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k–Super Harmonic Mean Labeling Of Some Graphs

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Abstract :
In this paper, we introduce the concept of k-Super harmonic mean labeling and investigate k-super harmonic mean of some graphs. Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called k-Super harmonic mean labeling if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a k-Super harmonic mean labeling is called k-Super harmonic mean graph.

key words: Super harmonic mean labeling, Super harmonic mean graph, k-Super harmonic mean labeling, k-Super harmonic mean graph.

1. Introduction

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3].

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J. A. Gallian (2016) can be found in [1]. Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced Super mean labeling of graphs in [5].S. Somasundram and S.S. Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7,8,9].S. Sandhya and C. David Raj introduced Super harmonic labeling in [9].

In this paper, we investigate k-Super harmonic mean labeling of some graphs.

Definition 1.1:
Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, 3 \dots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called *Super harmonic mean labeling* if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, \dots, p+q\}$. A graph that admits a *Super harmonic mean labeling is called Super harmonic mean graph.*

Definition 1.2:
Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil$ or $\left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$, then f is called *k-Super harmonic mean labeling* if $f(V) \cup \{f^*(e) : e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$. A graph that admits a *k-Super harmonic mean labeling* is called *k-Super harmonic mean graph.*

Definition 1.3:
P_n^+ is a graph obtained by joining a single pendant edge to each vertex of a path P_n.

Definition 1.4:

If G has order n, the *corona* of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i th vertex of G with an edge to every vertex in the i th copy of H.

Definition 1.5:

If G is a graph, then S (G) is a graph obtained by *subdividing* each edge of G by a vertex.

Definition 1.6:

The *middle graph* $M(G)$ of a graph G is the graph whose vertex set is V(G) ∪E(G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

2.Main results

Theorem 2.1:

The path P_n is a k-super harmonic mean graph for all n ≥ 2.

Proof:

Let $V(P_n) = \{v_i ; 1 \leq i \leq n\}$ and $E(P_n) = \{e_i = (v_i, v_{i+1}) ; 1 \leq i \leq n-1\}$

Define a function $f: V(P_n) \rightarrow \{k, k+1, k+2, \ldots., k+2n-2\}$ by

$$f(v_i) = k + 2i - 2 \quad 1 \leq i \leq n$$

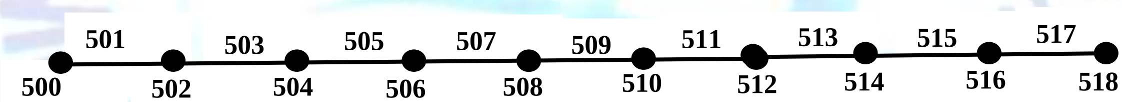
Then the induced edge labels are

$$f^*(e_i) = k + 2i - 1 \quad 1 \leq i \leq n-1$$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+2n-2\}$.

Hence P_n is a k -Super harmonic mean graph.

Example2.2:



500-super harmonic mean labeling of P_{10}

Theorem 2.3:

The comb P_n^+ is a k-super harmonic mean graph for all n ≥ 2.

Proof:

Let $V(P_n^+) = \{u_i, v_i ; 1 \leq i \leq n\}$

$E(P_n^+) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i' = (u_i, v_i); 1 \leq i \leq n\}$

Define a function $f: V(P_n^+) \rightarrow \{k, k+1, k+2, \ldots., k+4n-2\}$ by

$$f(u_i) = k + 4i - 2 \qquad 1 \leq i \leq n$$

$$f(v_1) = k$$

$$f(v_i) = k + 4i - 5 \qquad 2 \leq i \leq n$$

Then the induced edge labels are

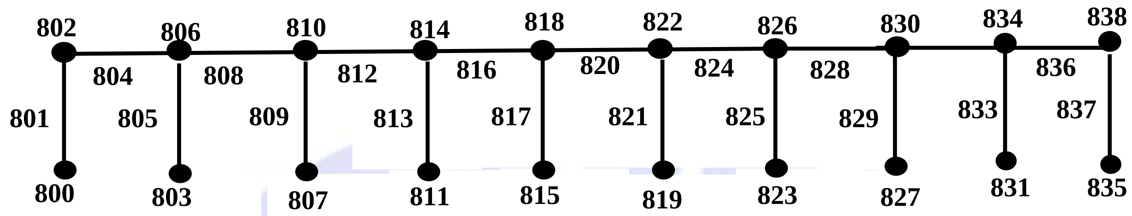
$$f^*(e_i) = k + 4i \qquad 1 \leq i \leq n-1$$

$$f^*(e_i') = k + 4i - 3 \qquad 1 \leq i \leq n$$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \ldots., k+4n-2\}$.

