COMBINED EFFECT OF HALL CURRENT, AND CHEMICAL REACTION ON THE MHD FLOW THROUGH POROUS MEDIA PAST A VERTICAL MOVING PLATE WITH CONSTANT WALL TEMPERATURE AND VARIABLE MASS DIFFUSION

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ABSTRACT
In the present paper we have investigated the combined effect of Hall current and chemical reaction on unsteady MHD flow through porous media past a vertical moving plate with constant wall temperature and variable mass diffusion. The fluid considered is an electrically conducting. The Laplace transform technique has been used to find the solutions for the velocity and skin friction. The effect of parameters involved in governing flow model is shown graphically and the values of the skin-friction and sherwood number for different parameters have been tabulated.

Keywords: MHD flow, constant temperature, mass diffusion and Hall current.

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1.0 INTRODUCTION

The study of unsteady incompressible MHD flow past a vertical plate with heat and mass transfer plays an important role in engineering and astrophysics. Hall current is also significant in many cases. MHD flow models with Hall effects have been studied by a number of researchers. Ibrahim and Makinde [3] have studied chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. Maripala and Naikoti [5] have analyzed Hall effects on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation. Muthucumarswamy and Ganesan [6] have studied first order chemical reaction on flow past an impulsively started vertical plate with uniform heat and mass flux. Ahmed et. al [1] have studied unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. Nandkeolyar et. al [8] have presented unsteady MHD convective flow within a parallel plate rotating channel with thermal source/sink in a porous medium under slip boundary conditions. Rajput and Gaurav, [2] have studied chemical reaction effect on unsteady MHD flow past an impulsively started oscillating inclined plate with variable temperature and mass diffusion in the presence of Hall current. N. Mahapatra et . al [4] have analyzed effects of chemical reaction on free convection flow through a porous medium bounded by vertical surface. Rajput and Sahu [7] have studied thermal diffusion and chemical reaction effects on free convection MHD flow through a porous medium bounded by a vertical surface with constant heat flux. We are considering combined effect of Hall current and chemical reaction on unsteady MHD flow through porous media past a vertical moving plate with constant wall temperature and variable mass diffusion. The results are shown with the help of graphs and table.

2.0 MATHEMATICAL ANALYSIS

A vertical moving plate with constant wall temperature is considered here. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \leq 0$ the temperature of the fluid and the plate are same as T_∞, and the concentration of the fluid is taken as C_∞

. At time t > 0, the temperature of the plate and the concentration of the fluid are raised to T_w and C_w, respectively. Using the relation $\nabla \cdot B = 0$, for the magnetic field $\overline{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. Due to Hall effect, there will be two cases of the momentum equation, which are given below. The usual assumptions have been taken in to consideration. The fluid model is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(u + mw) - \frac{\upsilon}{K}u, \quad (1)$$

$$\frac{\partial w}{\partial t} = \upsilon \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0{}^2}{\rho(1+m^2)}(w - mu) - \frac{\upsilon}{K}w, \quad (2)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_C(C - C_\infty), \quad (3)$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2}, \quad (4)$$

The initial and boundary conditions are:

$$\left.\begin{array}{l} t \le 0 : u = 0, w = 0, C = C_\infty, T = T_\infty, \text{for all the values of y}, \\ t > 0 : u = u_0, w = 0, T = T_W, C = C_\infty + (C_W - C_\infty)\dfrac{u_0{}^2 t}{\upsilon} \text{ at y} = 0, \\ u \to 0, w \to 0, C \to C_\infty, \text{T} \to T_\infty \quad \text{as y} \to \infty. \end{array}\right\} \quad (5)$$

