensionless form.

$$\left.\begin{array}{l} \bar{y}=\dfrac{yu_0}{\upsilon}, \bar{t}=\dfrac{tu_0^2}{\upsilon}, \bar{u}=\dfrac{u}{u_0}, \theta=\dfrac{T-T_\infty}{T_w-T_\infty}, \bar{C}=\dfrac{C-C_\infty}{C_w-C_\infty}, Sc=\dfrac{\upsilon}{D},\ M=\dfrac{\sigma B_0^2\upsilon}{\rho u_0^2}, K_O=\dfrac{K_C\upsilon}{u_0^2}, \\ Gr=\dfrac{g\beta\nu(T_w-T_\infty)}{u_0^3},\ \mu=\upsilon\rho,\ Gm=\dfrac{g\beta^*\upsilon(C_w-C_\infty)}{u_0^3},\ Sr=\dfrac{D_m K_T(T_w-T_\infty)}{\upsilon T_m(C_w-C_\infty)}, \Pr=\dfrac{\mu C_p}{k}, \end{array}\right\} \quad (5)$$

Here $\bar{u}$ is dimensionless velocity, $\bar{t}$ - dimensionless time, *Pr*- Prandtl number, *Sc*- Schmidt number, *Gr* - thermal Grashof number, *Gm*- mass Grashof number, θ - dimensionless temperature, $\bar{C}$ - dimensionless concentration, *M* -the magnetic field parameter, μ - the coefficient of viscosity, $\bar{K}$ – permeability parameter, K_o - dimensionless chemical reaction parameter and *Sr* - Soret number. Then model is transformed into the following non dimensional form of equations:

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+GrCos(\xi)\theta+GmCos(\xi)\bar{C}-M\bar{u}, \quad (6)$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}+Sr\frac{\partial^2\theta}{\partial\bar{y}^2}-\mathrm{K_O}\bar{\mathrm{C}}. \quad (7)$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\bar{y}^2}, \quad (8)$$

The initial and boundary condition become:

$$\left.\begin{array}{l} \bar{t}\le 0; \bar{u}=0, \theta=0, \bar{C}=0 \text{ for each value of } \bar{y}, \\ \bar{t}>0; \bar{u}=1, \theta=\bar{t}, \bar{C}=1 \text{ at } \bar{y}=0, \\ \bar{u}\to 0, \theta\to 0, \bar{C}\to 0 \text{ as } \bar{y}\to\infty. \end{array}\right\} \quad (9)$$

Dropping bars in the above equations, we get:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+GrCos(\xi)\theta+GmCos(\xi)C-Mu, \quad (10)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+Sr\frac{\partial^2\theta}{\partial y^2}-\mathrm{K_O C}. \quad (11)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}, \quad (12)$$

The initial and boundary condition become:

$$\left.\begin{array}{l} t \le 0; u = 0, \theta = 0, C = 0 \text{ for each value of } y, \\ t > 0; u = 1, \theta = t, C = 1 \text{ at } y = 0, \\ u \to 0, \theta \to 0, C \to 0 \text{ as } y \to \infty. \end{array}\right\} \quad (13)$$

Now the solution of equations (10), (11) and (12) under the boundary conditions (13) are obtained by the Laplace - transform technique. The exact solutions for fluid temperature θ, species concentration *C*, and fluid velocity *u* are respectively:

$$\theta = \left(-t - \frac{\Pr y^2}{2}\right)A_7 - \frac{A_{41}\sqrt{\Pr t}\, y}{\sqrt{\pi}}$$

$$C = \frac{1}{2K_O}[\Pr(2(A_{17}-1) + A_{10}B_{26} + A_{13}^{-1}B_7A_{14} - A_{14}A_{13}^{-1}(1 + A_{13}^2 - A_{19}A_{13}^2 + A_{18}))Sr] + \frac{A_{10}}{2}(1 + A_{11} + A_{12}A_{10}^{-2})$$

