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Impact of Heat and Mass Transfer Effects on Radiative MHD Flow of A Micropolar Fluid With Joule Heating and Partial Slip Under Convective Boundary Conditions

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Abstract

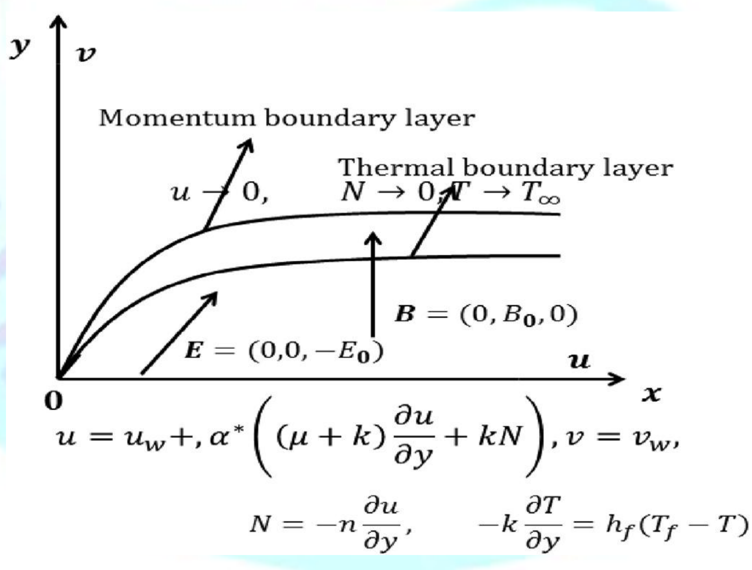
The present paper deals the heat and mass transfer flow of micropolar fluid past a permeable stretching sheet under the influence of Joule heating, thermal radiation, magnetohydrodynamics with convective boundary conditions. The governing non-linear partial differential equations have been transformed into a set of non-linear ordinary differential equations, which are solved numerically by applying shooting iteration technique along with the sixth order Runge-Kutta integration scheme. Numerical results for the local skin friction coefficient, the local Nusselt number and also the local Sherwood number as well as the dimensionless velocity, the microrotation, the temperature and the concentration profiles have been presented for different physical parameters. Obtained values are also compared with the existing literature in the limiting case and good coincidence is observed.

Keywords: Magneto hydrodynamic, Joule heating, thermal radiation, partial slip, convective boundary conditions, Viscous dissipation.

1. Introduction

Mechanics of non-Newtonian fluids has been one of the most challenging and interesting areas in research for engineers, physicists, mathematicians and numerical simulists. In the last few decades, interest in non-Newtonian fluids has increased tremendously, mainly due to its association with applied sciences and its vital role both in theory and in many industrial applications. The theory of micropolar fluids was first devolped by Eringen [1, 2]. A review of the subject and applications of micropolar fluid mechanics was mentioned by Peddison and and McNitt [3], Khonsari and Brewe [4], Qukaszewicz [5], Ariman et al. [6] and Bachok et al. [7].Combined heat and mass transfer phenomena are found everywhere in nature and it is important in all the branches of science and technology. In many practical applications, there exist significant temperature and concentration variations between the surface of the hot body and the free stream. It is a major area of interest and it has assumed practical importance in engineering devices like heat exchangers, solar collectors, nuclear reactors and electronic equipments. Mohanty et al. [8] studied the heat and mass transfer effect on micropolar fluid over a stretching sheet by using the numerical method. Heat and mass transfer on a stretching sheet in the presence of suction or injection was carried out by Gupta and Gupta [9].Analytical studies on MHD oscillatory flow of a micropolar fluid over a vertical porous plate were done by Kim and Lee [10]. El-Amin [11] explored mass transfer effects on MHD free convective flow of a micropolar fluid in the presence of constant suction. Magnetohydrodynamic free convection flow past an infinite vertical plate with constant suction and heat absorption was discussed by Sahoo et al. [12].Ishak et al.[13] studied magnetohydrodynamic flow of a micropolar fluid in the direction of a stagnation point on a vertical surface. MHD free convection flow of micropolar fluid between two parallel porous vertical plates by using numerical method was carried out by Bhargava et al.[14]. Siddheshwar and Mahabakeshwar [15] derived analytical solution to the magnetohydrodynamic flow of a micropolar fluid over a linear stretching sheet. Convective heat transfer in an electrically conducting micropolar fluid at a stretching surface with a uniform free stream was illustrated by Abo-Eldahab and Ghonaim [16].Radiation effects on convective heat transfer and MHD flow problems have gained more and more importance in electrical power generation, astrophysical flows, solar power technology, space vehicle reentry, and some other industrial areas. Since the solution to convection and radiation equations is too complicated, there are very few studies about the simultaneous effect of convection and radiation for internal flows. Mahmoud [17] has considered thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface in presence of variable thermal conductivity.Heat and mass transfer effects on MHD free convection of a micropolar fluid over a truncated cone in the presence of radiation was studied by Chamkha et al. [18]. Ishak [19] deliberated micropolar fluid flow over a stretching sheet under the influence of thermal radiation. M. Gnaneswara Reddy [20] presented Mass transfer effects on unsteady MHD radiative convection flow of micropolar fluid past a vertical porous plate with variable heat and mass fluxes. Rahman and Sattar [21] argued about the influence of thermal radiation effects on convective flow of a micropolar fluid past a continuously moving vertical porous plate. The viscous dissipation effects and Joule heating usually are characterized by the Eckert number, and the product of the Eckert number and the magnetic parameter, respectively, and both effects are important in nuclear engineering. The impact of viscous dissipation and radiation on magnetohydrodynamic mixed convection flow of micropolar fluid was investigated by Ibrahim et al. [22]. Hakiem et al. [23] presented joule heating effects on magnetohydrodynamic natural convection flow of a micropolar fluid. Haque et al. [24] presented Joule heating and viscous dissipation effects on steady MHD free convection flow with constant heat and mass fluxes. Effects of viscous dissipation on lubrication characteristics of micropolar fluid were examined by Khonsari and Brewe [25]. Rahman [26] discussed convective flows of micropolar fluids flow in the presence of radiation, viscous dissipation and joule heating effects. Effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet observed by Machireddy Gnaneswara Reddy, Polarapu Padma and Bandari Shankar []27]In some practical situations, it will be of importance to replace the no-slip condition by the partial slip condition because when a fluid flows in micro electro mechanical system (MEMS), the no-slip boundary condition at the solid-fluid interface will no longer be applicable. The region near non-equilibrium interface is more accurately described by the slip flow model. Mukhopadyay [28] proposed the analyses of boundary layer flow over a porous nonlinear stretching sheet in presence of a partial slip at boundary. Mahmood and Waheed [29] derived heat transfer effects on MHD flow of a micropolar fluid over a stretching surface in the presence of heat generation and slip velocity. MHD micropolar fluid flow over a moving plate under slip condition with lie group analysis was carried out by Das et al. [30]. Slip effects on MHD mixed convection flow of a micropoloar fluid were discussed by Das [31]. Zhang [32] illustrated the slip and buoyancy lift effects on the mixed flow of convection and transfer of radiation heat of a micropolar fluid towards a vertical permeable plate. Unsteady MHD free convection flow embedded in a porous medium in a polar fluid in the presence of radiation and variable suction with a slip flow regime was demonstrated by Chaudary et al [33].In the past few decades, most of the researchers well considered problems of convective heat transfer either with constant wall temperature (CWT), constant heat flux (CHF), or with Newtonian heating (NH) in a Newtonian and/or non-Newtonian fluid. Recently, a novel mechanism for the heating process has invited the involvement of many researchers, namely, convective boundary condition (CBC), where the heat is supplied to the convecting fluid from a bounding surface with an unlimited heat capacity. Further, this results in the heat transfer rate through the surface which is proportional to the local difference in temperature with ambient conditions. Besides, it is general and realistic, particularly in several technologies and industrial operations such as transpiration cooling process, textile drying, and heating of laser pulse. Aziz [34] proposed a similarity solution for laminar thermal boundary layer over a flat plate in the presence of convective surface boundary conditions. Similarity solutions for flow and heat transfer over a permeable surface in the presence of convective boundary condition were studied by Ishak [35]. Yao et al. [36] proposed heat transfer of a generalized stretching wall problem with convective boundary conditions. Very recently Ramzan et al. [37] have elaborated the effects of radiation and joule heating on a steady MHD flow of a micropolar fluid in the presence of partial slip and convective boundary conditions.The present communication has considered the effects of mass transfer on MHD flow of a micropolar fluid as shown in [37]. It studies numerically the effects of heat and mass transfer on MHD flow of a micropolar fluid under the presence of thermal radiation, joule heating, partial slip and convective boundary conditions. The set of governing equations and the boundary conditions are reduced to non-linear ordinary differential equations with suitable boundary conditions. Furthermore, the similarity equations are solved numerically by using the shooting technique with the sixth-order Runge-Kutta integration scheme. Numerical results of the local skin friction coefficient and also the local Nusselt number as well as the velocity, temperature and concentration profiles are presented for different physical parameters.

