Some Properties of Fuzzy Soft Prime Ideals

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Abstract: In this paper by using of fuzzy ideals and fuzzy prime ideals it's presented the definition of fuzzy soft prime ideals and then it's conversed some theorems on this field.

Keywords: Soft sets, Soft ideals, Fuzzy soft ideals, Fuzzy soft prime ideals

1.Introduction:

It is obvious that in real life situation most of the problems have various uncertainties. Molodtsov [6] initiated the theory of soft sets as a mathematical tool for dealing with uncertainties. Later other authors Maji et al. [7, 8, 9] have further studied the theory of soft sets and also introduced the concept of fuzzy soft set, which is a combination of fuzzy set [10] and soft set. In addition, Aktas and Cagman [1] have introduced the notion of soft groups. Aygunoghlo and Aygun [2] have generalized the concept of Aktas and Cagman [1] and introduce fuzzy soft group.

In this paper we introduce the notion of soft prime ideals and fuzzy soft prime ideals and some of their algebraic properties.

1.Some preliminary concepts

1.1. Definition: [3] Let U be a universe set and E is a set of parameters, let P(U) be the power set of U, the pair of $(\mathcal{F}, A)$ is a soft set on U, where $\mathcal{F}$ is a mapping as the following form:

$$\mathcal{F}: E \longrightarrow P(U)$$

1.2. Definition: [3] Let U be a universe set and E is a set of parameters and $A \subseteq E$, the pair of ($\mathcal{F}$,A) is called a fuzzy soft set on U, where $\mathcal{F}: A \longrightarrow I^U$ is a mapping , such that I^U is the collection of all fuzzy subsets of U.

1.3. Definition: [3] The binary operation of $*: [0,1] \times [0,1] \longrightarrow [0,1]$ is a continuous t-norm, if * satisfying in the following conditions:

i) * is commutative and associative
ii) * is Continuous
iii) $a * 1 = a$ for all $a \in [0,1]$
iv) $a * b \leq c * d$ if $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

2. 4. Example: Some of the continues t-norms are as the followings:

i) $a * b = ab$
ii) $a * \mathrm{b} = \min\{a , b\}$
iii) $a * \mathrm{b} = min\{a + b - 1 , 0\}$

2. 5. Definition: [4] The binary operation of $\boxtimes: [0,1] \times [0,1] \longrightarrow [0,1]$is a continues t-conorm if $\boxtimes$ satisfying in the following conditions:

i) $\boxtimes$ is commutative and associative.
ii) $\boxtimes$ is continuous.
iii) $a \boxtimes 0 = a$, for all $a \in [0,1]$

iv) $a \boxtimes b \leq c \boxtimes d$, if $\leq c$, $b \leq d$, for all $a, b, c, d \in [0,1]$.

2.6. Definition: [4] Let X be a group , ($\mathcal{F}$,A) is a soft set on X, then ($\mathcal{F}$,A) is called a soft group on X if and only if $\mathcal{F}(a)$ is a subgroup of X for all $a \in A$.

2.7. Definition: [5] Let X be a group , ($\mathcal{F}$,A) is a fuzzy soft set on X, then ($\mathcal{F}$,A) is called a fuzzy soft group on X if and only if for all $a \in A$ and $x, y \in X$ $we\ have$:

i)$\mathcal{F}_a(x, y) \geq \mathcal{F}_a(x) * \mathcal{F}_a(y)$

ii) $\mathcal{F}_a(x^{-1}) \geq \mathcal{F}_a(x)$.

Where $\mathcal{F}_a$ is a fuzzy subset of X equivalence to the parameter of$a \in A$.

2.8. Definition: [5] Let f and g are two arbitrary fuzzy subset of ring R, thenfog is a fuzzy subset or R as the following:

$$(fog)(z) = \begin{cases} sup_{(z=x.y)}\{min\{(f(x), g(y)\}, & if\ z = x.y \\ 0 & , if\ z \neq x.y \end{cases}$$

Where, $x, y, z \in R$

2.9. Definition: [4] Let X be an original universe set and E is a set of parameters, the pair of (F, E) is called a soft set on X if and only if F is a mapping of E onto the set of all subsets of X, i.e.,

$$f: E \longrightarrow P(X)$$

$Where\ P(X)\ is\ the\ power\ set\ of\ X.$

2.10. Note: The set of F(e) for each $e \in E$ may be explained as the set of e-elements of the soft set of (F,E), i.e.,

$$(F, E) = \{F(e) | e \in E\}$$

2.11. Definition: [4] Let I^X is the set of all fuzzy sets on X and $\subseteq E$, the pair of (f , A) is called the fuzzy soft set on X, where f is a mapping of A onto I^X as the following:

$$f(a) = f_a: X \longrightarrow I , \quad is\ a\ fuzzy\ subset\ on\ X \quad , \forall\ a \in A$$

2.12. Definition: [4] For every two fuzzy soft sets (f, A) , (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A) \subseteq (g, B)$ if:

i) $A \subseteq B$

ii) For each $a \in A$, $f_a \leq g_a$, that is ,f_a is a fuzzy subset of g_a.

2.13. Definition: [4] Two fuzzy sets (f, A) and (g, B) over a common universe X are said to be equal if $(f, A) \subseteq (g, B)$and $(g, B) \subseteq (f, A)$.