$$u = -\frac{M\Pr Sc\sqrt{\pi}(K_O Sc - M)Gm}{A_{36}}C_{12} + \frac{A_{29}B_{21}GmA_{28}A_{30}}{2(K_O Sc - M)} + C_{10}\frac{A_{28}A_{20}A_{29}B_{13}Gm}{2(K_O Sc - M)} - C_{11} - K_O ScB_{20}$$
$$-\frac{Gm\Pr Sr\sqrt{Sc}}{A_{36}}(C_{15} - C_{14}) + Gm\frac{A_{10}(1 + A_{11} + A_{12}A_{10}^{-2})}{2(K_O Sc - M)} + B_8 - \frac{GmSr}{A_{42}}(2\sqrt{\pi}(1 - A_{17})B_{27} - C_{13}$$
$$+ A_{14}A_{13}^{-1}\sqrt{\pi}\Pr B_{27}(-\Pr M + Sc))$$

The expressions for the constants involved in the above equations are given in the appendix.

SKIN- FRICTION

We calculate the non-dimensional form of skin friction (τ) from the velocity field as:

$$\tau = \left(-\frac{\partial u}{\partial y}\right)_{y=0}$$

And numerical values of τ for different parameters are given in table-1.

NUSSELT NUMBER

From temperature field, we study non dimensional form of rate of heat transfer (*Nu*) which is given as:

$$Nu = \left(-\frac{\partial \theta}{\partial y}\right)_{y=0}$$

Numerical values of *Nu* for different parameters are given in table-2.

RESULTS AND DISCUSSION

The numerical values of velocity, concentration and temperature are computed for different parameters like thermal Grashof number *Gr,* mass Grashof number *Gm*, magnetic field parameter *M*, Prandtl number *Pr*, Schmidt number *Sc*, inclination *ξ*, permeability parameter *K,* Soret number *Sr,* chemical reaction parameter *Ko* and time *t*. The values of the parameters considered are

M= 4, 9, 16, *Pr* = 0.71, 2.00, 7.00, $\xi = 30°, 45°, 60°$

Gm = 40, 50, 60, *Gr* = 5, 50, 200, , K_o = 2, 4, 6

Sr = 2, 5 , 8, *t* = 0.2, 0.3, 0.4. *Sc* = 1.50, 2.01, 3.00.

Figures 1, 2, 3, 4 and 5 show that velocity increases when one of the following parameters, *Sr, Pr, Gm, t, and Gr* are increased keeping others constant. Figures 6, 7, 8 and 9 show that velocity decreases when *ξ, M, Ko* and *Sc* are increased keeping others constant.

Figure 11 shows that temperature increases when *t* is increased; and Figure 10 shows that temperature decreases when *Pr* is increased.

Skin- friction is given in table-1. The value of skin-friction increases with increasing the values of *M, Sc, ξ, Ko;* and decreases with increasing the values of *Sr*, *Gm, t* , *Pr* and *Gr*.

Nusselt number is given in table-2 and its value increases with increasing the values of *t* and *Pr*.