Here *u* is velocity of the fluid in *x*- direction, *w* - the velocity of the fluid in *z* - direction, *m* -the Hall parameter, *g* - acceleration due to gravity, *β*- volumetric coefficient of thermal expansion, *β**- volumetric coefficient of concentration expansion, *t* - time, C_∞ - the concentration in the fluid far away from the plate, *C* - species concentration in the fluid , C_w - species concentration at the plate, *D* - mass diffusion, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, *T* - the temperature of the fluid , *k* - the thermal conductivity, υ - the kinematic viscosity, ρ - the fluid density, K_c -chemical reaction parameter, σ - electrical conductivity, μ - the magnetic permeability, and C_P is specific heat at constant pressure. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{aligned}&\bar{u}=\frac{u}{u_0},\bar{w}=\frac{w}{u_0},\bar{y}=\frac{yu_0}{\upsilon},Sc=\frac{\upsilon}{D},Pr=\frac{\mu C_P}{k},\bar{K}=\frac{Ku_0}{\upsilon^2},\\&M=\frac{\sigma B_0^2\upsilon}{\rho u_0^2},\bar{t}=\frac{tu_0{}^2}{\upsilon},Gm=\frac{g\beta^*\upsilon(C_w-C_\infty)}{u_0{}^3},Gr=\frac{g\beta\upsilon(T_w-T_\infty)}{u_0{}^3},\\&\bar{C}=\frac{C-C_\infty}{C_w-C_\infty},\theta=\frac{(T-T_\infty)}{(T_w-T_\infty)},\mathrm{K}_0=\frac{\upsilon\mathrm{K_c}}{\mathrm{u}_0^2}.\end{aligned}\right\}\qquad(6)$$

Here the symbols used are:

u is the dimensionless velocity, $\overline{w}$ - dimensionless velocity, ϑ - the dimensionless temperature, $\bar{C}$ - the dimensionless concentration, *Gr* - thermal Grashof number, *Gm* - mass Grashof number, μ - the coefficient of viscosity, *Pr*- the Prandtl number, *Sc* - the Schmidt number, *K* - permeability of the medium, *M* - the magnetic parameter K_0 - the chemical reaction parameter.

The dimensionless forms of equation (1), (2), (3) and (4) are as follows

$$\frac{\partial\bar{u}}{\partial\bar{t}}=\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+Gr\theta+Gm\bar{C}-\frac{M(\bar{u}+m\bar{w})}{(1+m^2)}-\frac{1}{K}u,\qquad(7)$$

$$\frac{\partial\bar{w}}{\partial\bar{t}}=\frac{\partial^2\bar{w}}{\partial\bar{y}^2}-\frac{M(\bar{w}-m\bar{u})}{(1+m^2)}-\frac{1}{K}w,\qquad(8)$$

$$\frac{\partial\bar{C}}{\partial\bar{t}}=\frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2}-K_0\bar{\mathrm{C}},\qquad(9)$$

$$\frac{\partial\theta}{\partial\bar{t}}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial\bar{y}^2}.\qquad(10)$$

The corresponding boundary conditions are:

$$\left.\begin{aligned}&\bar{t}\le 0,\bar{u}=0,w=0,\bar{C}=0,\theta=0,\text{ for all the values of y},\\&\bar{t}>0,\bar{u}=1,\bar{w}=0,\theta=1,\bar{C}=\bar{t}\text{ at }\bar{\mathrm{y}}=0,\\&u\to 0,w\to 0,\bar{C}\to 0,\theta\to 0\quad\text{as }\bar{\mathrm{y}}\to\infty.\end{aligned}\right.\qquad(11)$$

Dropping the bars and combining the equations (7) and (8), we get

$$\frac{\partial q}{\partial\bar{t}}=\frac{\partial^2 q}{\partial y^2}+Gr\theta+GmC-\left(\frac{M}{1+m^2}(1-mi)+\frac{1}{K}\right)q,\qquad(12)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-K_0 C, \tag{13}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{14}$$

Where $q = u + iw,$ and corresponding boundary conditions become

$$\begin{aligned} &t \le 0, q=0, C=0, \theta=0, \text{for all the values of y},\\ &t>0, q=1, \theta=1, C=t \text{ at } y=0,\\ &q\to 0, C\to 0, \theta\to 0 \quad \text{as } y\to\infty. \end{aligned} \tag{15}$$

