A FUZZY INVENTORY MODEL WITH WEIBULL DETERIORATING RATE UNDER PERMISSIBLE DELAY IN PAYMENTS USING AGREEMENT INDEX METHOD

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Abstract - ***In this paper, a fuzzy deterministic inventory model with two parameter weibulldeterioration rate, exponential demand rate under a permissible delay in payments,demand is completely backlogged have been proposed. The two-level credit policy is being discussed, ie., the permissible periods,which are less than or equal to and greater than the time at which the inventory becomes zero. The classical total inventory costs are obtained. The cost parameters are represented by Lower - Upper (L-U) bud fuzzy number. The fuzzytotal inventory costs are obtained and defuzzifiedunder agreement index technique. The analytical expressions for the optimum order quantity and shortage quantity are derived. A numerical example is given to illustrate both the proposed crisp model and fuzzy model.***

Keywords **-weibull deterioration rate, two-level credit policy, L-Ubud fuzzy number, agreement index technique.**

AMS Subject Classification (2010) **-90B05, 90C70.**

Introduction

Inventory System is one of the main streams of the Operations Research which is essential in business enterprises and Industries. Inventory is the essence as blood in the body for any business organization. The inventory theory deals with the determination of optimal stocks of commodities to meet demands in the foreseeable future. In a business organization, inventory management is one of the major core competencies to compete in the global market place. Deterioration cannot be avoided in business scenario. Deterioration is defined as change, damage, decay, spoilage obsolescence and a loss of utility or a loss of original value in a commodity that results in the decreasing usefulness from the original product. Owing to this fact, controlling and maintaining the inventory of deteriorating items become a challenging problem for decision makers.

In the traditional inventory model, it tacitly assumed that the retailer must pay to the supplier for the items purchased as soon as the retailer received the items. However, the most prevailing practice is that the supplier adopts a marketing strategy of permissible delay in payments to stimulate the retailer's demand. Therefore, the research about the trade credit terms has been paid much attention. Thus, the delay in the payment offered by the supplier is a kind of price discount, since paying later indirectly reduces the purchase cost and encourages the retailers to increase their order quantity. Credit terms are influenced by the nature of the business.

In recent periods, researchers are developing the inventory models for deteriorating items by introducing new concepts. There are a few inventory models in association with salvage value or disposal cost or sales revenue etc. Similarly researchers have developed trade credit with new ideas. This trade credit means that the supplier offers the customer a permissible delay in payments to attract new customers to increase sales. Case (i) : the permissible delay period less than or equal to replenishment cycle period for settling the account and Case (ii) : the permissible delay period is greater than replenishment cycle period for settling the account.

An EOQ inventory models under permissible delay in payments was cited by many researchers in different ways. Recently, Hardik Soni, Nita H. Shah and Chandra K. Jaggi (2010) [5] proposed a review of inventory models and trade credit.Amutha, Chandrasekaran (2013) [1] Kuo-Lung Hou Li-Chiao Lin [8] and Horng–Jinh Chang (2010) [18] considered an inventory model with constant demand rate. Tripathi, Misra, Shukla (2010) [10] discussed a cash flow model, Trailokyanath Singh, Hadibandhu Pattnayak (2012) [14] discussed exponentially declining demand rate, Raman Patel (2013) [13], Trailokyanath singh (2013) [15] and Venkateswarlu, Reddy (2014) [16] considered an EOQ model for a deteriorating item with time-dependent quadratic demand. Babu Krishnaraj, Ramasamy (2013) [2] discussed linear deterioration rate, Horng–Jinh Chang (2002) [6] and Jiang Wu, Ya-Lan Chan (2014) [7] proposed an inventory model for variable deterioration rate, Liu Na and Hu Jinsong (2010) [11] considered fuzzy deteriorate rate, Lian Zhu and Jin Zhang (2008) [9] and Xiao-song Ding, Ji-hong Zhang (2012) [17] discussed an inventory model with permissible delay in payments. K. Dhanam and W. Jesintha discussed L-U bud fuzzy number and Agreement index method in [3] & [4].Recently [12] Tripathi (2012) presented an inventory model with shortage and exponentially demand rate under permissible delay in payments. In this paper demand rate considered as exponentially increasing function of time, weibull deterioration rate and shortages are allowed.

Assumptions and Notations

The following assumptions and notations are used throughout this paper :

Assumptions :

1. The inventory system involves only one item.
2.The demandrateistimedependentandexponential function of time and is given by,

$$R(t) = \lambda_0 \, e^{at} \; ; \; 0 < a < 1 \text{ and } \; 0 \le t \le T .$$

3. The deterioration rate is time dependent and weibull distribution (with two parameter) function of time and is given by,

$$\theta(t) = \alpha \, \beta \, t^{\beta-1}; 0 < \alpha < 1; 0 < \beta < 1.$$

4. Shortages are allowed and completely backlogged.
5.Duringtimet_1,theinventorylevelbecomeszeroduetodemand and deterioration. Attime t_1, the shortagesstartoccurring.
6. Delay payment is allowed.
7. Lead time is zero.
8. The planning period is of infinite length.