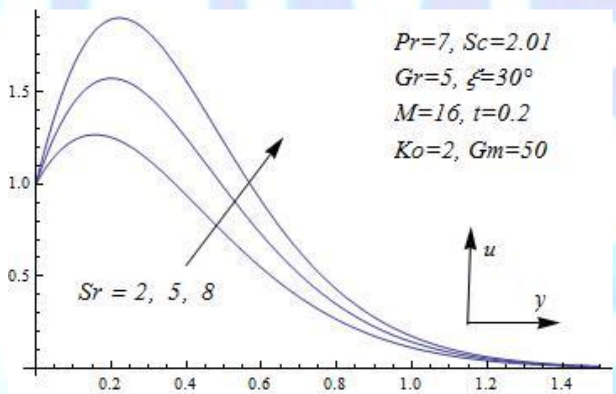

Figure-1: Velocity profile for different values of *Sr*

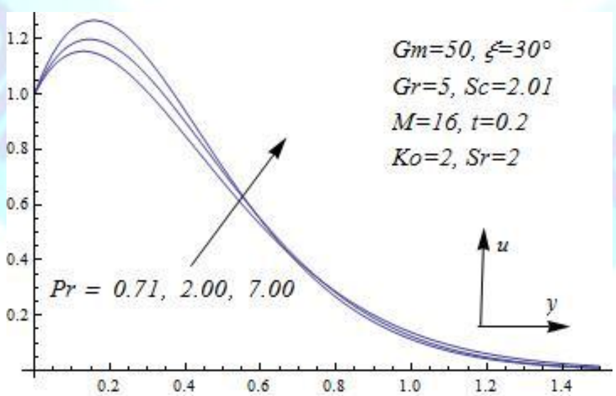

Figure-2: Velocity profile for different values of *Pr*

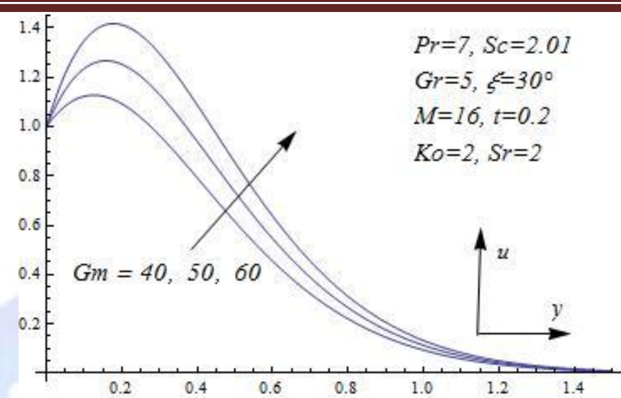

Figure-3: Velocity profile for different values of *Gm*

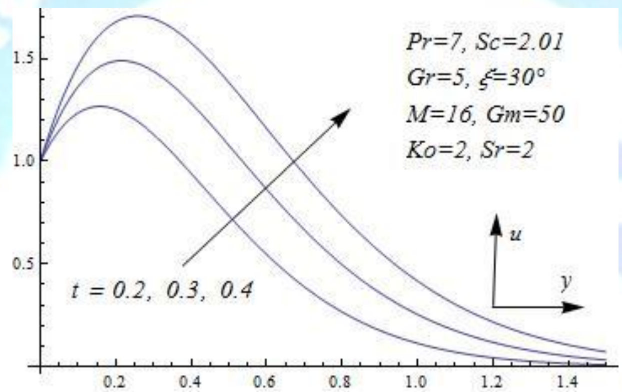

Figure-4: Velocity profile for different values of *t*

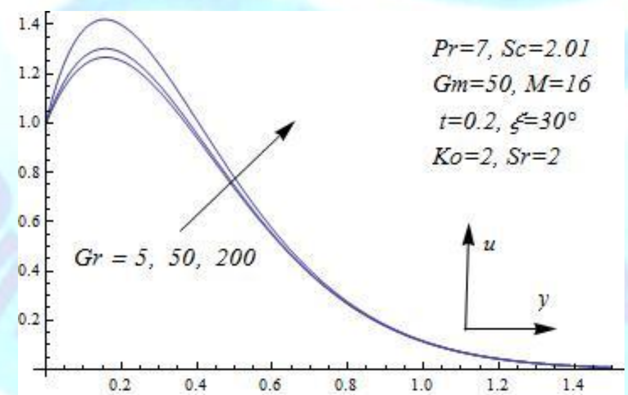