Hence P_n^+ is a k -Super harmonic mean graph.

Example 2.4:



800-super harmonic mean labeling of P_{10}^+

Theorem2.5 :

nP_m is a k-super harmonic mean graph for all m ≥ 2, n ≥ 1.

Proof:

Let $V(\mathrm{n}P_m) = \{v_{i,j}, 1 \le i \le n, 1 \le j \le m\}$.

Then $E(\mathrm{n}P_m) = \{(v_{i,j}, v_{i,j+1}); 1 \le i \le n, 1 \le j \le m-1\}$.

Define a function $f: V(nP_m) \to \{k, k+1, k+2, \ldots, k+n(2m-1)-1\}$ by

$$f(v_{i,j}) = k + (2m-1)(i-1) + 2j - 2 \; ; \quad 1 \le i \le n, 1 \le j \le m$$

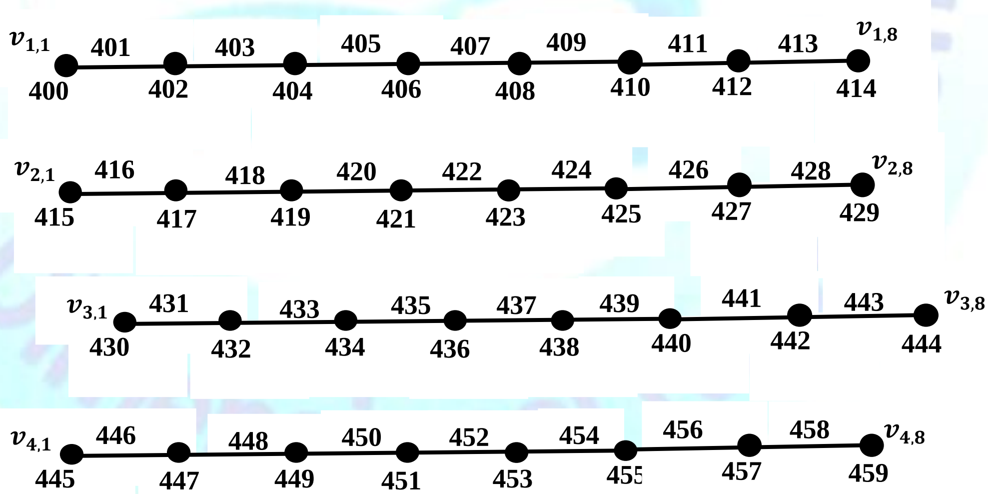
Then the induced edge labels are

$$f^*(v_{i,j}, v_{i,j+1}) = k + (2m-1)(i-1) + 2j - 1 \; ; \quad 1 \le i \le n, 1 \le j \le m-1.$$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+n(2m-1)-1\}$

Hence nP_m is a k -Super harmonic mean graph.

Example 2.6 :



400-super harmonic mean of $4P_8$

Theorem 2.7:

$P_n \odot \overline{K_2}$ is a k-super harmonic mean graph for all n ≥ 2.