Notations:

$I_1(t)$ - inventory level at any instant of time ‘t’, $0 \le t \le t_1$.

$I_2(t)$ - inventory level at any instant of time ‘t’, $t_1 \le t \le T$.

Q - the order quantity during a cycle of length T.

$\tilde{k}$ - fuzzy ordering cost per order.

$\tilde{p}_c$ - fuzzy purchase cost per unit per unit time.

$\tilde{h}_c$ - fuzzy holding cost per unit per unit time.

$\tilde{d}_c$ - fuzzy holding cost per unit per unit time.

$\tilde{s}_c$ -fuzzy shortage cost per unit per unit time.

I_e - the interest earned per unit time.

I_r - the interest charges invested in inventory .
m -permissible delay in settling the account, $0 \le m \le T$.
T - length of the replenishment cycle.
t_1 -time when inventory level comes down to zero, $0 \le t_1 \le T$.
$Z(t_1,T)$ - average total inventory cost per unit time when permissible delay in payment is m.

$$Z(t_1,T) - \begin{cases} z_1(t_1,T) & for \quad t_1 \ge m \\ z_2(t_1,T) & for \quad t_1 < m \end{cases}$$

Mathematical Model in Crisp Environment

The inventory level I_0 units at time t=0; from t=0 to t=t_1, the inventory level reduces, owing to both demand and deterioration, until it reaches zero level at time t=t_1.At this time, shortage is accumulated which is completely backlogged. At the end of the cycle, the inventory reaches a maximum shortage level so as to clear the backlogged and again raises the inventory to I_0 .

The rate of change of the inventory during the positive stock period (0,t_1) and shortage period (t_1,T) is governed by the following differential equations

$$\frac{dI_1(t)}{dt} + \alpha\,\beta\, t^{\beta-1} I_1(t) = -\lambda_0\, e^{at} \quad , 0 \le t \le t_1 \ldots\ldots\ldots\ldots(1)$$

$$\frac{dI_2(t)}{dt} = -\lambda_0\, e^{at} \quad , t_1 \le t \le T \ldots\ldots\ldots\ldots(2)$$

with the boundary conditions $I_1(0)=I_0$, $I_1(t_1)=0$, $I_2(t_1)=0$.
From equation (1),

$$I_1(t) \quad e^{\int \alpha\,\beta\, t^{\beta-1} dt} = -\int \lambda_0 e^{at} e^{\int \alpha\,\beta\, t^{\beta-1} dt} dt + c_1$$

using boundary condition $I_1(t_1) = 0$, Inventory level during the period (0,t_1), $I_1(t)$ is derived

$$I_1(t) = \lambda_0 \left\{ \begin{array}{l} \left[\begin{array}{l} (t_1 - t) + \frac{a}{2}(t_1^2 - t^2) + \frac{a^2}{6}(t_1^3 - t^3) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}(t_1^{2\beta+1} - t^{2\beta+1}) + \\ \frac{a\alpha}{(\beta+2)}(t_1^{\beta+2} - t^{\beta+2}) \end{array} \right] \\ -\alpha \left[\begin{array}{l} (t_1 t^\beta - t^{\beta+1}) + \frac{a}{2}(t_1^2 t^\beta - t^{\beta+2}) + \frac{a^2}{6}(t_1^3 t^\beta - t^{\beta+3}) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} t^\beta - t^{2\beta+1}) + \\ \frac{\alpha^2}{2(2\beta+1)}(t_1^{2\beta+1} t^\beta - t^{3\beta+1}) + \frac{a\alpha}{(\beta+2)}(t_1^{\beta+2} t^\beta - t^{2\beta+2}) \end{array} \right] \\ + \frac{\alpha^2}{2} \left[\begin{array}{l} (t_1 t^{2\beta} - t^{2\beta+1}) + \frac{a}{2}(t_1^2 t^{2\beta} - t^{2\beta+2}) + \frac{a^2}{6}(t_1^3 t^{2\beta} - t^{2\beta+3}) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} t^{2\beta} - t^{3\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}(t_1^{2\beta+1} t^{2\beta} - t^{4\beta+1}) + \\ \frac{a\alpha}{(\beta+2)}(t_1^{\beta+2} t^{2\beta} - t^{3\beta+2}) \end{array} \right] \end{array} \right\}$$

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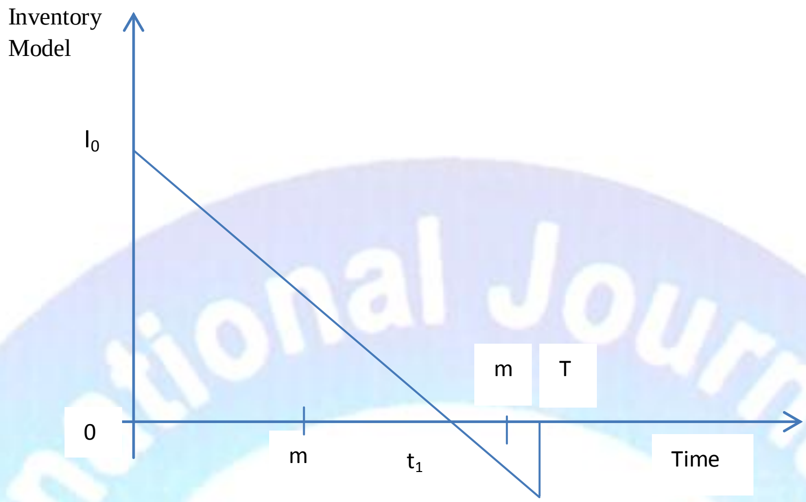


