Type – 2 Fuzzy Soft Sets and Decision Making

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Abstract

In this work, we study the Type-2 fuzzy soft sets by integrating the Type-2 fuzzy set theory and Soft set theory. We have applied the notion of distance based similarity measure between Type-2 fuzzy soft sets to obtain a solution of a decision making problem. A numerical example is solved tofind a solution of a decision making problem in an uncertain environment.

Key words: Decision Making Problem, Distance Based Similarity Measure, Type Reduction Method, Type – 2 Fuzzy Soft Sets, Various Distances.

1: Introduction

The decision making process is of key importance for functions such as inventory control, investments, personal actions, new-product development and allocation of resources as well as many others. Decision making is broadly defined as include any choice or selection of alternatives and is therefore of importance in many fields in both “soft” social sciences and the “hard” disciplines of natural sciences.

Classical decision making generally deals with set of alternative states of nature (outcomes, results), a set of alternative actions. A decision is made under the conditions of certainty when the outcome for each action can be determined and ordered precisely. When the decision making problem becomes an optimization problem, the decision is made under uncertainty. This is the prime domain for fuzzy decision making. Decision making under uncertainty is perhaps the most important category of decision making problems.

The theory of fuzzy sets was introduced by L.A.Zadeh[7] in 1965. But there exist a difficulty to set the membership function in each particular case. We should not impose only one way to set the membership function. For this reason we have the inadequacy of the parametrization tool of the theory. In 1999, D. Molodsov [5] initiated the concept of soft set theory as a mathematical tool for dealing with uncertainties which is free from the above difficulties.

Maji et al [4], first defined the soft sets into the decision making problem. To deal with the fuzziness of problem parameters, Roy and Maji introduced the concept of a fuzzy soft set. They also proposed the concept of fuzzy soft sets and its application in decision making under imprecise environment in the year 2007. The concept of type-2 fuzzy set was introduced by L.A.Zadeh[8] in 1975. Type-2 fuzzy sets can improve certain kinds of inference better than do fuzzy sets with increasing imprecision, uncertainty, fuzziness in information. To represent the fuzziness of evaluation of parameters directly, Zhiming Zhang and Shouhua Zhang [9] present the concept of type-2 fuzzy soft sets in 2012.Similarity measure have extensive application in many areas including decision making. Dr. P. Rajarajeswari and P.Dhanalakshmi[6s] used the notion of similarity measure with three approaches to make decision.

In this paper we have introduced the concept of distance based similarity measure into Type – 2 fuzzy soft sets. Further it is applied to decision making problems.In Section 2, some basic definitions of fuzzy set theory is given. In section 3, various distances and the distance based similarity measure of Type – 2 fuzzy soft set is defined. In section 4, a numerical example is given and a decision is made according to the decision maker's choice.

2: Basic Definitions

Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function μ_A such that the value assigned to the elements of the universal set X fall within a specified range [0,1].That is $\mu_A : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A.

The function μ_A is called the membership function and the set $A = \{(x, \mu_A(x)) : x \in X\}$ defined by μ_A for each $x \in X$ is called a fuzzy set.

Definition 2.2

A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$ where $P(U)$ is a power set of U.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A,\ F(\varepsilon)$ is regarded as the set of ε - approximates of the soft set (F, A).

Definition 2.3

Let U be a finite and non-empty set, which is referred to as the universe. A Type– 2 fuzzy set, denoted by $\tilde{A}$, is characterized by a Type – 2 membership function $\mu_{\tilde{A}}(x,u) : U \times I \to I$ where $x \in U$, I = [0, 1] and $u \in J_x \subseteq I$ that is $\tilde{A} = \{((x,u); \mu_{\tilde{A}}(x,u) / x \in U, u \in J_x \subseteq I)\}$, where $0 \le \mu_{\tilde{A}}(x,u) \le 1$.

$\tilde{A}$ can also be expressed as $\tilde{A} = \int_{x \in U} \int_{u \in J_x} \frac{\mu_{\tilde{A}}(x,u)}{(x,u)} = \int_{x \in U} \frac{f_x(u)/u}{x}, J_x \subseteq I$, where $f_x(u) = \mu_{\tilde{A}}(x,u)$.

The class of all Type – 2 fuzzy set of the universe U is denoted by $F_{T2}(U)$.