The solution of the above equations is obtained by the Laplace transform method, which is an under

$$\theta = Erfc\left[\frac{\sqrt{\Pr}y}{2\sqrt{t}}\right],$$

$$C=\frac{1}{4\sqrt{ScK_0}}e^{-y\sqrt{ScK_0}}[B_{11}](-Scy+2t\sqrt{ScK_0})+e^{2y\sqrt{ScK_0}}$$
$$[B_{12}](Scy+2t\sqrt{ScK_0}),$$

$$q=\frac{1}{2}e^{\sqrt{a}y}(A_0)+\frac{1}{2a}(-e^{-\sqrt{a}y}(A_0)+e^{\frac{at}{-1+Pr}-y\sqrt{\frac{at}{-1+Pr}}}(A_{13}+e^{2y\sqrt{\frac{at}{-1+Pr}}}A_{14}))Gr$$

$$+\frac{1}{4(a-K_0Sc)^2}yGm(\frac{2e^{-\sqrt{a}y}A_{01}}{y}(1-a_0)+\sqrt{a}e^{-\sqrt{a}y}(A_0-e^{2\sqrt{a}y}A_{12})$$

$$+\frac{1}{y}2e^{\frac{at}{-1+Sc}-\frac{tK_0Sc}{-1+Sc}-yA_{17}}(-1-e^{2yA_{17}}+A_{15}+e^{2yA_{17}}A_{16})[1-Sc]+\frac{1}{y}2e^{-\sqrt{ay}}$$

$$(-1-e^{\sqrt{ay}}-A_{11}+e^{2\sqrt{ay}}A_{12})Sc[1-tK_0]-\frac{1}{\sqrt{a}}e^{-\sqrt{ay}}(A_0-e^{2\sqrt{ay}})K_0Sc)$$

$$-\frac{1}{2a}(e^{\frac{at}{-1+Pr}-y\sqrt{\frac{at}{-1+Pr}}}(1+A_{18}+e^{2y\sqrt{\frac{aPr}{-1+Pr}}}A_{19})-2Erfc[\frac{y\sqrt{Pr}}{2\sqrt{t}}])Gr$$

$$+\frac{1}{4(a-K_0Sc)^2}yGm\sqrt{Sc}(-\frac{1}{\sqrt{K_0}}ae^{-y\sqrt{K_0Sc}}(-1-e^{2y\sqrt{K_0Sc}}+A_{20}+e^{2y\sqrt{K_0Sc}}A_{21})$$

$$+\frac{1}{y\sqrt{Sc}}2e^{-y\sqrt{K_0Sc}}(-1-e^{2y\sqrt{K_0Sc}}+A_{20}+e^{2y\sqrt{K_0Sc}}A_{21})[1-t-Sc]$$

$$-\frac{1}{y\sqrt{Sc}}2e^{\frac{at}{-1+Sc}-yA_{17}\frac{tK_0Sc}{-1+Sc}}(-1-e^{2yA_{17}}-A_{22}+e^{2y\sqrt{\frac{a-K_0}{-1+Sc}}\sqrt{Sc}}A_{23})[1-Sc]$$

$$+e^{-y\sqrt{K_0Sc}}[\frac{1}{y}2t((-1-e^{2y\sqrt{K_0Sc}}-A_{20}+e^{2y\sqrt{K_0Sc}}A_{21})K_0Sc)$$

$$+(1-e^{2y\sqrt{K_0Sc}}+A_{20}+e^{2y\sqrt{K_0Sc}}\sqrt{K_0}Sc)].$$

The symbols involved in the above equations are mentioned in the appendix.