2.Mathematical Formulation



We have Considered the MHD two dimensional convection flow of an incompressible micropolar fluid flow over a permeable stretching sheet under slip velocity. Uniform magnetic and electrical fields are also used to study the flow character with heat and mass transfer. Here we have considered the thermal radiation and effects of Joule heating owing to magnetic and electric fields in this flow. The governing equations under the boundary layer approximations (Hayat et al. [38]):

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(\upsilon+\frac{k}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y}+\frac{\sigma}{\rho}\left(E_0B_0-B_0^2u\right), \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2}-\frac{k}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa_1}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\left(uB_0-E_0\right)^2\sigma}{\rho c_p}+\frac{16\sigma^*}{3k^*\rho c_p}T_\infty^3\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{5}$$

The corresponding boundary conditions are

$$u=ax+\alpha^*\left(\left(\mu+k\right)\frac{\partial u}{\partial y}+kN\right),\quad v=v_w,\quad N=-n\frac{\partial u}{\partial y},\quad -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right),$$

$$-D_m\frac{\partial C}{\partial y}=k_m\left(C_f-C\right)\quad \text{at } y=0, \tag{6}$$

$$u\to 0,\quad N\to 0,\quad T\to T_\infty,\ C\to C_\infty \text{ as } y\to\infty$$

In this case u and v represent the components of velocity in x and y directions. μ is the viscosity of fluid, N is the micro-rotation or angular velocity of micropolar fluid, k is vortex viscosity, ρ is the density of fluid, j is the microinertia density, γ^* is the spin gradient viscosity, σ is the electrical conductivity, B_0 is the applied magnetic field strength, E_0 is applied electric field, k_1 is the thermal conductivity, c_p is the specific heat at constant pressure, α^* is the slip coefficient, v_w is the suction/injection velocity, σ^* is the Stefan-Boltzmann constant, k^* is the mean absorption coefficient, h_f is the heat transfer coefficient, k_m is the mass transfer coefficient, D_m is the molecular diffusivity, *T* is the temperature of fluid, *C* is the concentration of fluid, T_∞ is ambient temperature of fluid and C_∞ is the ambient concentration of fluid. Further, n is a constant, $n = \frac{1}{2}$ indicates that the anti-symmetrical part of the tensor of stress vanishes which tends to weak concentration. The case $n=1$ is for turbulent flows. Here viscous case is recovered by putting $k=0$.

In order to obtain similarity solution to the problem, the following non-dimensional variables are introduced

$$\eta=\sqrt{\frac{a}{\upsilon}}y,\quad \psi=\sqrt{a\upsilon x}\,f(\eta),\quad N=ax\sqrt{\frac{a}{\upsilon}}h(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty} \tag{7}$$

where η is the similarity variable and $\psi(x,y)$ is the stream function defined by

$u=\frac{\partial\psi}{\partial y}=U_w f'(\eta)$, $v=-\frac{\partial\psi}{\partial x}$, which identically satisfies equation (1). Similarity variables (7), equations (2), (3), (4) (5) and (6) are used to obtain the following set of ordinary differential equations:

$$(1+K)f'''+ff''-f'^2+Kh'-M^2f'+M^2E=0 \tag{11}$$

$$\left(1+\frac{K}{2}\right)h''+\begin{vmatrix} f & f' \\ h & h' \end{vmatrix}-K(2h+f'')=0 \tag{12}$$

$$\left(1+\frac{4}{3}R\right)\theta''+\Pr\begin{vmatrix} f & f' \\ \theta & \theta' \end{vmatrix}+\Pr Ecf''^2+\Pr M^2Ec(f'-E)^2=0 \tag{13}$$

$$\phi''+Sc\begin{vmatrix} f & f' \\ \phi & \phi' \end{vmatrix}=0 \tag{14}$$

The corresponding boundary conditions are

$$f(0)=f_w,\qquad f'(0)=1+\alpha(1+K)f''(0),\qquad h(0)=-nf''(0),\qquad \theta'(0)=-\gamma_1(1-\theta(0)),$$
$$\phi'(0)=-\gamma_2(1-\phi(0))$$
$$f'(\infty)=0,\quad h(\infty)=0,\quad \theta(\infty)=0,\quad \phi(\infty)=0 \tag{15}$$

where the notation primes show differentiation with respect to η and the parameters are defined as: $u_w=ax$ is the velocity, $K=\frac{k}{\mu}$ is the material parameter, $M^2=\frac{\sigma B_0^2}{\rho a}$ is the magnetic field parameter,