Definition 2.4

A pair (F, A) is called a Fuzzy Soft Set over U, where F is a mapping given by $F : A \to F_{T1}(U)$ where $F_{T1}(U)$ denotes the set of all fuzzy sub sets of$U$and $A \subseteq E$ a set of parameters.

Definition 2.5

Let U be an initial universe and $A \subseteq E$ a set of parameters, a pair $(\mathscr{F}, A)$ is called a Type – 2 Fuzzy Soft Set over U, where $\mathscr{F}$ is a mapping given by $\mathscr{F} : A \rightarrow F_{T2}(U)$. It can be written as

$$\mathscr{F}(\varepsilon) = \int_{x \in U} \int_{u \in J_x} \frac{\mu_{\mathscr{F}(\varepsilon)}(x,u)}{(x,u)} = \int_{x \in U} \frac{\int_{u \in J_x} f_x(u)/u}{x}, J_x \subseteq I .$$

Here u, $\mu_{\mathscr{F}(\varepsilon)}(x,u)$ are respectively the primary membership degree and secondary membership degree that object x holds on parameter ε .

2.6: Type Reduction

If $\tilde{A}$ is a type – 2 fuzzy set $A \subseteq X$ in discrete case, the centroid type reduction can be defined as follows:

$$C_{\tilde{A}} = \int_{u_1 \in J_{x_1}} . \int_{u_2 \in J_{x_2}} ..... \int_{u_n \in J_{x_n}} \left[f_{x_1}(u_1). f_{x_2}(u_2).....f_{x_n}(u_n) \right] \Bigg/ \frac{\sum_{j=1}^{n} x_j \mu_A(x_j)}{\sum_{j=1}^{n} \mu_A(x_j)}$$

Where $\tilde{A} = \sum_{j=1}^{n} \left[\sum_{u \in J_{x_j}} f_{x_j}(u)/u \right] \Big/ x_j$.

3: Distances for Type – 2 Fuzzy Soft Sets

i. Hamming distance: $d_H(\tilde{A}, \tilde{B}) = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} \left| \mathscr{F}(\mathrm{e}_{ij}) - \mathscr{G}(\mathrm{e}_{ij}) \right|$.

ii. Normalized hamming distance : $d_{NH}(\tilde{A}, \tilde{B}) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left| \mathscr{F}(\mathrm{e}_{ij}) - \mathscr{G}(\mathrm{e}_{ij}) \right|$

iii. Euclidean distance : $d_E(\tilde{A}, \tilde{B}) = \frac{1}{m} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| \mathscr{F}(\mathrm{e}_{ij}) - \mathscr{G}(\mathrm{e}_{ij}) \right|^2 \right)^{1/2}$

iv. Normalized Euclidean distance : $d_{NE}(\tilde{A}, \tilde{B}) = \frac{1}{nm} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| \mathscr{F}(\mathrm{e}_{ij}) - \mathscr{G}(\mathrm{e}_{ij}) \right|^2 \right)^{1/2}$

v. Generalized normalized distance : $d(\tilde{A}, \tilde{B}) = \frac{1}{mn} \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left| \mathscr{F}(\mathrm{e}_{ij}) - \mathscr{G}(\mathrm{e}_{ij}) \right|^q \right)^{1/q}$

Note :

- If q=1 then (v) reduces to normalized hamming distance and if q=2 then (v) reduces to normalized Euclidean distance.

3.1. Distance Based Similarity Measure for Type – 2 Fuzzy Soft Set:

The similarity measure between ($\mathscr{F}$,E) and ($\mathscr{G}$,E) denoted by $S(\mathscr{F},\mathscr{G})$ is defined as $S(\mathscr{F},\mathscr{G}) = 1 - d(\mathscr{F},\mathscr{G})$.

4. Numerical Example

We would call any two type-2 fuzzy soft sets ($\mathscr{F},\tilde{E}$) and ($\mathscr{G},\tilde{E}$) in the fuzzy soft class (U,E) to be significantly similar if $S\left(\left(\mathscr{F},\tilde{E}\right),\left(\mathscr{G},\tilde{E}\right)\right) > \frac{3}{4} = 0.75$.

Suppose the decision maker want to determine and identify the patient those who are affected by malaria fever. There are two patients P_1, P_2in a hospital with symptoms temperature, head ache …. Let the universal set contains only two objects (elements) yes 'y' , no 'n' and U=(y, n). Here the set of parameters E is the set of certain approximations determined by the hospital.