2.1 SKIN FRICTION

The dimensionless skin friction at the plate $y = 0$ is computed by

$$\left(\frac{dq}{dy}\right)_{y=0}=\tau_x+i\tau_z.$$

2.2 SHERWOOD NUMBER

The dimensionless Sherwood number at the plate $y = 0$ is computed by

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=0} = -\frac{1}{4}e^{-y\sqrt{ScK_0}}(B_{11}(-Scy+2t\sqrt{ScK_0})+e^{2y\sqrt{ScK_0}}$$

$$(B_{12}(Scy+2t\sqrt{ScK_0}))+\frac{1}{4\sqrt{ScK_0}}e^{-y\sqrt{ScK_0}}(-ScB_{11}e^{2y\sqrt{ScK_0}}ScB_{12})$$

$$-\sqrt{\frac{Sc}{\pi t}}(e^{B_{11}^2}(-Sc+2t\sqrt{ScK_0})+e^{-B_{12}^2+2y\sqrt{ScK_0}}(Sc+2t\sqrt{ScK_0}))+$$

$$2e^{2y\sqrt{ScK_0}}B_{12}(Sc+2t\sqrt{ScK_0}).$$

3.0 RESULTS AND DISCUSSION

The numerical values of velocity, Sherwood number and skin friction are computed for different parameters. Figures 1, 2, 3, 7 and 9 show that u increases when *Gm, Gr, m, t and, K* are increased. Figures 4, 5, 6 and 8 show that u decreases when *M, Sc, Pr* and, K_0 are increased. Further, it is observed from figures 10, 11, 13, 16 and 18 that w increases when *Gm, Gr, M, t* and *K* are increased. Figures 12, 14, 15 and 17 show that w decreases when *m, Pr, Sc* and K_0 are increased. Further, Figures 19 and 20 show that concentration decreases when K_0, and*Sc*are increased. From table – 1, it is observed that τ_x decreases with increase in *Pr, M, Sc and* K_0, and it increases along with *m, Gm, Gr, K* and *t*. Further, τ_z increases with increase in *Gr, t, Gm, K,* and *M;* and it decreases when *Sc, Pr,* K_0 and *m* are increased. From table – 2, it is observed that Sherwood number decreases with increase in *Sc,* K_0, and t.

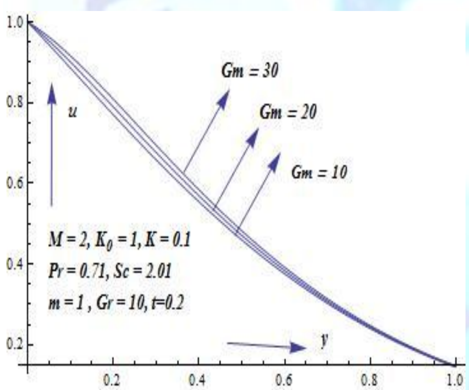

Fig. 1.The effect of Gmon velocity u .

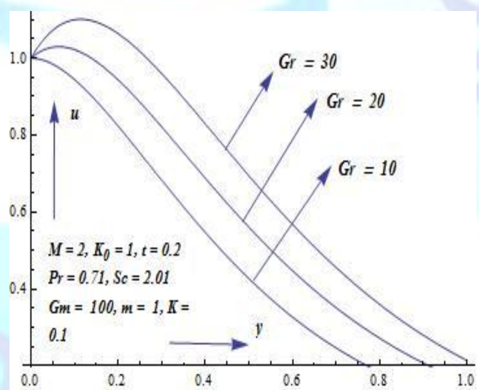

Fig. 2.The effect of Gr on velocity u.

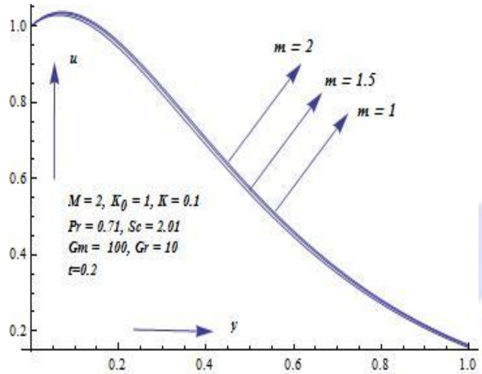

Fig. 3.The effect of m on velocity u.