Proof:

Let $V(P_n \odot \overline{K_2}) = \{u_i, v_i, w_i\,; 1 \le i \le n\}$

$E(P_n \odot \overline{K_2}) = \{(u_i\,, u_{i+1}\,); 1 \le i \le n-1\} \cup \{(u_i, v_i\,); 1 \le i \le n\} \cup \{(u_i\,, w_i\,); 1 \le i \le n\}$

Define a function $f: V(P_n \odot \overline{K_2}) \to \{k, k+1, k+2, \ldots, k+6n-2\}$by

$$f(u_i) = k + 6i - 4 \qquad 1 \le i \le n$$
$$f(v_i) = k + 6i - 6 \qquad 1 \le i \le n$$
$$f(w_i) = k + 6i - 1 \qquad 1 \le i \le n-1$$
$$f(w_n) = k + 6n - 2$$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + 6i - 2 \qquad 1 \le i \le n-1$

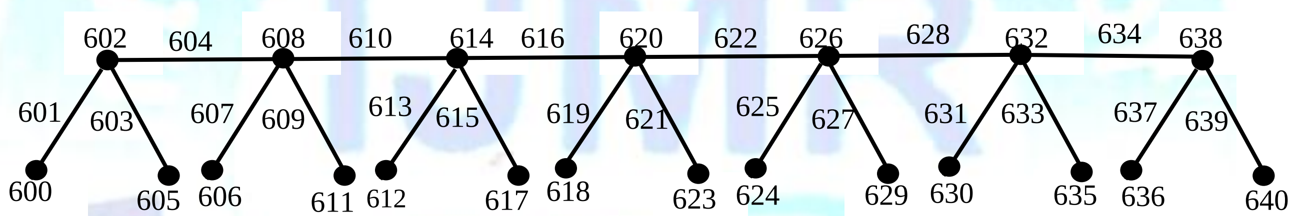
$f^*(u_i v_i) \quad = k + 6i - 5 \qquad 1 \le i \le n$

$f^*(u_i w_i) \quad = k + 6i - 3 \qquad 1 \le i \le n$

Thus$f(V) \cup \{f^*(e); e\epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+6n-2\}$.

Hence $P_n \odot \overline{K_2}$ is a k -Super harmonic mean graph.

Example 2.8:



600-super harmonic mean of $P_7 \odot \overline{K_2}$

Theorem 2.9:

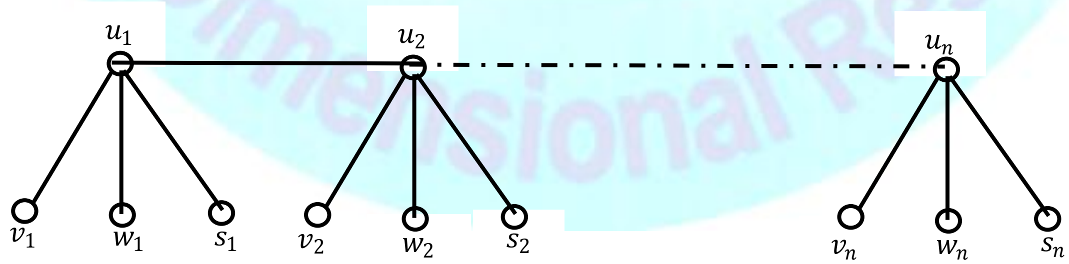
$P_n \odot \overline{K_3}$ is a k-super harmonic mean graphfor all n ≥ 2.

Proof:

Let $V(P_n \odot \overline{K_3}) = \{u_i, v_i, w_i, s_i; 1 \le i \le n\}$

$E(P_n \odot \overline{K_3}) = \{(u_i, u_{i+1}\,); 1 \le i \le n-1\} \cup \{(u_i, v_i\,); 1 \le i \le n\} \cup$
$\{(u_i, w_i\,); 1 \le i \le n\} \cup \{(u_i\,, s_i\,); 1 \le i \le n\}$

The ordinary labeling of $P_n \odot \overline{K_3}$ is given below



Define a function $f: V(P_n \odot K_3) \to \{k, k+1, k+2, \ldots, k+8n-2\}$ by

$$f(u_i) = k + 8i - 1 \qquad 1 \le i \le n$$
$$f(v_i) = k + 8i - 8 \qquad 1 \le i \le n$$
$$f(w_i) = k + 8i - 6 \qquad 1 \le i \le n$$
$$f(s_i) = k + 8i - 2 \qquad 1 \le i \le n$$