$E=\dfrac{E_0}{u_w B_0}$ is the electrical parameter, $f_w=-(a\nu)^{-\frac{1}{2}}\nu_w$ is the suction/blowing parameter, $\alpha=\alpha^*\mu\sqrt{\dfrac{a}{\nu}}$ is the slip parameter, $\Pr=\dfrac{\mu c_p}{\kappa_1}$ is the Prandtl number, $R=\dfrac{4\sigma^* T_\infty^3}{3k^* k_1}$ is the thermal radiation, $Ec=\dfrac{u_w{}^2}{c_p\left(T_f-T_\infty\right)}$ is the Eckert number , $Sc=\dfrac{\upsilon}{D_m}$ is the Schmidt number, thermal Biot number $\gamma_1=\dfrac{h_f}{k}\sqrt{\dfrac{\upsilon}{a}}$, and concentration Biot number $\gamma_2=\dfrac{k_m}{D_m}\sqrt{\dfrac{\upsilon}{a}}$.

The quantities of physical interest in this problem are local skin friction, local couple stress, the local Nusselt number, and also the local Sherwood number are defined as

$$C_{fx}=\frac{2\tau_w}{\rho(ax)^2},\quad M_x=\frac{\nu M_w}{xa^2},\quad Nu_x=\frac{xq_w}{k\left(T_f-T_\infty\right)},\quad Sh_x=\frac{xq_m}{k\left(C_f-C_\infty\right)}$$

where τ_w (wall shear stress), M_w (surface couple stress), q_w (heat flux) and q_m (mass flux) are given by

$$\tau_w=\left((\mu+k)\frac{\partial u}{\partial y}+kN\right)_{y=0},\quad M_w=\left(\frac{a}{\nu}\right)^{-1}\left(\frac{\partial N}{\partial y}\right)_{y=0}\quad q_w=-\kappa_1\left(\frac{\partial T}{\partial y}\right)_{y=0},\quad q_m=-D_m\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Dimensionless form of the above equation becomes

$$\frac{1}{2}C_{fx}\mathrm{Re}_x^{1/2}=\left(1+(1-n)K\right)f''(0),\quad M_x\big/\mathrm{Re}_x^{1/2}=-h'(0)\quad Nu_x\big/\mathrm{Re}_x^{1/2}=-\theta'(0),\quad Sh_x\big/\mathrm{Re}_x^{1/2}=-\phi'(0)$$

where the local Reynolds number is given by $\mathrm{Re}_x=\dfrac{u_w x}{\upsilon}$.

3. Numerical methods for solution

Eqs. (11) to (14) constitute the problem of a highly non-linear coupled boundary value of the third and second-order. So we have developed very effective numerical shooting technique with sixth-order Runge-Kutta integration algorithm. To select η_∞ we have begun with some initial guess value and solved the problem with some particular set of parameters to obtain $f''(0), h(0), \theta'(0)\, and\, \varphi'(0)$. The solution process is repeated with another larger value of η_∞ until two successive values of

$f''(0), h(0), \theta'(0) \, and \, \varphi'(0)$ differ only after desired digit signifying the limit of the boundary along η. The last value of η_∞ has been selected as appropriate value for that particular simultaneous equations of first order for seven unknowns following the method of superposition [37]. To solve this system we require seven initial conditions whilst we have only two initial conditions $f'(0)$ and $f(0)$ on f,two initial conditions on each on *h,* θ and ϕ. Still there are three initial conditions $f''(0), h(0), \theta'(0) \, and \, \varphi'(0)$ which are not prescribed. Now, we have used the numerical shooting technique where these two ending boundary conditions are utilized to produce two unknown initial conditions at $\eta = 0$. In this calculation, the step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution with $\eta_{\max} = 6$ and five-decimal accuracy as the condition for convergence.

4. Results and Discussion

The purpose of this section is to analyze the graphical results for the effects of magnetic field parameter *M*, material parameter *K*, electric parameter *E*, Prandtl number *Pr*, radiation parameter *R*, Eckert number *Ec*, thermal Biot number γ_1, concentration Biot number γ_2, suction velocity f_w, slip parameter α and Schmidt number *Sc* on the velocity, microrotation, temperature and concentration fields. Hence, Figures 1to21 have been displayed.

For numerical computations we have fixed the non-dimensional values as *K = 0.2, M = 0.2, n = 0.5, R = 0.2, Pr = 0.71, E = 0.2,* $E_c = 0.2$, γ_1 *= 0.1,* γ_2 *= 0.1,* f_w *= 0.1, Sc = 0.22, and* α *= 0.1.* These values are preserved as common in the whole analysis excluding the different values and they are presented in definite figures and tables.Characteristics of magnetic field parameter M, material parameter K, electrical parameter E, slip parameter α and suction velocity f_w on velocity distribution are shown in the figs. 1 to 5.Fig.1. shows the nature of magnetic field parameter M on the velocity distribution. We have observed that velocity distribution is reduced through larger magnetic field parameter M. Impact of material parameter K on velocity distribution is described in fig. 2. Here velocity distribution reduces via increasing material parameter. Fig. 3 describes the relation of velocity distribution and electrical parameter E. from Fig. 3 we have noticed larger values of electrical parameter boost the velocity. Figs. 4 & 5 are prepared to analyze the effects of different values of slip parameter α and suction velocity f_w on velocity distribution. We have found that the velocity and its boundary layer thickness are decreasing the functions of slip parameter and suction velocity.

Figs. 6 to 10 are sketched for the different values of magnetic field parameter M, material parameter K, electrical parameter E, slip parameter α and suction velocity f_w on the micro-rotation velocity distribution. Fig. 6 is outlined for behavior of magnetic field parameter on micro-rotation. It is noticed that micro-rotation is increasing with magnetic field parameter. In Figs. 7 to 10, the effects of material parameter K, Electrical parameter E, slip parameter and suction velocity on micro-rotation are displayed, respectively. With an increase in the values of material parameter, Electrical parameter and slip parameter, the micro-rotation decreases, while the same increases with an increase in suction velocity.