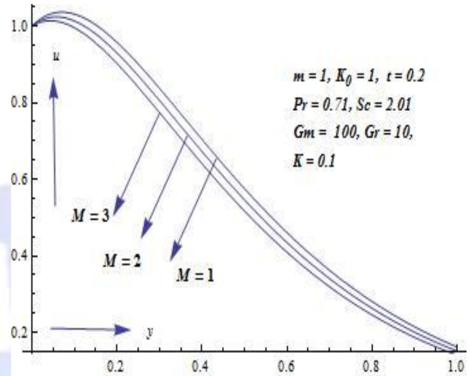

Fig. 4.The effect of M on velocity u.

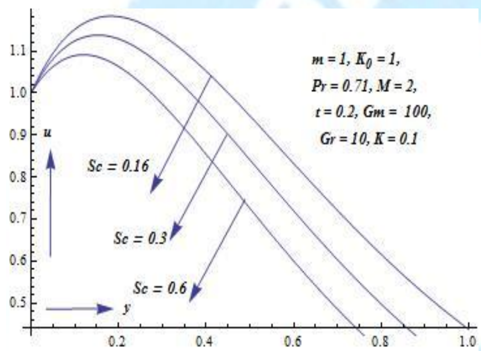

Fig. 5.The effect of Scon velocity u.

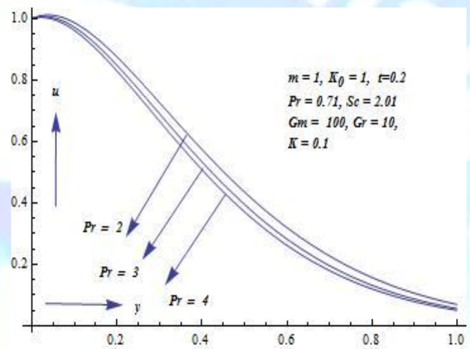

Fig. 6.The effect of Pr on velocity u.

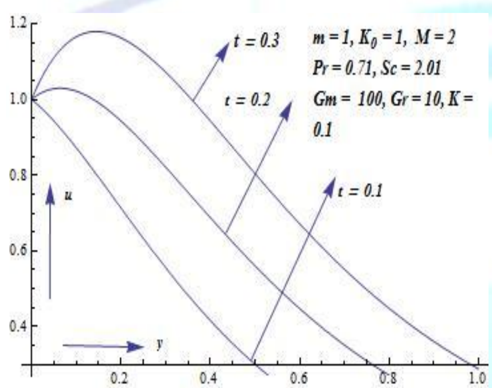

Fig. 7.The effect of t on velocity u.

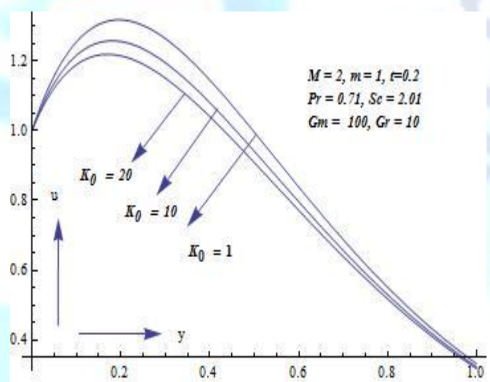

Fig. 8.The effect of K_0 on velocity u.

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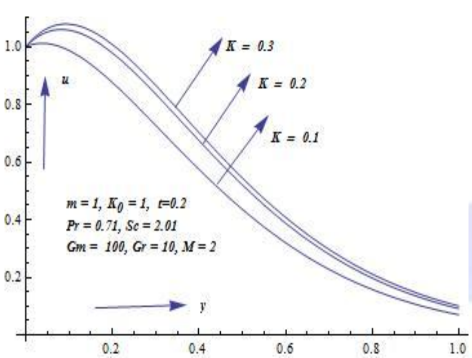

Fig. 9.The effect of K on velocity u.

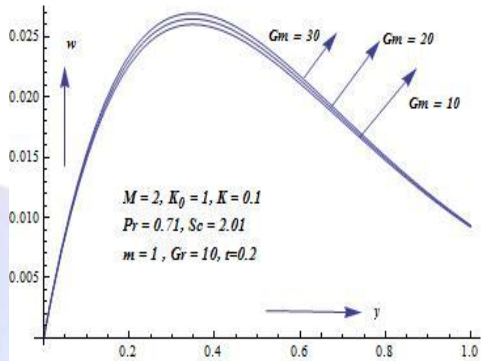

Fig. 10.The effect of Gm on velocity w.