Then the induced edge labels are

$f^*(u_i u_{i+1}) = k + 8i - 4 \qquad 1 \le i \le n-1$

$f^*(u_i v_i) = k + 8i - 7 \qquad 1 \le i \le n$

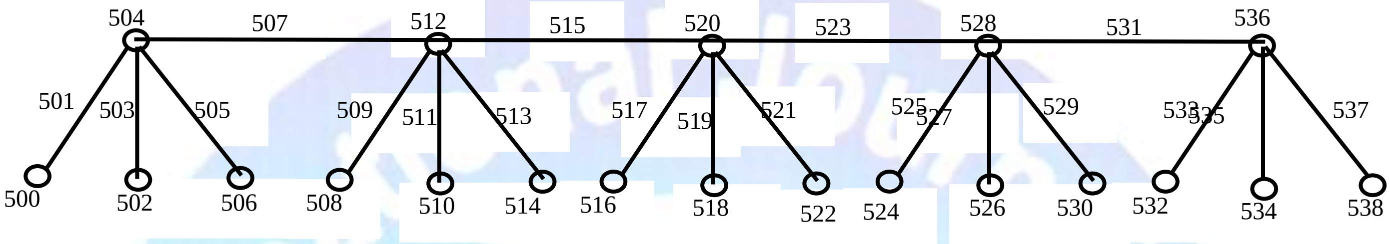
$f^*(u_i w_i) = k + 8i - 5 \qquad 1 \le i \le n$

$f^*(u_i s_i) = k + 8i - 3 \qquad 1 \le i \le n$

Thus $f(V) \cup \{f^*(e): e \epsilon E(G)\} = \{k, k+1, k+2, \dots, k+8n-2\}$.

Hence $P_n \odot \overline{K_3}$ is a k -Super harmonic mean graph.

Example 2.10



500 *super mean labeling of* $P_5 \odot \overline{K_3}$

Theorem 2.11 :

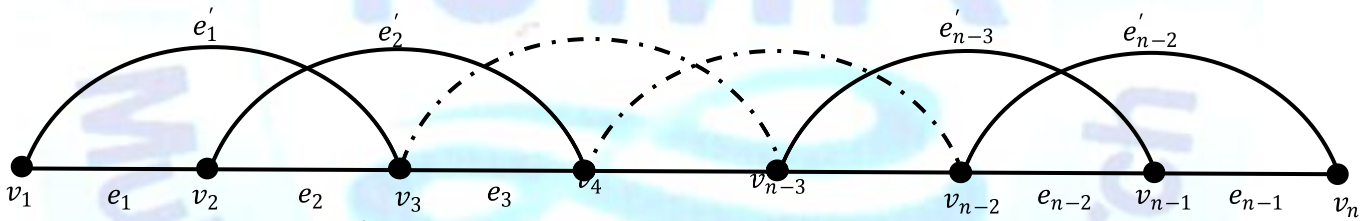
P_n^2 is a k-super harmonic graph for all $n \ge 4$.

Proof:

Let $V(P_n^2) = \{v_i\,;\, 1 \le i \le n\}$

$E(P_n^2) = \{e_i = (v_i, v_{i+1}); 1 \le i \le n-1\} \cup \{e_i' = (v_i, v_{i+2}); 1 \le i \le n-2\}$

The ordinary labeling of the graph P_n^2 is given below



First we label the vertices as follows:

Define $f: V(P_n^2) \to \{k, k+1, k+2, \dots, k+3n-4\}$ by

$f(v_i) = k + 3i - 3, \qquad 1 \le i \le n-1,$

$f(v_n) = k + 3n - 4$

Then the induced edge labels are

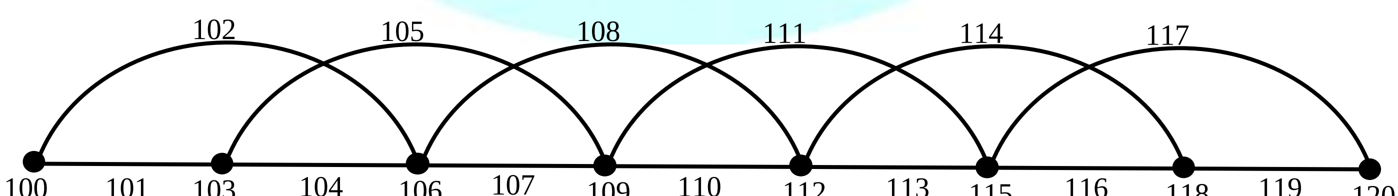
$f^*(e_i) = k + 3i - 2, \qquad 1 \le i \le n-1$

$f^*(e_i') = k + 3i - 1, \qquad 1 \le i \le n-2$

Thus $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, \dots, k+3n-4\}$.