Figs. 11 to 17 are prepared to analyze the behavior of different emerging parameters such as Prandtl number, radiation parameter R, Eckert number, Electrical parameter E, thermal Biot number γ_1, slip parameter and suction velocity on temperature distribution.

Fig. 11 plots Prandtl number variation on temperature distribution. We have concluded that the temperature distribution and associated thermal boundary layer thickness decrease through larger Prandtl number. Figs. 12 & 13 depict the variation of temperature distribution against radiation parameter and Eckert number. It is noted that the temperature distribution monotonically increases with either rise in radiation R or Eckert number values. For various values of the Electrical parameter E, the temperature profiles are plotted in Fig. 14. It is clear that the existence of the Electrical parameter E decreases the temperature distribution.The impact of thermal Biot number γ_1, slip parameter and suction velocity on temperature distribution profiles are displayed in Figs. 15 to 17. It is observed that by increasing either value of thermal Biot number γ_1 or slip parameter, the thermal boundary layer thickness enhances. However, the temperature distribution decreases with an increase in the suction velocity.

Behavior of Schmidt number Sc, concentration Biot number γ_2, suction velocity and slip parameter on concentration distribution are depicted in the Figs. 18 to 21.

Fig. 18 displays the result of the concentration distribution for various values of Schmidt number. it is obvious from the figure that the concentration distribution decreases with an increase of Schmidt number Sc. The concentration distribution for various physical parameters such as concentration Biot number γ_2, suction velocity and slip parameter are shown in Figs. 19 to 21. Concentration profiles increase with increasing the values of concentration Biot number γ_2 and slip parameter while, it decreases with an increase in the values of suction velocity.To validate the present results, comparisons for Nusselt number with existing literature are shown in table 1. From this table, it can be seen that the current results show a favorable agreement with existing literature. The variation in skin-friction coefficient, the rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number, for different parameters are shown in tables 1 & 2, and hence they are not studied any further to keep brevity.

5. **Conclusion Remarks :**

In Heat and mass transfer analysis in the presence of thermal radiation, Joule heating is analyzed for the steady MHD flow of an incompressible micropolar fluid under partial slip and convective boundary conditions. A series solutions for velocity, angular momentum, temperature and concentration fields are developed and discussed. The behavior of the embedded parameters is studied. The main results of the present analysis can be listed below:

- The velocity profile is an increasing function of energy parameter.
- The micro-rotation profile is a decreasing function of energy parameter.
- The temperature profile decreases when Pradtl number increases.
- The concentration field decreases when Schmidt number increases.
- The magnitude of skin-friction coefficient decreases for energy and material parameters.
- Nusselt number intensifies with the growing values of Prandtl number and energy parameters, whereas it reduces when magnetic field parameter and radiation parameters increase.
- The magnitude of Sherwood number increases when Schmidt number increases.