From equation (2),

$$I_2(t) = -\lambda_0 \int e^{a\,t} dt + c_2 \Rightarrow \quad I_2(t) = -\lambda_0 \left[\frac{e^{a\,t}}{a} \right] + c_2$$

using boundary condition $I_2(t_1) = 0$, Inventory level during the period (t_1, T), $I_2(t)$ is derived

$$I_2(t) = \frac{\lambda_0}{a}\left[a(t_1 - t) + \frac{a^2}{2}(t_1^{\,2} - t^2) \right] \qquad t_1 \le t \le T\,,$$

using the boundary condition $I_1(0) = I_0$, Inventory level during the period (0,T), I_0 is derived

$$I_0 = I_1(0) = \lambda_0 \left[(t_1) + \frac{a}{2}(t_1^{\,2}) + \frac{a^2}{6}(t_1^{\,3}) + \frac{\alpha}{\beta+1}(t_1^{\,\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}(t_1^{\,2\beta+1}) + \frac{a\alpha}{(\beta+2)}(t_1^{\,\beta+2}) \right]$$

From I_0 and $I_2(T)$, the order quantity Q is derived

$$Q = I_0 + I_2(T)$$

$$= \lambda_0 \left[2t_1 + a(t_1^{\,2}) + \frac{a^2}{3}(t_1^{\,3}) + \frac{\alpha}{\beta+1}(t_1^{\,\beta+1}) + \frac{\alpha^2}{2(2\beta+1)}(t_1^{\,2\beta+1}) + \frac{a\alpha}{(\beta+2)}(t_1^{\,\beta+2}) - T - \frac{a}{2}(T^2) - \frac{a^2}{6}(T^3) \right]$$

Inventory is available in the system during the time interval $(0,t_1)$. Hence, the cost for holding inventory in stock is computed for time period $(0,t_1)$ as follows:

$$HC = h\int_0^{t_1} I(t)\,dt$$

$$= h\,\lambda_0\int_0^{t_1}\left\{\begin{array}{l}\left(\left(t_1-t\right)+\frac{a}{2}\left(t_1^2-t^2\right)+\frac{a^2}{6}\left(t_1^3-t^3\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_1^{2\beta+1}-t^{2\beta+1}\right)+\right.\\ \left.\frac{a\alpha}{(\beta+2)}\left(t_1^{\beta+2}-t^{\beta+2}\right)\right)-\alpha\left(\left(t_1t^{\beta}-t^{\beta+1}\right)+\frac{a}{2}\left(t_1^2t^{\beta}-t^{\beta+2}\right)+\frac{a^2}{6}\left(t_1^3t^{\beta}-t^{\beta+3}\right)+\frac{\alpha}{\beta+1}\right.\\ \left.\left(t_1^{\beta+1}t^{\beta}-t^{2\beta+1}\right)+\frac{\alpha^2}{2(2\beta+1)}\left(t_1^{2\beta+1}t^{\beta}-t^{3\beta+1}\right)+\frac{a\alpha}{(\beta+2)}\left(t_1^{\beta+2}t^{\beta}-t^{2\beta+2}\right)\right)\\ +\frac{\alpha^2}{2}\left(\left(t_1t^{2\beta}-t^{2\beta+1}\right)+\frac{a}{2}\left(t_1^2t^{2\beta}-t^{2\beta+2}\right)+\frac{a^2}{6}\left(t_1^3t^{2\beta}-t^{2\beta+3}\right)+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}t^{2\beta}-t^{3\beta+1}\right)\right.\\ \left.+\frac{\alpha^2}{2(2\beta+1)}\left(t_1^{2\beta+1}t^{2\beta}-t^{4\beta+1}\right)+\frac{a\alpha}{(\beta+2)}\left(t_1^{\beta+2}t^{2\beta}-t^{3\beta+2}\right)\right)\end{array}\right\}dt$$

On simplification,

$$= h\,\lambda_0\left[\begin{array}{l}\left(\frac{t_1^{\,2}}{2}+\frac{a t_1^3}{3}+\frac{a^2 t_1^4}{8}+\frac{\alpha t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2 t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha t_1^{\beta+3}}{\beta+3}\right)-\alpha\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a t_1^{\,\beta+3}}{(\beta+1)(\beta+3)}+\right.\\ \left.\frac{a^2 t_1^{\,\beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha t_1^{\,2\beta+2}}{2(\beta+1)^2}+\frac{\alpha^2 t_1^{\,3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha t_1^{\,2\beta+3}}{(\beta+1)\,2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}+\right.\\ \left.\frac{a t_1^{\,2\beta+3}}{(2\beta+1)(2\beta+3)}+\frac{a^2 t_1^{\,2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha t_1^{\,3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2 t_1^{\,4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha t_1^{\,3\beta+3}}{(2\beta+1)(3\beta+3)}\right)\end{array}\right]$$