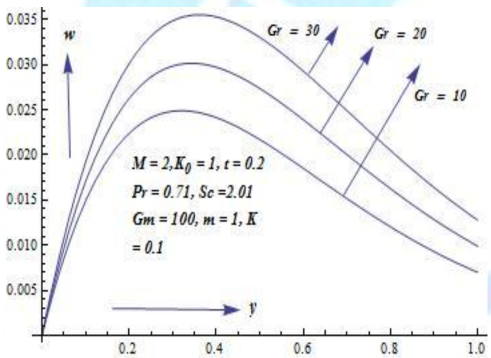

Fig. 11.The effect of Gr on velocity w.

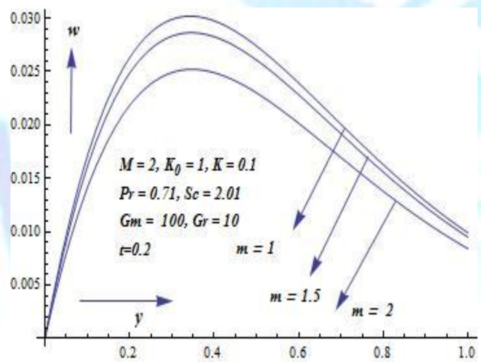

Fig. 12.The effect of m on velocity w.

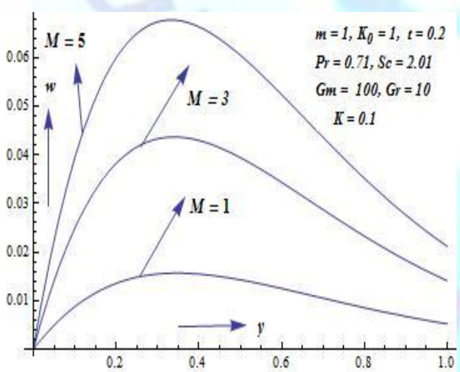

Fig. 13.The effect of M on velocity w.

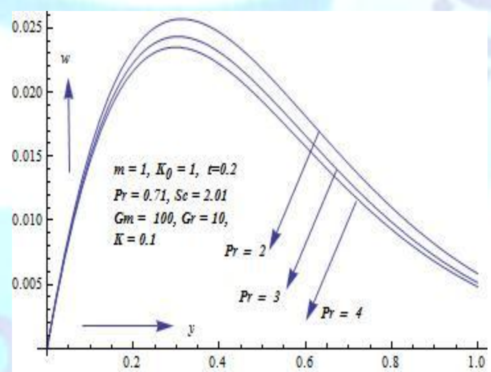

Fig. 14. The effect of Pr on velocity w.

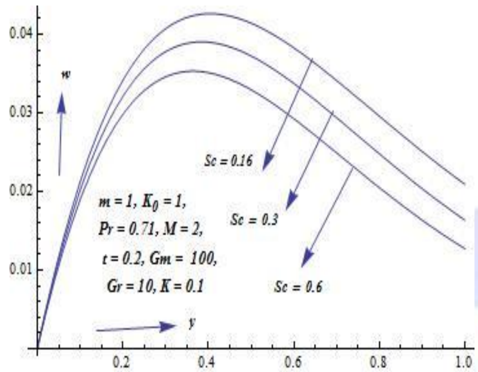

Fig. 15.The effect of Sc on velocity w.

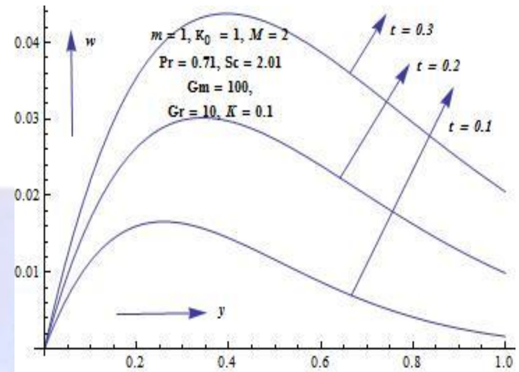

Fig. 16.The effect of t on velocity w.