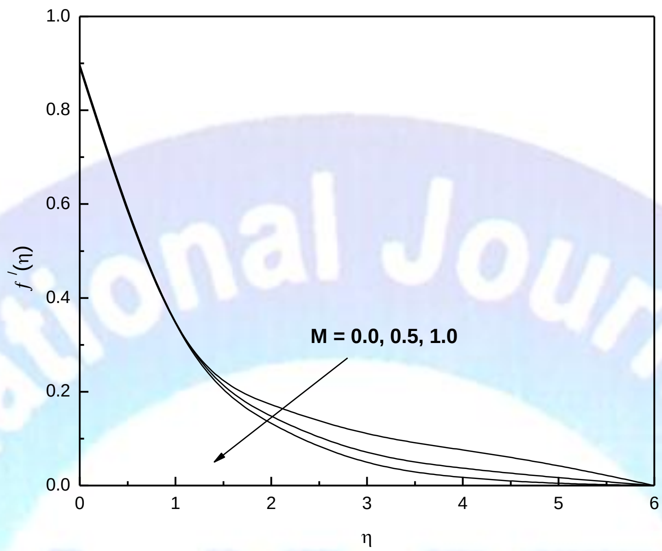


Fig.1 Velocity profile for various values of *M*

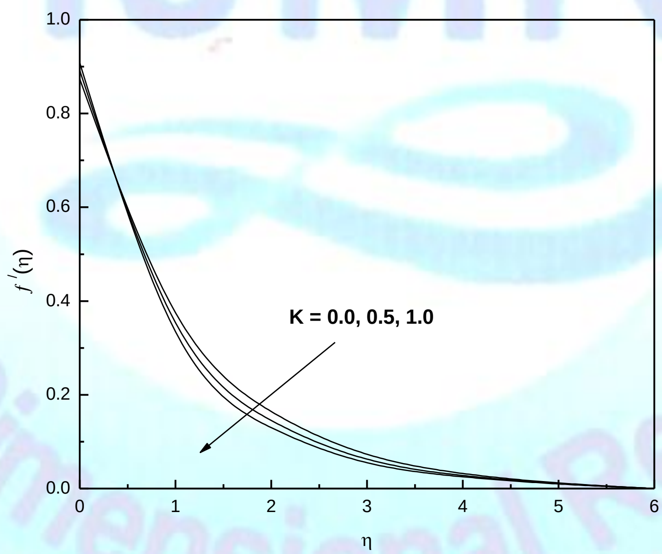


Fig.2 Velocity profile for various values of *K*

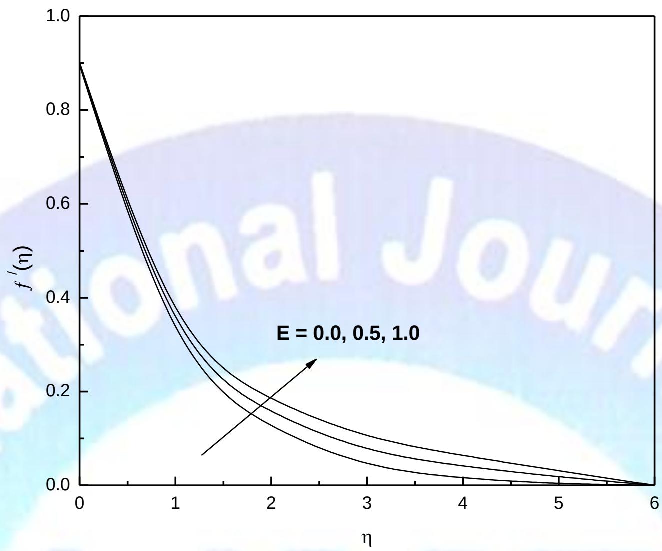


Fig.3 Velocity profile for various values of *E*

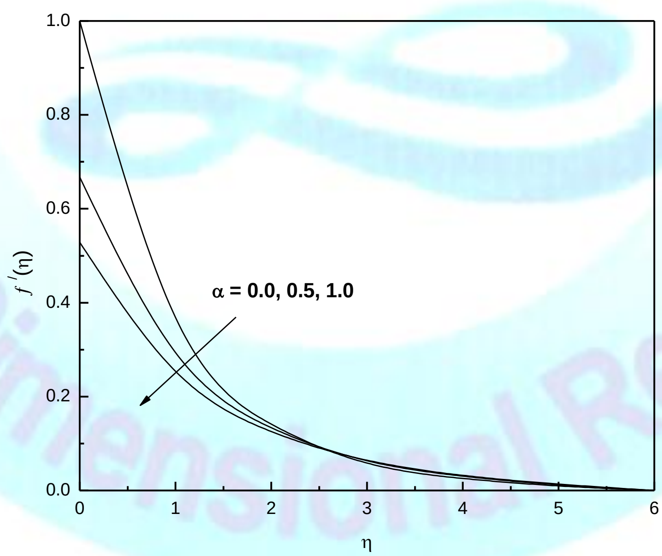


Fig.4 Velocity profile for various values of α

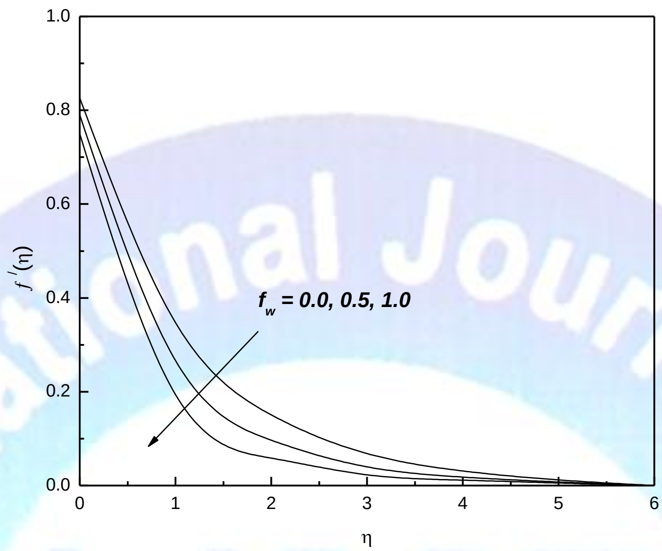


Fig.5 Velocity profile for various values of f_w

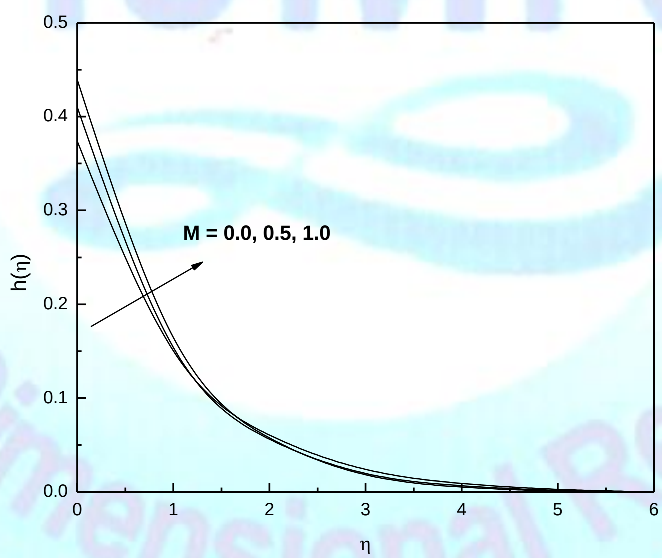


Fig.6 Micro-rotation velocity profile for various values of M

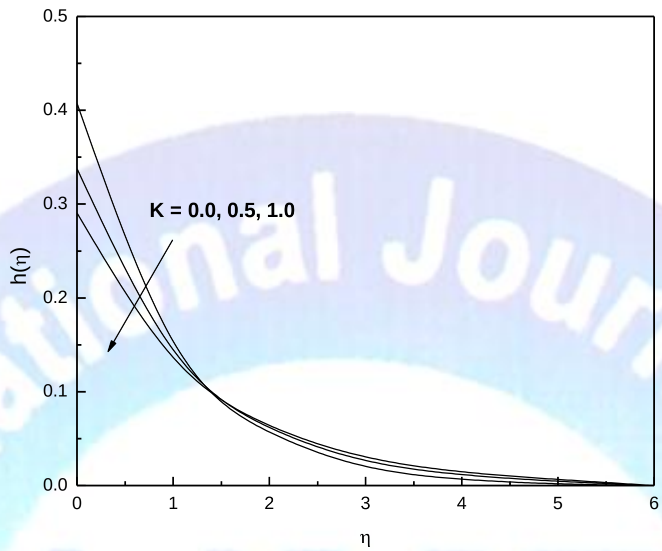


Fig.7 Micro-rotation velocity profile for various values of *K*

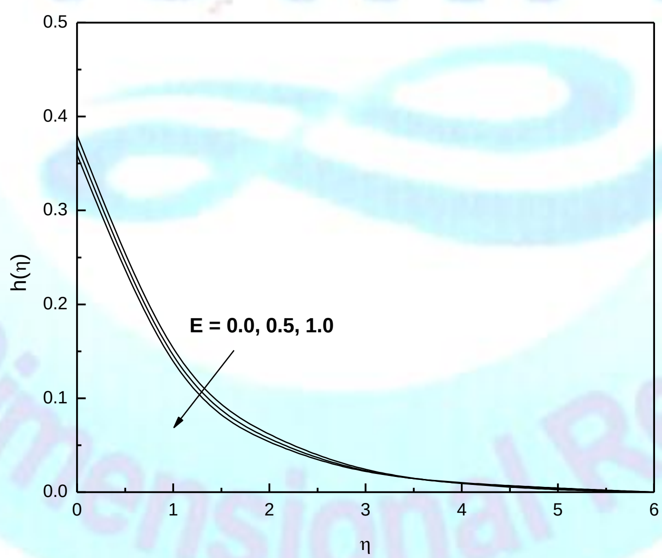


Fig.8 Micro-rotation velocity profile for various values of *E*

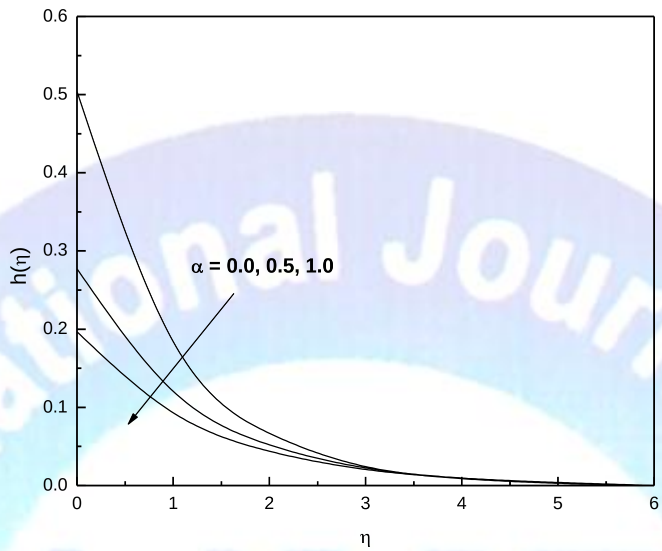


Fig.9 Micro-rotation velocity profile for various values of α

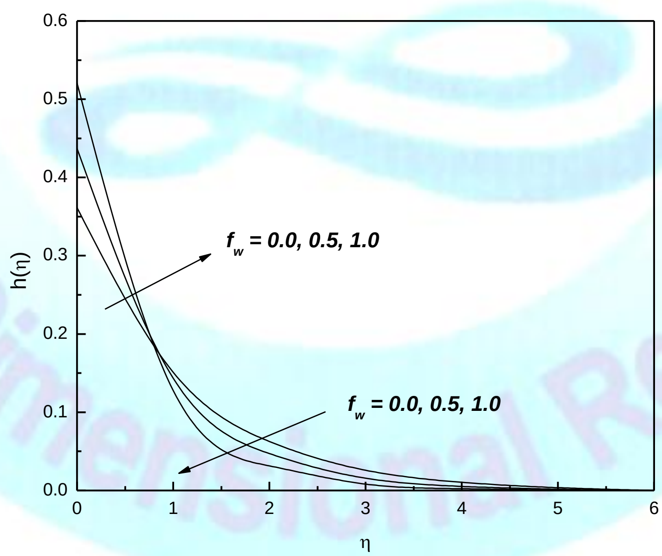


Fig.10 Micro-rotation velocity profile for various values of f_w

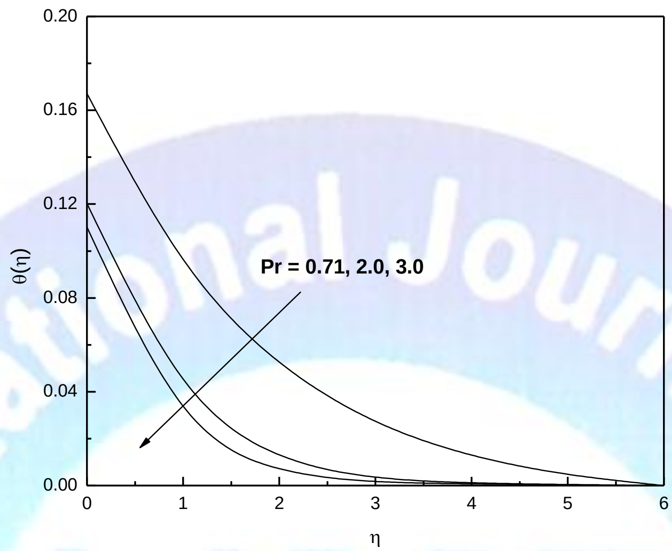