.......................(3)

$$NDU = Q-\int_0^{t_1} D(t)\,dt$$

$$=\lambda_0\left[\begin{array}{l}t_1+\frac{a}{2}t_1^{\,2}+\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}\\ +\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\end{array}\right]-\left[\frac{\lambda_0\,e^{a t}}{a}\right]_0^{t_1}$$

On simplification,

$$=\lambda_0\left[\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right]$$

Deterioration cost is given by

$$DC = d\lambda_0\left[\frac{a^2}{6}t_1^{\,3} + \frac{\alpha}{\beta+1}t_1^{\,\beta+1} + \frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1} + \frac{a\alpha}{\beta+2}t_1^{\,\beta+2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

The total shortage cost during the time period (t_1,T) is determined

$$= \frac{s\lambda_0}{a}\left[\frac{a}{2}(t^2 - t_1^{\,2}) - at_1(t - t_1) - (t - t_1)\frac{a^2 t_1^{\,2}}{2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Regarding interest payable and earned, the following two possible cases are based on the values of t_1&m .

<u>Case(i)</u>: $m \le t_1$,

Since the length of period with positive stock is larger than the credit period, the buyer can use the sales revenue to earn interest at an annual rate I_e in $(0, t_1)$.

The interest earned E_1 is derived

$$E_1 = p\,I_e\int_0^{t_1}\lambda_0\,e^{at}(t_1 - t)\,dt$$

$$= p\,I_e\,\lambda_0\left[(t_1 - t)\frac{e^{at}}{a} - (-1)\frac{e^{at}}{a^2}\right]_0^{t_1}$$

On simplification,

$$E_1 = \frac{p\,I_e\,\lambda_0\,t_1^{\,2}}{2} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

Beyond the credit period,the unsold stock is assumed to be financed with an annual rate and the interest payable I is derived

$$I = p\,I_r\int_m^{t_1}\lambda_0\,e^{at}(t_1 - t)\,dt$$

$$= p\,I_r\,\lambda_0\left[(t_1 - t)\frac{e^{at}}{a} - (1)\frac{e^{at}}{a^2}\right]_m^{t_1}$$

On simplification,

$$I = p\,I_r\,\lambda_0\left[\frac{m^2}{2} + \frac{t_1^{\,2}}{2} - \frac{at_1m^2}{2} + \frac{a\,m^3}{2} - t_1\,m\right] \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Thus, the average total cost of $z_1(t_1,T)$ per unit time is given by $ATC\left(z_1(t_1,T)\right)$

$$= \frac{\text{ordering cost} + \text{holding cost} + \text{shortage cost} + \text{deterioration cost} + I - E_1}{T}$$

$Min\ ATC(z_1(t_1,T))$

$$= \frac{1}{T}\left[\begin{array}{l} k+\left\{h_c\,\lambda_0\left(\begin{array}{l}\left(\frac{t_1^{\,2}}{2}+\frac{a\,t_1^3}{3}+\frac{a^2\,t_1^4}{8}+\frac{\alpha\,t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2\,t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha\,t_1^{\beta+3}}{\beta+3}\right)-\alpha\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\,t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{a^2\,t_1^{\beta+4}}{2(\beta+1)(\beta+4)}\right. \\ \left.+\frac{\alpha\,t_1^{2\beta+2}}{2(\beta+1)^2}+\frac{\alpha^2\,t_1^{3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha\,t_1^{2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{2\beta+2}}{(2\beta+1)(2\beta+2)}+\frac{a\,t_1^{2\beta+3}}{(2\beta+1)(2\beta+3)}+\right. \\ \left.\frac{a^2\,t_1^{2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha t_1^{3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2\,t_1^{4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha\,t_1^{3\beta+3}}{(2\beta+1)(3\beta+3)}\right)\end{array}\right)\right\} \\ +\left\{\frac{s_c\lambda_0}{a}\left(-at_1(t-t_1)-(t-t_1)\frac{a^2t_1^{\,2}}{2}-\frac{a}{2}(t^2-t_1^{\,2})\right)\right\}+\left\{d_c\,\lambda_0\left(\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right)\right\} \\ +\left\{p_c\,I_r\,\lambda_0\left(\frac{m^2}{2}+\frac{t_1^{\,2}}{2}-\frac{at_1m^2}{2}+\frac{a\,m^3}{2}-t_1\,m\right)\right\}-\frac{p_c\,I_e\,\lambda_0\,t_1^{\,2}}{2} \end{array}\right]$$

The optimal value of t_1 and T can be obtained from $ATC(z_1(t_1,T))$ in the case of $m \le t_1$ by using MATLAB Software.