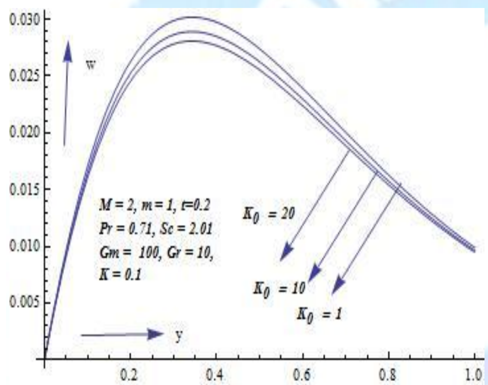

Fig. 17.The effect of K_0 on velocity w.

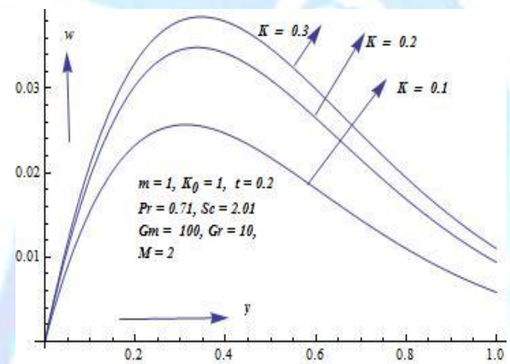

Fig. 18.The effect of K on velocity w.

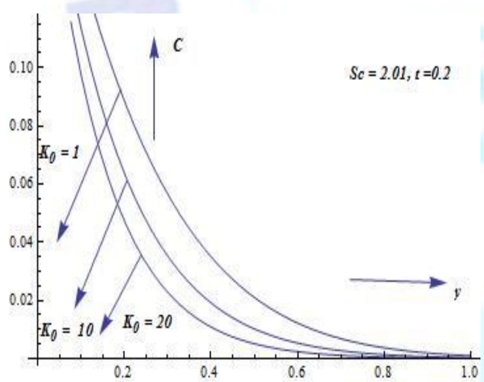

Fig. 19. The effect of K_0 on concentration.

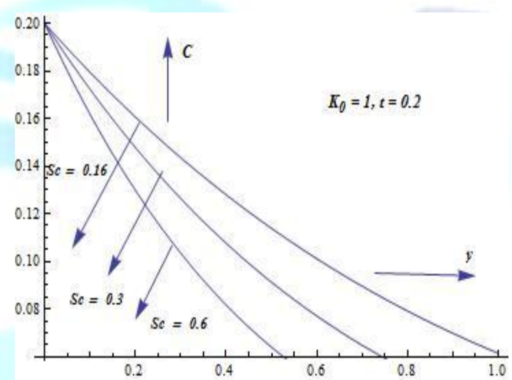

Fig. 20: The effect of Sc on concentration.

Table 1: Skin friction for different parameters

m	Gr	Gm	M	Sc	K_0	K	Pr	t	τ_x	τ_z
1.0	20	100	2	2.01	1.0	0.1	0.71	0.2	2.9965	0.2619
1.0	30	100	2	2.01	1.0	0.1	0.71	0.2	4.9958	0.3068
1.0	10	20	2	2.01	1.0	0.1	0.71	0.2	-0.8730	0.1955
1.0	10	30	2	2.01	1.0	0.1	0.71	0.2	-0.6392	0.1982
1.5	10	100	2	2.01	1.0	0.1	0.71	0.2	1.0820	0.2037
2.0	10	100	2	2.01	1.0	0.1	0.71	0.2	1.1307	0.1782
1.0	10	100	3	2.01	1.0	0.1	0.71	0.2	0.8844	0.3187
1.0	10	100	5	2.01	1.0	0.1	0.71	0.2	0.6607	0.5091
1.0	10	100	2	2.01	1.0	0.1	3.00	0.2	0.4862	0.1958
1.0	10	100	2	2.01	1.0	0.1	4.00	0.2	0.3788	0.1923
1.0	10	100	2	0.30	1.0	0.1	0.71	0.2	2.0585	0.2488
1.0	10	100	2	0.60	1.0	0.1	0.71	0.2	1.6994	0.2366
1.0	10	100	2	2.01	1.0	0.2	0.71	0.2	2.2144	0.2748
1.0	10	100	2	2.01	1.0	0.3	0.71	0.2	2.692	0.3011
1.0	10	100	2	2.01	1.0	0.1	0.71	0.1	-0.9349	0.1589
1.0	10	100	2	2.01	1.0	0.1	0.71	0.3	2.8727	0.2691
1.0	10	100	2	2.01	10	0.1	0.71	0.2	0.6516	0.2109
1.0	10	100	2	2.01	20	0.1	0.71	0.2	0.4073	0.2066