Hence P_n^2 is a k-super harmonic mean graph.

Example 2.12



100 *super harmonic mean labeling of* P_8^2

Theorem 2.13 :

A graph obtained by identifying the central vertex of $K_{1,2}$at each pendent vertex of a comb $P_n^+$is a k-super harmonic mean graphfor all n ≥ 2.

Proof:

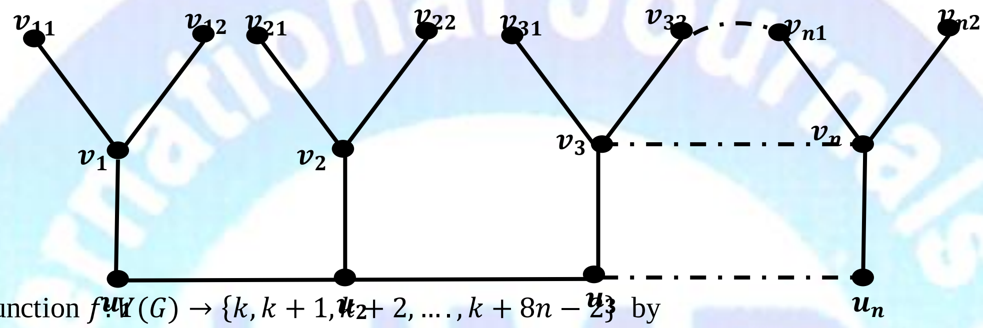
Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n.

Let v_i be a vetex adjacent to u_i ,$1 \le i \le n$ .The resulting graph is P_n^+ .

Let w_i, v_{i1}, v_{i2} be the vertices of i^{th} copy of $K_{1,2}$with $w_i$is the central vertex.Identify the vertex w_i with $v_i$we get the required graph G whose edge set is

$$E = \{u_i u_{i+1} ;\ 1 \le i \le n-1\} \cup \{u_i v_i ,\quad v_i v_{i1} , v_i v_{i2} ; 1 \le i \le n\}$$

The ordinary labeling is given below



Define a function $f: V(G) \to \{k, k+1, k+2, \ldots, k+8n-2\}$ by

$$f(u_1) = k+6$$
$$f(u_i) = k+8i-4 \qquad 2 \le i \le n$$
$$f(v_1) = k+4$$
$$f(v_i) = k+8i-2 \qquad 2 \le i \le n$$
$$f(v_{11}) = k$$
$$f(v_{i1}) = k+8i-9 \qquad 2 \le i \le n$$
$$f(v_{12}) = k+2$$
$$f(v_{22}) = k+8$$
$$f(v_{i2}) = k+8i-7 \qquad 3 \le i \le n$$