<u>Case(ii)</u>: $m > t_1$

In this case, the buyer pays no interest but earns interest at an annual rate during the period. Interest earned is derived

$$E_2 = p\,I_e\int_0^{t_1}\lambda_0\,e^{at}(m-t)\,dt$$

$$= p\,I_e\,\lambda_0\left[\frac{m\,e^{at}}{a}-t\left(\frac{e^{at}}{a}\right)+\left(\frac{e^{at}}{a^2}\right)\right]_0^{t_1}$$

$$E_2 = \frac{p\,I_e\,\lambda_0}{a^2}\left[\left(a\,m-m\right)+a^2\,m\,t_1+\left(\frac{a^2\,m}{2}-\frac{a^2}{2}\right)t_1^{\,2}-\frac{a^3}{2}t_1^{\,3}\right]$$

…………… (9)

Thus, the average total cost of $z_2(t_1,T)$ per unit time is given by

$$ATC\left(z_2(t_1,T)\right)$$

$$=\frac{\text{ordering cost}+\text{holding cost}+\text{shortage cost}+\text{ deterioration cost}-E_2}{T}$$

$$Min\,ATC\left(z_2(t_1,T)\right)$$

$$=\frac{1}{T}\Bigg[k+h_c\ \lambda_0\left(\frac{t_1^{\ 2}}{2}+\frac{a t_1^3}{3}+\frac{a^2 t_1^4}{8}+\frac{\alpha t_1^{\ \beta+2}}{\beta+2}+\frac{\alpha^2 t_1^{\ 2\beta+2}}{4(\beta+1)}+\frac{a\alpha t_1^{\beta+3}}{\beta+3}\right)-\alpha\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{a t_1^{\ \beta+3}}{(\beta+1)(\beta+3)}+\frac{a^2 t_1^{\ \beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha t_1^{\ 2\beta+2}}{2(\beta+1)^2}+\right.$$

$$\left.\frac{\alpha^2 t_1^{\ 3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha t_1^{\ 2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\ 2\beta+2}}{(2\beta+1)(2\beta+2)}+\frac{a t_1^{\ 2\beta+3}}{(2\beta+1)(2\beta+3)}+\frac{a^2 t_1^{\ 2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha t_1^{\ 3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2 t_1^{\ 4\beta+2}}{2\left(2\beta+1\right)(4\beta+2)}\right.$$

$$\left.+\frac{a\alpha t_1^{\ 3\beta+3}}{\left(2\beta+1\right)(3\beta+3)}\right)+\left\{\frac{s_c\lambda_0}{a}\left(-at_1(t-t_1)-(t-t_1)\frac{a^2 t_1^{\ 2}}{2}-\frac{a}{2}(t^2-t_1^{\ 2})\right)\right\}+\left\{d_c\ \lambda_0\left(\frac{a^2}{6}t_1^{\ 3}+\frac{\alpha}{\beta+1}t_1^{\ \beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\ 2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\ \beta+2}\right)\right\}-$$

$$\left.\frac{p_c\, I_e\, \lambda_0}{a^2}\left\{\left(\left(a\,m-m\right)+a^2\, m t_1+\left(\frac{a^2\, m}{2}-\frac{a^2}{2}\right)t_1^{\ 2}-\frac{a^3}{2}t_1^{\ 3}\right)\right\}\right]$$

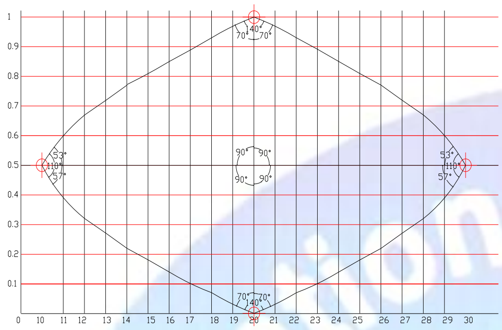
The optimal value of t_1 and T can be obtained from ATC($z_2(t_1$,T)) in the case of m > t_1, by using MATLAB Software.

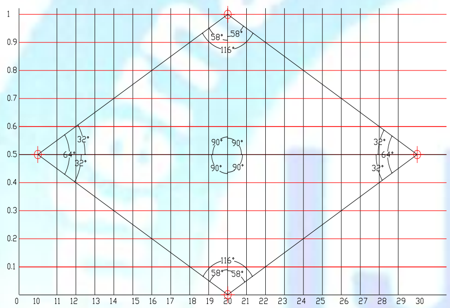
LU-bud Fuzzy Number and its agreement index method

Definition :(L-Ubud fuzzy number)

A LU-bud fuzzy number $\tilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows

$$\mu_{\tilde{A}}(x)=\begin{cases} 0.5 & at \quad x=a,c \\ \frac{1}{2}\left\{1-\left(\frac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b \\ \frac{1}{2}\left\{1-\left(\frac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c \\ \frac{1}{2}\left\{1+\left(\frac{x-a}{b-a}\right)^{\frac{n}{n+1}}\right\} & at \quad a\le x\le b \\ \frac{1}{2}\left\{1+\left(\frac{c-x}{c-b}\right)^{\frac{n}{n+1}}\right\} & at \quad b\le x\le c \\ 0 \text{ and } 1 & at \quad x=b \end{cases}$$





This type of fuzzy number be denoted as $\tilde{A} = [a, b, c; w_A]$, where $w_A = 0.5$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}$ is a continuous mapping from R to the closed interval [0,1].
2. $\mu_{\tilde{A}}$ is a convex function.
3. $\mu_{\tilde{A}} = 0, 1 \;\; at \;\; x = b.$
4. $\mu_{\tilde{A}} = 0.5 \;\; at \;\; x = a \,\&\, c.$
5. $\mu_{\tilde{A}}$ is strictly decreasing as well as increasing and continuous function on [a,b] & [b,c].