Fig.11 Temperature profile for various values of *Pr*

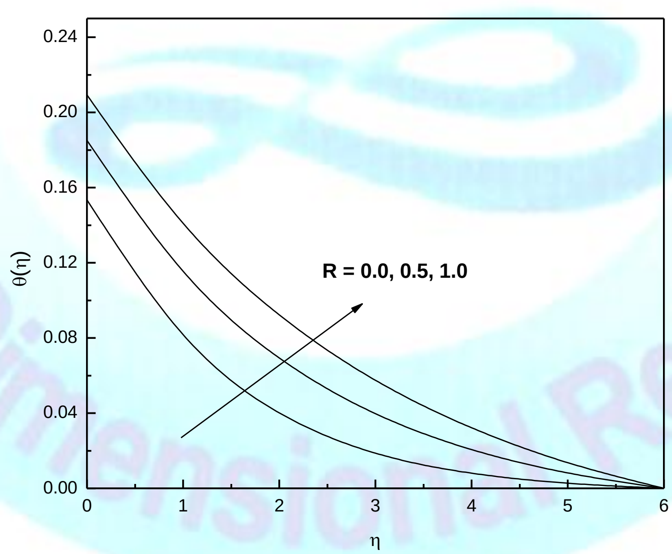


Fig.12 Temperature profile for various values of *R*

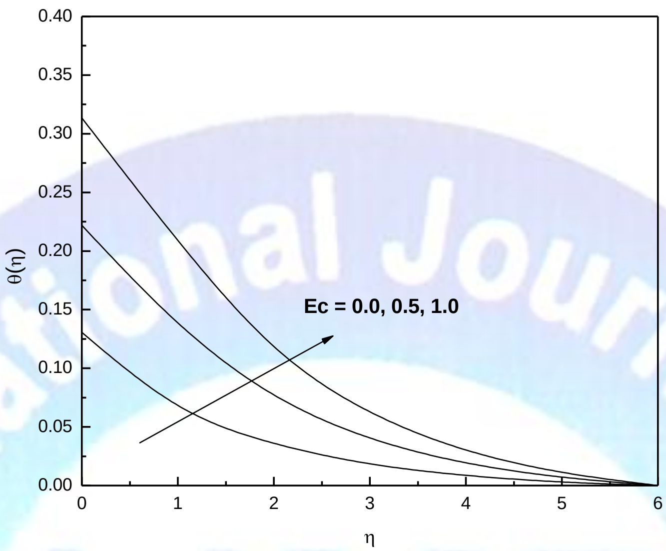


Fig.13 Temperature profile for various values of *Ec*

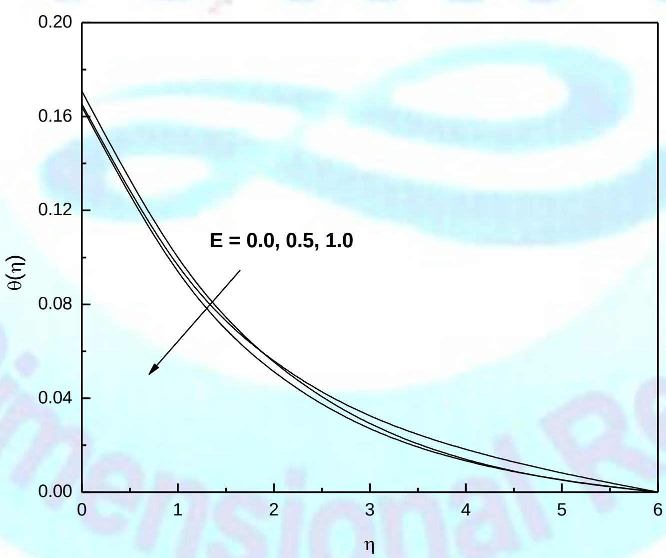


Fig.14 Temperature profile for various values of *E*

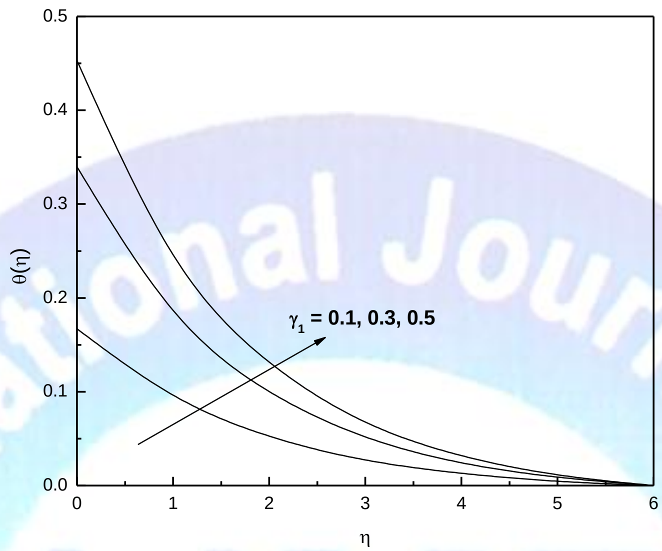


Fig.15 Temperature profile for various values of γ_1

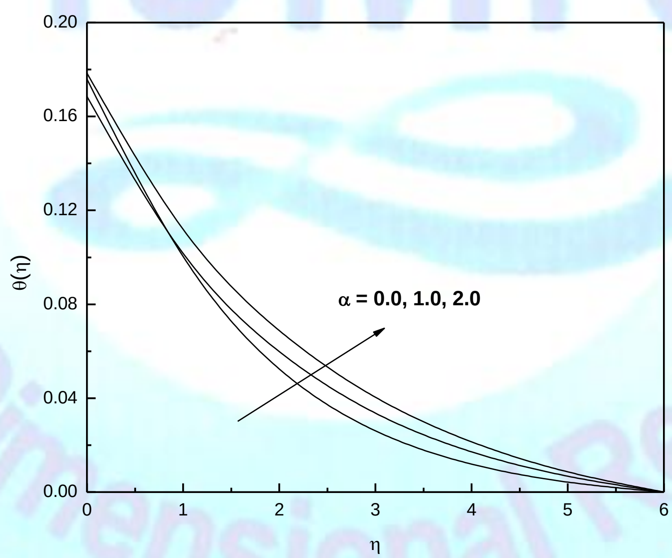


Fig.16. Temperature profile for various values of α

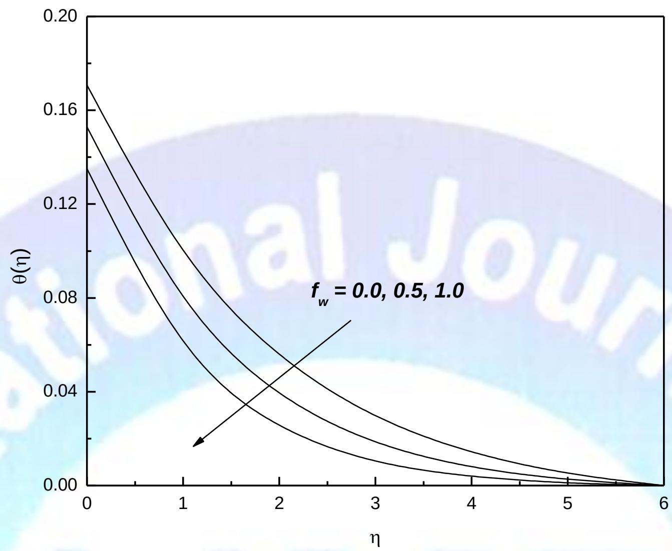


Fig.17 Temperature profile for various values of f_w

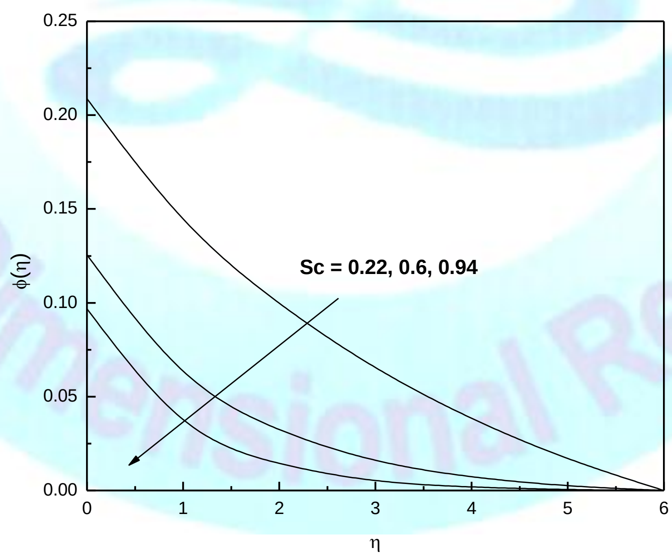


Fig.18 Concentration profile for various values of *Sc*

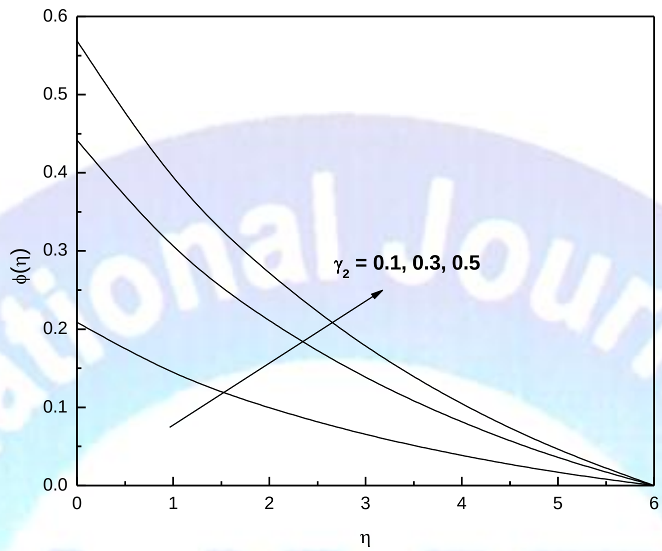


Fig.19 Concentration profile for various values of γ_2

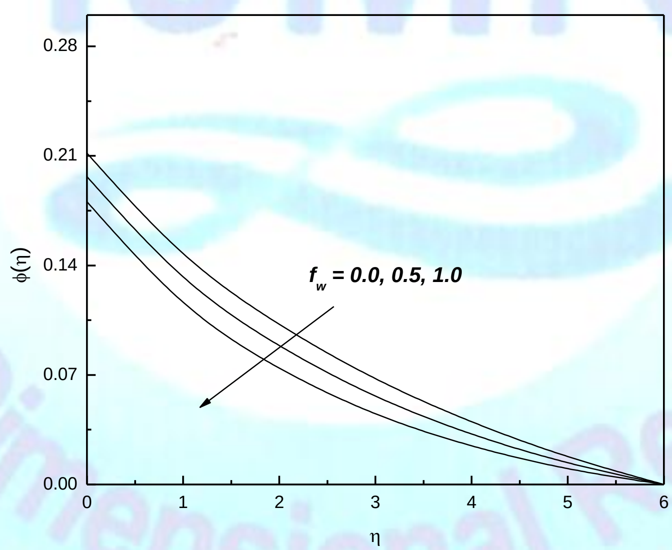


Fig.20 Concentration profile for various values of f_w

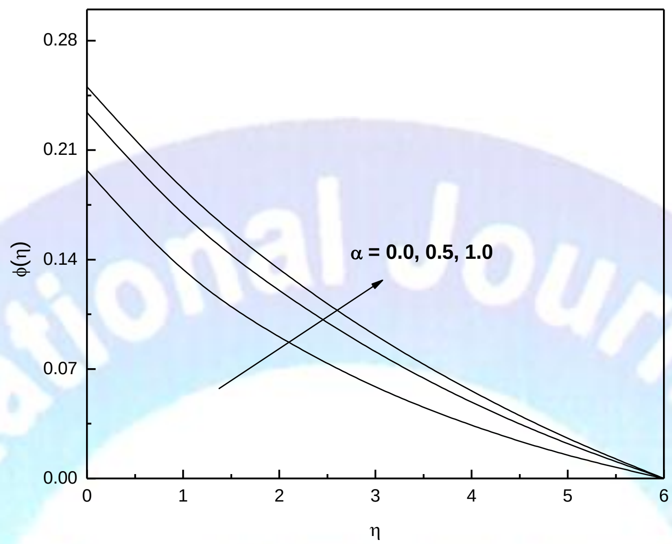


Fig.21 Concentration profile for various values of α