Let E={$e_1, e_2, e_3, e_4, e_5, e_6$} where e_1 =temperature , e_2 = head ache , e_3 = vomiting , e_4 = joint pain , e_5 = cough, e_6 =stomach pain.

Here the type-2 fuzzy soft set for malaria fever from the medical knowledge and the medical reports of the two patients against malaria fever ($\mathscr{F},\tilde{P}_1$) , ($\mathscr{F},\tilde{P}_2$) are tabulated as follows.

Table – 1: Type-2 fuzzy soft set for malaria fever from the medical knowledge

$\mathscr{F},\tilde{E}$	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.5/0.7	0.6/0.8	0.8/1	0.6/0.8	0.6/0.7	0.4/0.6
N	0.2/0.3	0.4/0.5	0.5/0.6	0.4/0.4	0.5/0.6	0.4/0.5

Table – 2:Type-2 fuzzy soft set for the first patient

$\mathscr{F},\tilde{P}_1$	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.9/1	0.1/0.2	0.2/0.3	0.1/0.2	0.2/0.2	0.8/0.1
N	0.8/0.9	0.9/1	0.1/0.1	0.9/1	1/1	0.7/1

Table – 3:Type-2 fuzzy soft set for the second patient

$\mathscr{T},\tilde{P}_2$	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.4/0.5	0.3/0.5	0.6/0.8	0.6/0.7	0.5/0.7	0.5/0.6
N	0.4/0.6	0.6/0.8	0.6/0.9	0.4/0.5	0.6/1	0.5/0.9

We can reduce these type-2 fuzzy soft sets into Type-1 Fuzzy soft sets using centroid type reduction. These type-reduced Fuzzy Soft Sets are tabulated below:

Table – 4: Type-1 fuzzy soft set for malaria fever from the medical knowledge

($\mathscr{T}$,E)	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.5	0.5	0.5	0.8	0.8	0.8
N	0.2	0.4	0.5	0.4	0.5	0.4

Table – 5:Type-1 fuzzy soft set for the first patient

($\mathscr{T}$,P_1)	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.9	0.1	0.2	0.1	0.2	0.8
N	0.8	0.9	0.1	0.9	1	0.7

Table – 6:Type-1 fuzzy soft set for the second patient

($\mathscr{T}$, P_2)	e_1	e_2	e_3	e_4	e_5	e_6
Y	0.4	0.3	0.6	0.6	0.5	0.5
N	0.4	0.6	0.6	0.4	0.6	0.5

Now we have to find the distance between these two type – 2 fuzzy soft sets using the above distances.

Hamming distance

Here i=6 and j=2

$$d_H(\tilde{E},\tilde{P}_1) = \frac{1}{6}\left[\sum_{i=1}^{2}\sum_{j=6}^{6}\left|\tilde{E}(e_{ij}) - \tilde{P}_1(e_{ij})\right|\right] = \frac{1}{6}[0.9167]$$

Now the distance based similarity measure

$S(\mathscr{F},\tilde{P}_1) = 1 - 0.9167 = 0.0833$

Similarly

$S(\mathscr{F},\tilde{P}_2) = 0.7667$

In the same way, we can calculate the other distances and its similarity measure. These values are presented in table – 7.

Table – 7: Various distances and its similarity measures

Measures	Hamming Distance	Normalized Hamming Distance	Euclidean Distance	Normalized Euclidean Distance
$S(\mathscr{F},\tilde{P}_1)$	0.0833	0.5417	0.7307	**0.8654**
Significance level (75%)	Not similar	Not similar	Not similar	Similar
$S(\mathscr{F},\tilde{P}_2)$	**0.7667**	**0.8833**	**0.9218**	**0.9609**
Significance level (75%)	Similar	Similar	Similar	Similar

From the above table, the Hamming distance, Normalized Hamming distance and Euclidean distance measure values of the second patient is significantly similar with the medical data considered. Whereas the same values are calculated for the first patient and so they are not significantly similar with the medical data. In Normalized Euclidean Distance measure value for both the patients are significantly similar with the medical data. So we conclude that both the patients are affected by malaria fever at 75% significant level.

5. Conclusion

In this paper, we have applied distance based similarity measure using various distances between Type-2 fuzzy soft sets to obtain a solution of a decision making problem. A numerical example is solved to illustrate this method and the centroid type reduction method is also used in this problem. We found that the Normalized Euclidean distance helps us to take decision inmore consistent manner.In future, other types of measures can also be used to find the solution of any decision making problem.

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