Figure-5: Velocity profile for different values of *Gr*

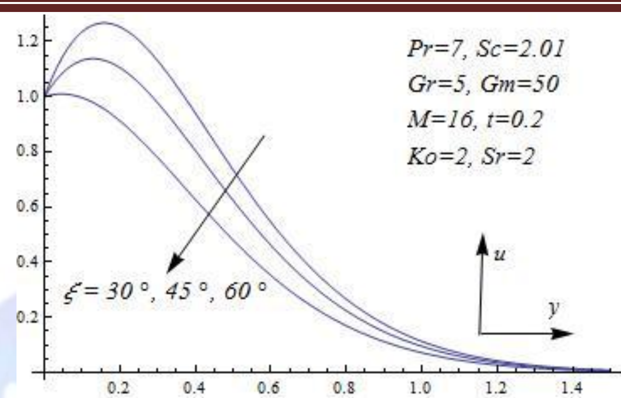

Figure-6: Velocity profile for different values of ξ

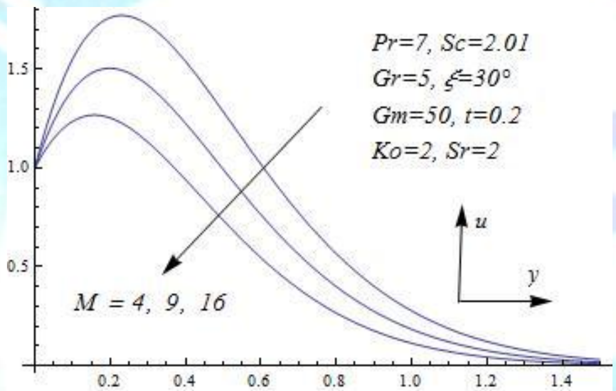

Figure-7: Velocity profile for different values of *M*

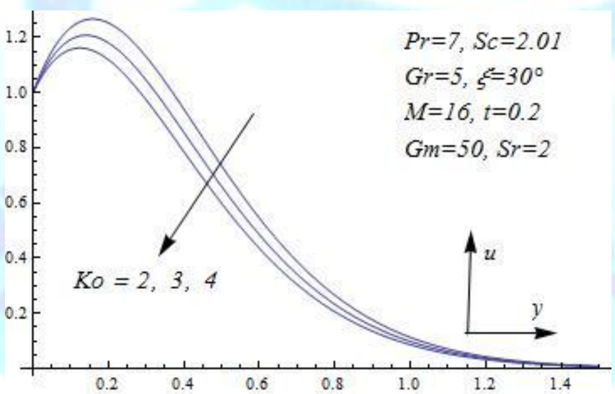

Figure-8: Velocity profile for different values of *Ko*

Vol.09 Issue-01, (January - June, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