Table 1. Comparative study of present results of $-\theta'(0)$ with Ramzan et al. [37] via *K* , *M* and *R* when $E = 0.2$, $\alpha = 0.1$, $\gamma_1 = 0.1, Ec = 0.2$, $f_w = 0.1$, $\Pr = 1.0$ *, and* $n = 0.5$.

K	*M*	*R*	Ramzan et al. [37]	Present
0.3	0.2	0.1	0.08591	0.0855242
0.6			0.08601	0.0861405
0.9			0.08608	0.0866247
	0.3		0.08595	0.0851909
	0.6		0.08490	0.0845054
		0.3	0.08540	0.0842906
		0.6	0.08351	0.0829247

Table 2 Variations in the local friction factor coefficient, local couple stress, local Nusselt and Sherwood numbers at different non-dimensional governing parameters

M	*E*	*K*	*Pr*	*R*	*Ec*	$f''(0)$	*h(0)*	$-\theta'(0)$	$-\phi'(0)$
0.2	0.2	0.2	0.71	0.2	0.2	-0.751495	0.343163	0.0832841	0.0791207
0.3						-0.756752	0.346832	0.0832648	0.0791908
0.5						-0.776023	0.358434	0.0830841	0.0793045
	0.3					-0.74719	0.342199	0.0834154	0.0792185
	0.5					-0.738748	0.340226	0.0835787	0.079403
		0.3				-0.723977	0.321635	0.0834917	0.0791656
		0.5				-0.675385	0.284978	0.0838505	0.0792468
			1.0			-0.751495	0.343163	0.0852107	0.0791207