Properties:

1. Opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at 90°.
3. In the horizontal and vertical diagonal, the base of the adjacent angles are equal.
4. The length of the horizontal diagonal, is twice of thelength of the vertical diagonal.
5. If n=0, the L-Ubud reduced to two lines, which intersectperpendicularly.
6. If $n \geq 8$, the L-Ubud shape reduced to the diamond shape andthesumoftheanglesof the diagonal is 360°.(Figure:b)

5. Agreement index method :

Let us define the concept of a fuzzy upper bound. A function $h(x)$; $x \in R$ such that

$$h(x)=\begin{cases} 1 & at \quad x \le x_1 \\ \dfrac{x_2 - x}{x_2 - x_1} & at \quad x_1 \le x \le x_2 \\ 0 & at \quad x \ge x_2 \end{cases}$$

$h(x)$ represent by a fuzzy subset $H \subset R.$ consider a fuzzy number $A \subset R,$ which we call the agreement index of A with regard to H, the ratio being defined as

$$i(A,H)=\frac{area\ of\ \ A\cap H}{area\ of\ \ A} \in [0,1].$$

When **A is nonfuzzy**, that is, $\mu(A)=\begin{cases}1, & x=n\\ 0, & x\neq n\end{cases}$

then, $i(A,H)=h(n).$

The agreement index of a LU-bud fuzzy number is of the form

When **H isnonfuzzy**, that is,

$h(x)=\begin{cases}1, & x\le x_1\\ 0, & x>x_1\end{cases}$ In this case compute the area of A to the left of x_1.

$$i_G(A,H)=\frac{I_1}{(c-a)}\left[I_2+aI_3+bI_4+cI_5+I_6\right]$$

where,

$$I_1=\frac{2n+1}{(n+1)}, I_2=\left(\frac{(\alpha_2-\alpha_1)(x_1+x_2)}{2}\right),$$

$$I_3=\left(\frac{n(2\alpha_2-1)^{\frac{2n+1}{n}}}{2(2n+1)}-\alpha_2\right),$$

$$I_4=-\left(\frac{n(2\alpha_2-1)^{\frac{2n+1}{n}}}{2(2n+1)}+\frac{n(1-2\alpha_1)^{\frac{2n+1}{n}}}{2(2n+1)}\right),$$

$$I_5=\left(\alpha_1-\frac{n(1-2\alpha_1)^{\frac{2n+1}{n}}}{2(2n+1)}\right), I_6=\frac{-n}{2(n+1)}.$$

Inventory Model in Fuzzy Environment

If the cost parameters are LU-bud fuzzy numbers then the problem (8) & (10) are transformed to average total cost in fuzzy environment

$$Min\ A\tilde{T}C(z_1(t_1,T))$$

$$=\frac{1}{T}\left[\begin{array}{l}\tilde{k}+\left\{\tilde{h}_c\,\lambda_0\left(\begin{array}{l}\left(\frac{t_1^{\,2}}{2}+\frac{a\,t_1^3}{3}+\frac{a^2\,t_1^4}{8}+\frac{\alpha\,t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2\,t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha\,t_1^{\,\beta+3}}{\beta+3}\right)-\\ \alpha\left(\frac{t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\,t_1^{\,\beta+3}}{(\beta+1)(\beta+3)}+\frac{a^2\,t_1^{\,\beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha\,t_1^{\,2\beta+2}}{2(\beta+1)^2}+\right.\\ \left.\frac{\alpha^2\,t_1^{\,3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha\,t_1^{\,2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}+\frac{a\,t_1^{\,2\beta+3}}{(2\beta+1)(2\beta+3)}\right.\\ \left.+\frac{a^2\,t_1^{\,2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha t_1^{\,3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2\,t_1^{\,4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha\,t_1^{\,3\beta+3}}{(2\beta+1)(3\beta+3)}\right)\end{array}\right)\right\}\\ +\left\{\frac{\tilde{s}_c\lambda_0}{a}\left(-at_1(t-t_1)-(t-t_1)\frac{a^2t_1^{\,2}}{2}-\frac{a}{2}(t^2-t_1^{\,2})\right)\right\}\\ +\left\{\tilde{d}_c\,\lambda_0\left(\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right)\right\}\\ +\left\{p_c\,I_r\,\lambda_0\left(\frac{m^2}{2}+\frac{t_1^{\,2}}{2}-\frac{at_1m^2}{2}+\frac{a\,m^3}{2}-t_1\,m\right)\right\}-\frac{p_c\,I_e\,\lambda_0\,t_1^{\,2}}{2}\end{array}\right]$$