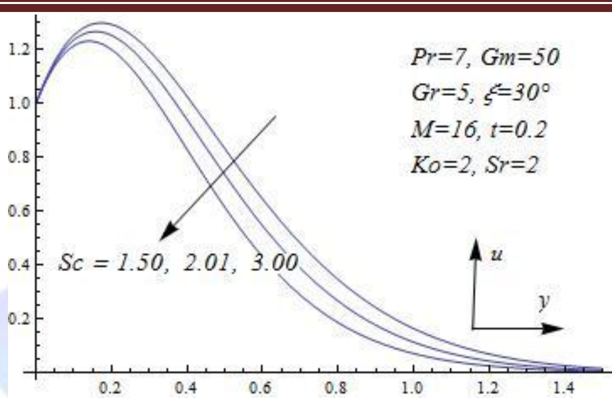

Figure-9: Velocity profile for different values of *Sc*

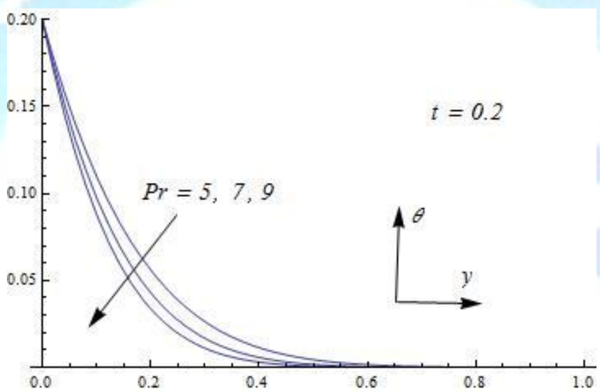

Figure-10: Temperature profile for different values of *Pr*

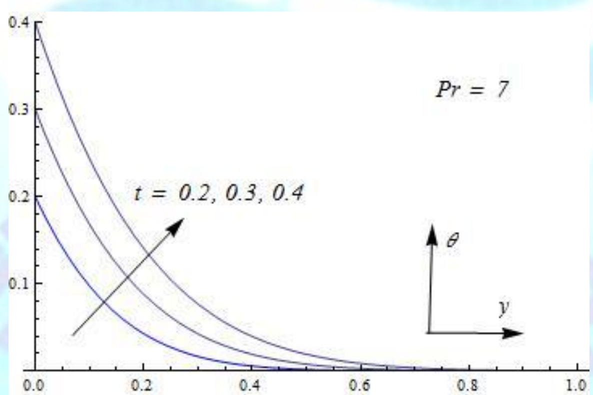

Figure-11: Temperature profile for different values of *t*

Table-1: Skin-Friction for different values of parameters

ξ (In degree)	*M*	*Pr*	*Sr*	*t*	*Ko*	*Gm*	*Gr*	*Sc*	τ
30	16	7	2	0.2	2	50	005	2.01	-3.6732

45	16	7	2	0.2	2	50	005	2.01	-2.2641
60	16	7	2	0.2	2	50	005	2.01	-0.4277
30	04	7	2	0.2	2	50	005	2.01	-7.4367
30	09	7	2	0.2	2	50	005	2.01	-5.6078
30	16	8	2	0.2	2	50	005	2.01	-3.7451
30	16	9	2	0.2	2	50	005	2.01	-3.8088
30	16	7	5	0.2	2	50	005	2.01	-5.9144
30	16	7	8	0.2	2	50	005	2.01	-8.1556
30	16	7	2	0.3	2	50	005	2.01	-5.0080
30	16	7	2	0.4	2	50	005	2.01	-6.0754
30	16	7	2	0.2	4	50	005	2.01	-3.2180
30	16	7	2	0.2	6	50	005	2.01	-2.8271
30	16	7	2	0.2	2	40	005	2.01	-2.1513
30	16	7	2	0.2	2	60	005	2.01	-5.1951
30	16	7	2	0.2	2	50	050	2.01	-4.2983
30	16	7	2	0.2	2	50	200	2.01	-6.3818
30	16	7	2	0.2	2	50	005	1.50	-3.8257
30	16	7	2	0.2	2	50	005	3.00	-3.4944

Table-2: Nusselt number for different values of parameters

Pr	*t*	*Nu*
7	0.2	1.3351
8	0.2	1.4273
9	0.2	1.5138
7	0.3	1.6351
7	0.4	1.8881

CONCLUSION

Some conclusions of study are as below:

- The velocity of the fluid increases with increasing the values of Soret number (*Sr*), *time (t)*, thermal Grashof number (*Gr*), mass Grashof number *(Gm)* and Prandtl number (*Pr).*
- The velocity of the fluid decreases with increasing the values of inclination (*ξ)*, magnetic field parameter (*M),* chemical reaction parameter *(Ko)* and Schmidt number (*Sc).*
- The values of skin friction increases with increasing the values of magnetic field parameter (*M),* chemical reaction parameter *(Ko),* inclination (*ξ)* and Schmidt number (*Sc).*

- The values of skin friction decreases with increasing the values of mass Grashof number *(Gm), time (t),* Soret number (*Sr),* thermal Grashof number (*Gr*) and Prandtl number (*Pr).*
- The temperature of the fluid increases with increasing the values of time (*t).*
- The temperature of the fluid decreases with increasing the values of Prandtl number (*Pr).*
- Nusselt number increases with increasing the values of time *(t)* and Prandtl number (*Pr).*