			3.0			-0.751495	0.343163	0.0889769	0.0791207
				0.3		-0.751495	0.343163	0.0826514	0.0791207
				0.5		-0.751495	0.343163	0.0814863	0.0791207
					0.3	-0.751495	0.343163	0.081455	0.0791207
					0.5	-0.751495	0.343163	0.0777969	0.0791207

Table 3 Shows variations in the local friction factor coefficient, local couple stress, local Nusselt and Sherwood numbers at various non-dimensional governing parameters

Sc	f_w	γ_1	γ_2	α	$f''(0)$	h(0)	$-\theta'(0)$	$-\phi'(0)$
0.22	0.1	0.1	0.1	0.1	-0.751495	0.343163	0.0832841	0.0791207
0.6					-0.751495	0.343163	0.0832841	0.0874431
0.94					-0.751495	0.343163	0.0832841	0.0903152
	0.3				-0.811092	0.408309	0.0839939	0.0797074
	0.5				-0.873974	0.484709	0.0847092	0.080315
		0.3			-0.858428	0.408905	0.198839	0.0796511
		0.5			-0.858428	0.408905	0.27587	0.0796511
			0.3		-0.858428	0.408905	0.0829831	0.169834

			0.5		-0.858428	0.408905	0.0829831	0.219551
				0.3	-0.670444	0.295297	0.083432	0.078667
				0.5	-0.554649	0.230141	0.0834911	0.0779177

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