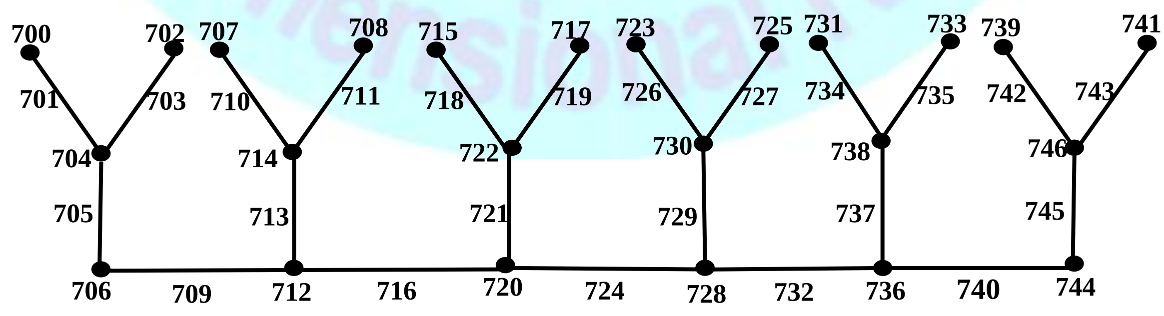
Then the induced edge labels are

$$f^*(u_1u_2) = f(u_1)+3$$
$$f^*(u_iu_{i+1}) = k+8i \qquad 2 \le i \le n-1$$
$$f^*(u_iv_i) = k+8i-3 \qquad 1 \le i \le n$$
$$f^*(v_{11}v_1) = k+1$$
$$f^*(v_{i1}v_i) = k+8i-6 \qquad 2 \le i \le n$$
$$f^*(v_{i2}v_i) = k+8i-5 \qquad 1 \le i \le n$$

Thus $f(V) \cup \{f^*(e); e\epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+8n-2\}$.

Hence G is a k -Super harmonic mean graph.

Example 2.14:



700 -Super harmonic mean graph of G

Theorem 2.15 :

$S(P_n \odot K_1)$ is a k-super harmonic mean graphfor all n ≥ 2.

Proof:

Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the vertices of $P_n \odot K_1$.

Let x_i $(1 \le i \le n)$ be the vertex which divides the edge $u_i\, v_i$ $(1 \le i \le n)$

and y_i $(1 \le i \le n-1)$ be the vertex which divides the edge $u_i\, u_{i+1}$ $(1 \le i \le n-1)$.

Then $V(S(P_n \odot K_1) = \{u_i, v_i, x_i, y_j \;;\; 1 \le i \le n, 1 \le j \le n-1\}$

Define a function $f: S(V(P_n \odot K_1)) \to \{k, k+1, k+2, \dots, k+8n-4\}$ by

$$f(u_1) = k+4$$
$$f(u_i) = k+8i-8 \qquad 2 \le i \le n$$
$$f(v_1) = k$$
$$f(v_i) = k+8i-3 \qquad 2 \le i \le n-1$$
$$f(v_n) = k+8n-4$$
$$f(x_i) = k+8i-6 \qquad 1 \le i \le n$$
$$f(y_i) = k+8i-2 \qquad 1 \le i \le n-1$$