………………(8)

where ~ represents the fuzzification of the parameters and $\tilde{k}=(k_1,k_2,k_3);\tilde{h}_c=(h_1,h_2,h_3);$ $\tilde{d}_c=(d_1,d_2,d_3);\tilde{s}_c=(s_1,s_2,s_3);$ By using the agreement index of LU-bud fuzzy number ATC(z_1(t_1,T)) isdefuzzified.

$$Min\ A\tilde{T}C(z_1(t_1,T))$$

$$=\frac{1}{T}\left[\begin{array}{l}i_G(k,H)+\left\{i_G(h_c,H)\lambda_0\left(\begin{array}{l}\left(\frac{t_1^{\,2}}{2}+\frac{a\,t_1^3}{3}+\frac{a^2\,t_1^4}{8}+\frac{\alpha\,t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2\,t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha\,t_1^{\,\beta+3}}{\beta+3}\right)-\\ \alpha\left(\frac{t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\,t_1^{\,\beta+3}}{(\beta+1)(\beta+3)}+\frac{a^2\,t_1^{\,\beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha\,t_1^{\,2\beta+2}}{2(\beta+1)^2}+\right.\\ \left.\frac{\alpha^2\,t_1^{\,3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha\,t_1^{\,2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}+\frac{a\,t_1^{\,2\beta+3}}{(2\beta+1)(2\beta+3)}\right.\\ \left.+\frac{a^2\,t_1^{\,2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha t_1^{\,3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2\,t_1^{\,4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha\,t_1^{\,3\beta+3}}{(2\beta+1)(3\beta+3)}\right)\end{array}\right)\right\}\\ +\left\{\frac{i_G(s_c,H)\lambda_0}{a}\left(-at_1(t-t_1)-(t-t_1)\frac{a^2t_1^{\,2}}{2}-\frac{a}{2}(t^2-t_1^{\,2})\right)\right\}\\ +\left\{i_G(d_c,H)\lambda_0\left(\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right)\right\}\\ +\left\{p_c\,I_r\,\lambda_0\left(\frac{m^2}{2}+\frac{t_1^{\,2}}{2}-\frac{at_1m^2}{2}+\frac{a\,m^3}{2}-t_1\,m\right)\right\}-\frac{p_c\,I_e\,\lambda_0\,t_1^{\,2}}{2}\end{array}\right]$$

The optimal value of t_1 and T can be obtained from min $A\tilde{T}C\left(z_1\left(t_1,T\right)\right)$ in the case of $m\le t_1$ by using MATLAB Software.

$$Min\, A\tilde{T}C\left(z_2(t_1,T)\right)$$

$$=\frac{1}{T}\left[\begin{array}{l}\tilde{k}+\tilde{h}_c\ \lambda_0\left(\frac{t_1^{\,2}}{2}+\frac{a\,t_1^3}{3}+\frac{a^2\,t_1^4}{8}+\frac{\alpha\,t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2\,t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha\,t_1^{\,\beta+3}}{\beta+3}\right)-\alpha\left(\frac{t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\,t_1^{\,\beta+3}}{(\beta+1)(\beta+3)}+\right.\\ \left.\frac{a^2\,t_1^{\,\beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha\,t_1^{\,2\beta+2}}{2(\beta+1)^2}+\frac{\alpha^2\,t_1^{\,3\beta+2}}{2(\beta+1)(3\beta+2)}+\frac{a\alpha\,t_1^{\,2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}+\right.\\ \left.\frac{a\,t_1^{\,2\beta+3}}{(2\beta+1)(2\beta+3)}+\frac{a^2\,t_1^{\,2\beta+4}}{2(2\beta+1)(2\beta+4)}+\frac{\alpha\,t_1^{\,3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2\,t_1^{\,4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha\,t_1^{\,3\beta+3}}{(2\beta+1)(3\beta+3)}\right)+\\ \left\{\frac{\tilde{s}_c\lambda_0}{a}\left(-at_1(t-t_1)-(t-t_1)\frac{a^2t_1^{\,2}}{2}-\frac{a}{2}(t^2-t_1^{\,2})\right)\right\}+\\ \left\{\tilde{d}_c\ \lambda_0\left(\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right)\right\}\\ -\frac{p_c\,I_e\,\lambda_0}{a^2}\left((a\,m-m)+a^2\,m\,t_1+\left(\frac{a^2\,m}{2}-\frac{a^2}{2}\right)t_1^{\,2}-\frac{a^3}{2}t_1^{\,3}\right)\end{array}\right]$$

where ~ represents the fuzzification of the parameters and

$\tilde{k}=(k_1,k_2,k_3);\tilde{h}_c=(h_1,h_2,h_3);$

$\tilde{d}_c=(d_1,d_2,d_3);\tilde{s}_c=(s_1,s_2,s_3);$

By using the agreement index of LU-bud fuzzy number ATC($z_2(t_1,T)$) isdefuzzified.