REFERENCES

[1] Bejan and K. R. Khair, “Heat and mass transfer by natural convection in a porous medium” Int. J. Heat Mass Transfer, vol. 28, pp. 909-918 (1985).
[2] Hossain M A and Shayo L K, “The skin-friction in the unsteady free convection flow past an exponentially accelerated plate” Astrophysics and Space Science, 125, 315-324 (1986).
[3] Attia H A and katb N A, “MHD flow between two parallel plate with Heat Transfer” Acta Mechanica, 117, 215-220 (1996).
[4] Alam M S, Rahman M M, and Samad M A, “Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium” Nonlinear Analysis modeling and control, vol. 11, no. 3, pp. 217-226 (2006).
[5] Postelnicu, “Influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects” Heat Mass Transfer, vol. 43, pp.595-602 (2007).
[6] Ibrahim a. abdallah, “analytic solution of heat and mass transfer over a permeable stretching plate affected by chemical reaction, internal heating, Dufour-Soret effect and hall effect” thermal science: vol. 13 no. 2, pp. 183-197 (2009).
[7] Reddy M G and Reddy N B, “Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation” Int. J. of Appl. Math and Mech., vol. 6, no. 1, pp. 1-12 (2010).
[8] Das K and Jana S “Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium” Bull. Soc. Math. Banja Luka, Vol. 17, pp.15-32 (2010).
[9] Rajesh V “MHD effects on free convection and mass transform flow through a porous medium with variable temperature” Int. J. of Appl. Math. and Mech., 6(14), pp.1-16 (2010).
[10]Dipak Sharma, Nazibuddin Ahmed and Pulak Kr Mahanta, “Soret and Dufour effects on steady MHD convective flow past a continuously moving porous vertical plate” Int. J. of Eng. Sci. and Tech., vol 4, no. 12, pp. 4757-4765 (2012).
[11]M D Enamul Karim, M D Abdus Samad and M D Maruf Hasan,“Dufour and Soret effect on steady MHD flow in presence of Heat generation and magnetic field past an inclined stretching sheet” Open Journal of Fluid Dynamics*,* vol. 2, pp. 91-100, (2012).
[12]M. Bhavana, D. Chenna Kesaraiah and A. Sudhakarraiah, “The Soret effect on free convective unsteady MHD flow over a vertical plate with heat source” *Int. J. of Innovative R. in Sci. Eng. And Tech*., vol. 2(5), pp. 1617-1628, (2013).
[13]Damala Chenna Kesavaiah and Satyanarayana P V, “Radiation absorption and dufour effects to MHD flow in vertical surface" Global Journal of engineering, Design & Technology, Vol 3(2); 51-57 (2014).
[14]U S Rajput and N K Gupta, “Dufour effect on unsteady free convection MHD flow past an exponentially accelerated plate through porous media with variable temperature and constant

mass diffusion in an inclined magnetic field" International research journal of engineering and technology -Vol 3(8), 2135-2140 (2016).

[15]U S Rajput and Gaurav Kumar, "Soret effect on unsteady MHD flow through porous medium past an oscillating inclined plate with variable wall temperature and mass diffusion" International research journal of engineering and technology -Vol 3(5), 2353-2358 (2016).

APPENDIX

$A_7 = Erfc\left[\frac{\sqrt{\Pr} y}{2\sqrt{t}}\right]$, $A_8 = Erf\left[\sqrt{\frac{\Pr tK_O}{\Pr - Sc}} + \frac{\sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_9 = Erf\left[\sqrt{tK_O} + \frac{\sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_{10} = e^{-\sqrt{K_O Sc} y}$, $A_{11} = Erf\left[\sqrt{tK_O} - \frac{\sqrt{Sc} y}{2\sqrt{t}}\right]$,

$A_{12} = Erfc\left[\sqrt{tK_O} + \frac{\sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_{13} = e^{\sqrt{\frac{\Pr Sc K_O}{\Pr - Sc}} y}$, $A_{14} = e^{\frac{tScK_O}{\Pr - Sc}}$ $A_{15} = Erf\left[\frac{2\sqrt{\frac{K_O \Pr}{\Pr - Sc}} t - \sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_{16} = Erfc\left[\frac{2\sqrt{\frac{K_O \Pr}{\Pr - Sc}} t + \sqrt{Sc} y}{2\sqrt{t}}\right]$,