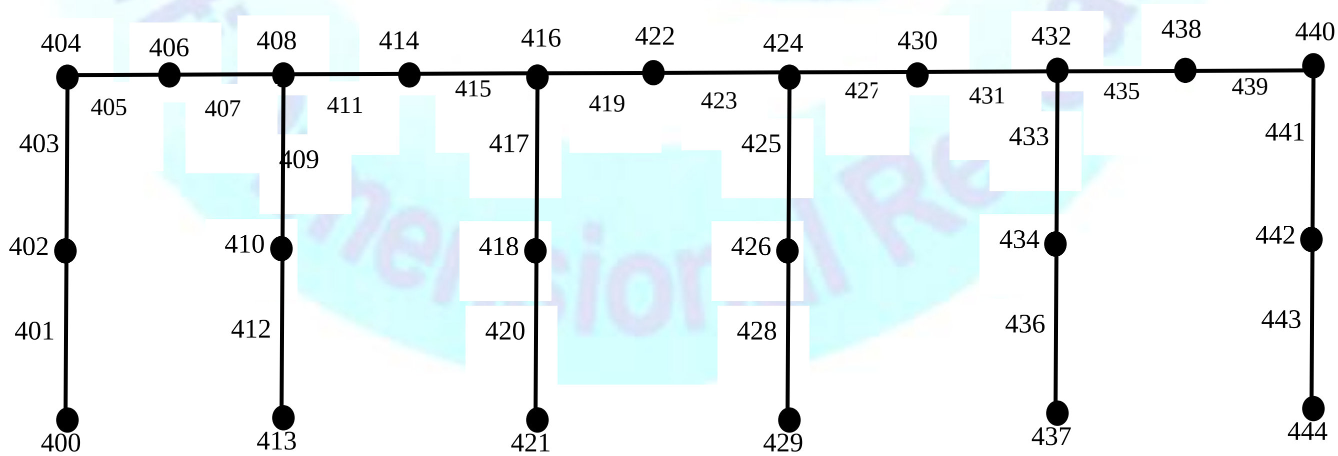
Then the induced edge labels are

$$f^*(u_1 y_1) = k+5$$
$$f^*(u_i y_i) = k+8i-5 \qquad 2 \le i \le n-1$$
$$f^*(y_1 u_2) = k+7$$
$$f^*(y_i u_{i+1}) = k+8i-1 \qquad 2 \le i \le n-1$$
$$f^*(u_1 x_1) = k+3$$
$$f^*(u_i x_i) = k+8i-7 \qquad 2 \le i \le n$$
$$f^*(x_1 v_1) = k+1$$
$$f^*(x_i v_i) = k+8i-4 \qquad 2 \le i \le n-1$$
$$f^*(x_n v_n) = f(x_n)+1$$

$Thus$ $f(V) \cup \{f^*(e); e \epsilon E(G)\} = \{k, k+1, k+2, \dots, k+8n-4\}$.

Hence $S(P_n \odot K_1)$ is a k -Super harmonic mean graph.

Example 2.16:



400-*super harmonic mean of* $S(P_6 \odot K_1)$

Theorem 2.17 :

Middle graph of a path P_n is $k-$super harmonic mean graph for all $n \geq 2$.

Proof:

Let $V(M(P_n)) = \{v_i, e_j ;\ 1 \leq i \leq n, 1 \leq j \leq n-1\}$

$E(M(P_n)) = \{v_i e_i; 1 \leq i \leq n-1\} \cup \{v_i e_{i-1}; 2 \leq i \leq n\} \cup \{e_i e_{i+1}; 1 \leq i \leq n-2\}$

$|V(M(P_n)| = 2n-1 |E(M(P_n)| = 3n-4$

Define a function $f: V(M(P_n)) \to \{k, k+1, k+2, \ldots, k+5n-6\}$ by

$$f(e_i) = k+5i-3 \qquad 1 \leq i \leq n-1$$
$$f(v_i) = k+5i-5 \qquad 1 \leq i \leq n-1$$
$$f(v_n) = k+5n-6$$

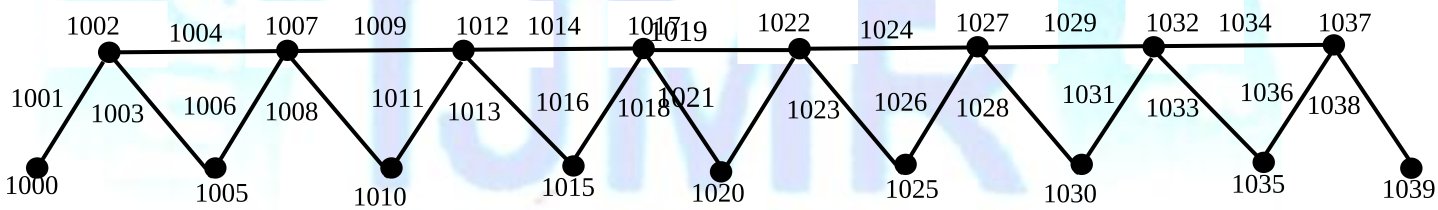
Then the induced edge labels are

$$f^*(e_i e_{i+1}) = k+5i-1 \qquad 1 \leq i \leq n-2$$
$$f^*(v_i e_i) = k+5i-4 \qquad 1 \leq i \leq n-1$$
$$f^*(v_i e_{i-1}) = k+5i-2 \qquad 2 \leq i \leq n$$

Thus $f(V) \cup \{f^*(e); e \epsilon E(G)\} = \{k, k+1, k+2, \ldots, k+5n-6\}$.

Hence $M(P_n)$ is a k -Super harmonic mean graph.

Example 2.18:



1000-super harmonic mean graph of $M(P_9)$

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