$$Min\, A\tilde{T}C\left(z_2(t_1,T)\right)$$

$$=\frac{1}{T}\left[\begin{array}{l}\left[i_G(k,H)+i_G(h_c,H)\ \lambda_0\left(\frac{t_1^{\,2}}{2}+\frac{a\,t_1^3}{3}+\frac{a^2\,t_1^4}{8}+\frac{\alpha\,t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha^2\,t_1^{\,2\beta+2}}{4(\beta+1)}+\frac{a\alpha\,t_1^{\,\beta+3}}{\beta+3}\right)-\right.\\ \alpha\left(\frac{t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}+\frac{a\,t_1^{\,\beta+3}}{(\beta+1)(\beta+3)}+\frac{a^2\,t_1^{\,\beta+4}}{2(\beta+1)(\beta+4)}+\frac{\alpha\,t_1^{\,2\beta+2}}{2(\beta+1)^2}+\frac{\alpha^2\,t_1^{\,3\beta+2}}{2(\beta+1)(3\beta+2)}+\right.\\ \left.\frac{a\alpha\,t_1^{\,2\beta+3}}{(\beta+1)2\beta+3}\right)+\frac{\alpha^2}{2}\left(\frac{t_1^{\,2\beta+2}}{(2\beta+1)(2\beta+2)}+\frac{a\,t_1^{\,2\beta+3}}{(2\beta+1)(2\beta+3)}+\frac{a^2\,t_1^{\,2\beta+4}}{2(2\beta+1)(2\beta+4)}+\right.\\ \left.\frac{\alpha\,t_1^{\,3\beta+2}}{(2\beta+1)(3\beta+2)}+\frac{\alpha^2\,t_1^{\,4\beta+2}}{2(2\beta+1)(4\beta+2)}+\frac{a\alpha\,t_1^{\,3\beta+3}}{(2\beta+1)(3\beta+3)}\right)+\frac{i_G(s_c,H)\lambda_0}{a}\left(-at_1(t-t_1)-\right.\\ \left.\left.(t-t_1)\frac{a^2t_1^{\,2}}{2}-\frac{a}{2}(t^2-t_1^{\,2})\right)\right\}+\left\{i_G(d_c,H)\lambda_0\left(\frac{a^2}{6}t_1^{\,3}+\frac{\alpha}{\beta+1}t_1^{\,\beta+1}+\frac{\alpha^2}{2(2\beta+1)}t_1^{\,2\beta+1}+\frac{a\,\alpha}{\beta+2}t_1^{\,\beta+2}\right)\right\}\\ -\frac{p_c\,I_e\,\lambda_0}{a^2}\left\{\left((a\,m-m)+a^2\,m\,t_1+\left(\frac{a^2\,m}{2}-\frac{a^2}{2}\right)t_1^{\,2}-\frac{a^3}{2}t_1^{\,3}\right)\right\}\end{array}\right]$$

The optimal value of t_1 and T can be obtained from $A\tilde{T}C\left(z_2\left(t_1,T\right)\right)$ in the case of m > t_1, by using MATLAB .

Numerical Example

The following numerical values of the parameter in proper unit were considered as input for the numerical analysis of the model,

$\lambda_0 = 100$; $\alpha = 0.2$; $\beta = 0.2$; $I_r = 0.08$; $I_e = 0.06$; $a = 0.1$; $m = 0.3$; $x_1 = 25$; $x_2 = 15$; $p = 800$; $\alpha_1 = 0.815$; $\alpha_2 = 0.14$;

$(k_1, k_2, k_3) = (0.13, 0.15, 0.17)$; $(h_1, h_2, h_3) = (0.10, 0.12, 0.14)$;

$(d_1, d_2, d_3) = (0.08, 0.09, 0.10)$; $(s_1, s_2, s_3) = (0.11, 0.13, 0.15)$;

Using the analytical expressions the time when the inventory level reaches zero, the time when the maximum shortages occur, order quantity, average total cost and total cost per unit in crisp and fuzzy environment, the following results are obtained.

Table 1: Comparison table of crisp and fuzzy environment

case (i)	k	h_c	d_c	s_c	t_1	T	Q	ATC
Crisp	0.13	0.10	0.08	0.11	2.39	3.24	234	830
Crisp	0.15	0.12	0.09	0.13	2.19	3.04	221	827
Crisp	0.17	0.14	0.10	0.15	2.18	3.01	233	820
Fuzzy	0.145	0.109	0.095	0.138	2.48	3.18	256	817

case (ii)	k	h_c	d_c	s_c	t_1	T	Q	ATC
Crisp	0.13	0.10	0.08	0.11	1.33	1.83	187	826
Crisp	0.15	0.12	0.09	0.13	1.35	1.85	184	823
Crisp	0.17	0.14	0.10	0.15	1.35	1.83	181	820
Fuzzy	0.145	0.109	0.095	0.138	1.155	2.00	188	815

Conclusion

The main purpose of this paper is to frame an inventory model with credit linked demand and deterioration that will help the retailer to determine the optimal replenishment policy for deteriorating items when the supplier offers a permissible delay in payments.From the analysis carried out some managerial insights are obtained. The following are the managerial implications: the retailer can reduce total inventory cost by ordering lower quantity when the supplier provides a permissible delay in payments. The fuzzy economic order quantity is high when compare with crisp value and the fuzzy average total cost is minimum value when compare with the crisp value.

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