$A_{17} = Erf\left[\frac{\sqrt{\Pr} y}{2\sqrt{t}}\right]$, $A_{18} = Erf\left[\frac{2\sqrt{\frac{K_O Sc}{\Pr - Sc}} t - \sqrt{\Pr} y}{2\sqrt{t}}\right]$, $A_{19} = Erf\left[\frac{2\sqrt{\frac{K_O Sc}{\Pr - Sc}} t + \sqrt{\Pr} y}{2\sqrt{t}}\right]$, $A_{20} = e^{-\sqrt{M} y}$, $A_{21} = Erf\left[\frac{2\sqrt{M} t - y}{2\sqrt{t}}\right]$,

$A_{22} = Erfc\left[\frac{2\sqrt{M} t + y}{2\sqrt{t}}\right]$, $A_{23} = Erf\left[\frac{2\sqrt{M} t + y}{2\sqrt{t}}\right]$, $A_{24} = e^{\sqrt{\frac{\Pr M}{\Pr - 1}} y}$, $A_{25} = e^{\frac{Mt}{\Pr - 1}}$, $A_{26} = Erf\left[\frac{2\sqrt{\frac{M \Pr}{\Pr - 1}} t - y}{2\sqrt{t}}\right]$, $A_{27} = Erf\left[\frac{2\sqrt{\frac{M \Pr}{\Pr - 1}} t + y}{2\sqrt{t}}\right]$,

$A_{28} = e^{\frac{-K_O Sct}{Sc - 1}}$, $A_{29} = e^{\sqrt{\frac{(M - Ko)Sc}{Sc - 1}} y}$, $A_{30} = e^{\frac{Mt}{Sc - 1}}$, $A_{31} = Erf\left[\frac{2\sqrt{\frac{(M - Ko)Sc}{Sc - 1}} t - y}{2\sqrt{t}}\right]$, $A_{32} = Erf\left[\frac{2\sqrt{\frac{(M - Ko)Sc}{Sc - 1}} t + y}{2\sqrt{t}}\right]$,

$A_{33} = e^{\sqrt{\frac{\Pr M - MSc + KoSc}{\Pr - Sc}} y}$, $A_{34} = Erf\left[\sqrt{\frac{M \Pr - MSc + KoSc}{\Pr - Sc}} \sqrt{t} - \frac{y}{2\sqrt{t}}\right]$, $A_{35} = Erf\left[\sqrt{\frac{M \Pr - MSc + KoSc}{\Pr - Sc}} \sqrt{t} + \frac{y}{2\sqrt{t}}\right]$,

$A_{36} = 2Ko\sqrt{\pi}(-M + KoSc)(KoSc(\Pr - 1) + M(Sc - \Pr))$, $A_{37} = Erf\left[\frac{2\sqrt{\frac{M}{\Pr - 1}} t - \sqrt{\Pr} y}{2\sqrt{t}}\right]$, $A_{38} = Erf\left[\frac{2\sqrt{\frac{M}{\Pr - 1}} t + \sqrt{\Pr} y}{2\sqrt{t}}\right]$,

$A_{39} = Erf\left[\frac{2\sqrt{\frac{M - Ko}{Sc - 1}} t - \sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_{40} = Erf\left[\frac{2\sqrt{\frac{M - Ko}{Sc - 1}} t + \sqrt{Sc} y}{2\sqrt{t}}\right]$, $A_{41} = e^{\frac{-\Pr y^2}{4t}}$, $A_{42} = 2MKo\sqrt{\pi}(A(\Pr - Sc) - KoSc(\Pr - 1))$,