Table - 2 Sherwood number

SC	K_0	t	Sh
0.16	1.0	0.2	-0.2150
0.3	1.0	0.2	-0.2944
2.01	10	0.2	-1.1182
2.01	20	0.2	-1.4264
2.01	1.0	0.2	-0.7621
2.01	1.0	0.3	-0.9613

4.0 CONCLUSION

Hall parameter has increasing effect on velocity along the plate and skin fraction. However velocity decreases due to chemical reaction. Further it is observed that chemical reactiondecreases the Sherwood number.

APPENDIX:

$$A_0 = 1 + A_{11} + e^{2\sqrt{a}y} A_{12},\ A_{01} = 1 + e^{2\sqrt{a}y} + A_{11} - e^{2\sqrt{a}y} A_{12},$$

$$A_{11} = Erf\{\frac{2a\sqrt{t} - y}{2\sqrt{t}}\},\ A_{12} = Erf\{\frac{2a\sqrt{t} - y}{2\sqrt{t}}\},\ A_{13} = Erf\{\frac{y - 2t\sqrt{a(1 + \frac{1}{-1+\Pr})}}{2\sqrt{t}}\},$$

$$A_{14} = Erf\{\frac{y + 2t\sqrt{a(1 + \frac{1}{-1+\Pr})}}{2\sqrt{t}}\},\ A_{15} = Erf\{\frac{y - 2t\sqrt{y - \frac{(a-K_0)Sc}{-1+Sc})}}{2\sqrt{t}}\},$$

$$A_{16} = Erf\{\frac{y + 2t\sqrt{y - \frac{(a-K_0)Sc}{-1+Sc})}}{2\sqrt{t}}\},\ A_{17} = \sqrt{\frac{(a-K_0)Sc}{-1+Sc}}),\ A_{18} = Erf\{\frac{2t\sqrt{\frac{a}{-1+\Pr}} - y\sqrt{\Pr}}{2\sqrt{t}}\},$$

$$A_{19} = Erf\{\frac{2t\sqrt{\frac{a}{-1+\Pr}} + y\sqrt{\Pr}}{2\sqrt{t}}\},\ A_{20} = Erf\{\frac{2t\sqrt{K_0} - y\sqrt{Sc}}{2\sqrt{t}}\},\ A_{21} = Erf\{\frac{2t\sqrt{K_0} + y\sqrt{Sc}}{2\sqrt{t}}\},$$

$$A_{22} = Erf\{\frac{2t\sqrt{\frac{a-K_0}{-1+Sc}} - y\sqrt{Sc}}{2\sqrt{t}}\},\ A_{23} = Erf\{\frac{2t\sqrt{\frac{a-K_0}{-1+Sc}} + y\sqrt{Sc}}{2\sqrt{t}}\},\ B_{11} = Erfc\{\frac{\sqrt{Sc}y - 2t\sqrt{K_0}}{2\sqrt{t}}\},$$

$$B_{12} = Erfc\{\frac{\sqrt{Sc}y + 2t\sqrt{K_0}}{2\sqrt{t}}\},\ a = \frac{M}{1+m^2}(1 - im) + \frac{1}{K}.$$

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