$B_7 = 1 + A_{15} + A_{13}^2 A_{16}$, $B_8 = \frac{1}{2} A_{20}(1 + A_{21} + A_{20}^{-2} A_{22})$, $B_9 = A_{30} A_{28} A_{29}^{-1} Ko\sqrt{\pi Sc}(1 - Sc)$, $B_{10} = 1 + A_{20}^{-2} + A_{21} - A_{20}^{-2} A_{23}$,

$B_{11} = 1 - A_{20}^{-2} + A_{21} + A_{20}^{-2} A_{23}$, $B_{12} = (-1 - A_{24}^2 - A_{26} + A_{24}^2 A_{27})(1 - \Pr)$,

$B_{13} = -A_{30} A_{20}^{-1} + A_{28}^{-1} A_{29} - A_{30} A_{20}^{-1} A_{29}^2 + A_{29} A_{28}^{-1} A_{20}^{-2} + A_{21} A_{29} A_{28}^{-1} - A_{31} A_{30} A_{20}^{-1} - A_{29} A_{23} A_{20}^{-2} A_{20}^{-2} + A_{30} A_{32} A_{20}^{-1} A_{29}^2$

$B_{14} = 1 - \Pr - Mt$, $B_{15} = M\Pr - ASc + KoSc - KoSc\Pr$, $B_{16} = A_{30} A_{28} A_{29}^{-1} Ko\sqrt{\pi} Sc(1 - Sc)(-1 - A_{29}^2 - A_{31} + A_{29}^2 A_{32})$

$B_{17} = A_{14} A_{33}^{-1}(-1 - A_{33}^2 - A_{34} + A_{33}^2 A_{35})$, $B_{18} = M\Pr - MSc - Sc\Pr Ko + Sc^2 Ko$,

$B_{19} = 2A_{41}\sqrt{t\Pr} y + \sqrt{\pi}\Pr y^2 (A_{17} - 1)$, $B_{20} = A_{25} A_{24}^{-1}\sqrt{\pi}\Pr(\Pr - 1)(1 + A_{24}^2 + A_{37} - A_{24}^2 A_{38})$,

$B_{21} = 1 + A_{29}^2 + A_{39} - A_{29}^2 A_{40}$, $B_{22} = -1 - A_{10}^{-2} - A_{11} + A_{10}^{-2} A_9$, $B_{23} = M(\sqrt{Sc} - \frac{\Pr}{\sqrt{Sc}}) + \sqrt{Sc}Ko(\Pr - 1)$

$B_{24} = -1 - A_{13}^2 - A_{15} + A_{13}^2 A_8$, $B_{25} = \frac{M\Pr}{\sqrt{Sc}} - \frac{\sqrt{Sc}}{M} - \sqrt{Sc}Ko\Pr + Sc^{3/2}Ko$, $B_{26} = M\Pr^2 - MSc\Pr + KoSc\Pr - Ko\Pr^2 Sc$

$B_{27} = -1 - A_{13}^2 - A_{18} + A_{13}^2 A_{19}$,

$C_{10} = \frac{Gr}{4M^2}(2A_{20}^{-1}B_{10}B_{14} + \sqrt{M}A_{20}^{-1}yB_{11} + 2A_{25}A_{24}^{-1}B_{12})$, $C_{11} = \frac{Gm\Pr Sr}{A_{36}}(-A_{20}\sqrt{\pi}B_{10}B_{15} - B_{16} - \sqrt{\pi}B_{17}B_{18})$,

$C_{12} = -A_{20}B_{10}B_{15} - A_{25}A_{24}^{-1}KoScB_{12} - M\Pr B_{17}(\Pr - Sc)$, $C_{13} = \frac{-Gr}{2A^2\sqrt{\pi}}(-2\sqrt{\pi}(-1 + A_{17})B_{14} + MB_{19} + B_{20})$

$C_{14} = A_{10}\sqrt{\pi}B_{22}B_{23} + A_{14}A_{13}^{-1}\sqrt{\pi}B_{24}B_{25}$, $C_{15} = A_{30}A_{28}A_{29}^{-1}Ko\sqrt{\pi}ScB_{21}(1 - Sc)$