2.14. Definition: [4] Union of Two fuzzy sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cup B$ and

$$h(c) = \begin{cases} f_c & , \quad if\ c \in A - B \\ g_c & , \quad if\ c \in B - A \\ f_c \vee g_c & , \quad if\ c \in A \cap B \end{cases} , \forall\ c \in C$$

It is denoted by $(f, A) \cup (g, B) = (h, C)$.

2.15. Definition: [4] Intersection of two fuzzy sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A \cap B$ and $h_c = f_c \wedge g_c$, $\forall c \in C$.

It is written as $(f, A) \cap (g, B) = (h, C)$.

2.16. Definition: [4] If (f, A) and (g, B) are two soft sets, then (f, A) **AND** (g, B) is denoted by $(f, A) \wedge (g, B)$.The$(f, A) \wedge (g, B)$ is defined as $(h, A \times B)$.

where $h(a, b) = h_{a,b} = f_a \wedge g_b$, $\forall (a, b) \in A \times B$.

2. Soft Substructures of Rings:

Throughout this paper, R will always denote a ring. A subgroup S of (R, +) is called a subring of R and denoted by $S < R$. A subgroup I of (R, +) is called a left ideal if $ri \in I$ (resp., right ideal if $ir \in I$) for all $r \in R$ and $i \in I$ denoted by $I \lhd_l R$ (resp., $I \lhd_r R$).

If I is both left and right ideals of R, then it is called an ideal of R and denoted by $I \lhd R$.

3.1. Definition: [5] Let S be a subring of R and (F, S) is a soft set on S, if for each $x, y \in S$ we have:

$S_1)\quad F(x-y) \supseteq F(x) \cap F(y)$

$S_2)\quad F(xy) \supseteq F(x) \cap F(y)$

Then (F, S) is called a soft subring on R and it's denoted by $(F, S) \precsim R($ or$F_S \precsim R$.

3.2. Example: Let $R = (Z_6, +, .)$ and $S_1 = \{0, 3\}$ and (F, S_1) is a soft set on R, where $F: S_1 \longrightarrow P(R)$ is a valuation set function by: $F(0) = \{0, 1,4,5\}$and $F(3) = \{0,4,5\}$. Then, it is obvious that:

$$F_{S_1} \precsim R$$

Again if $S_2 = \{0,2,4\}$ and (G, S_2) is a soft set on R, where $G: S_2 \longrightarrow P(R)$ is a valuation set function by the following:

$G(0) = \{0,1,3,4,5\}$and $G(2) = \{1,3\}$ and $G(4) = \{0,1,3,4\}$, then obviously we have:

$$G_{S_2} \precsim R$$

Now if we define (T, S_2) is a soft set on R, where $T: S_2 \longrightarrow P(R)$ is a valuation set function by the following:

$T(0) = \{0, ,1,3,4,5\}$and$T(2) = \{1,3\}$ and $T(4) = \{1,2\}$, then since:

$$T(2.2) = T(4) = \{1,2\} \not\supseteq T(2) \cap T(2) = T(2) = \{1,3\}$$

Therefore $(T, S_2)$is **not** a soft subring of R.

3.3. Definition: [5] Let I be an ideal of R and (F , I) is a soft set on R, if for each $x, y \in I$ and $r \in R$ we have:

i) $F(x-y) \supseteq F(x) \cap F(y)$

ii) $F(rx) \supseteq F(x)$

iii) $F(xr) \supseteq F(x)$

Then (F,I) is called a soft ideal on R and its denoted by $(F, I) \trianglelefteq R$ (or $F_I \trianglelefteq R$).

3.4. Definition: Let (I,A) is a soft ideal on (R , + , .), if for each $a, b \in A$ we have:

i) I(a) is an ideal of R.

ii) If $xy \in I(a)$ then: $x \in I(a)$ or $y \in I(a)$

Therefore (I, A) is a prime soft ideal on R.

3.5. Definition: Let (R, +, .) is a ring and E is a set of parameters and $A \subseteq E$, if $I: A \longrightarrow [0,1]^R$ is a set function, where $[0,1]^R$is the collection of fuzzy subsets of R. Then (I, A) is a fuzzy soft prime ideal on R if and only if for each $a \in A$ the fuzzy subset equivalence by$I_a: R \longrightarrow [0,1]$ is a fuzzy prime ideal of R. In other word the following assertions are satisfying:

i) $I_a(x-y) \geq I_a(x) * I_a(y)$

ii) $I_a(x.y) \geq \max\{I_a(x), I_a(y)\}, \quad \forall x, y \in R$

iii) If $I_a(xy) = I_a(0)$ then: $I_a(x) = I_a(0)$or $I_a(y) = I_a(0), \quad \forall\, x, y \in R$

3.6. Theorem:*Let (R, +,.) is a ring and E is a set of parameters and $A \subseteq E$, then (I, A) is a fuzzy soft prime ideal on R if and only if for each $a \in A$ the equivalence fuzzy subset of I_a of R is satisfying in the following conditions:*

i) $I_a(x-y) \geq I_a(x) * I_a(y)\ , \quad \forall x, y \in R$

ii) $\chi_R o I_a \leq I_a$, $I_a o \chi_R \leq I_a$ *, where χ_R is the characteristic function of R.*

iii) If $I_a(xy) = I_a(0)$, then $I_a(x) = I_a(0)$ or $I_a(y) = I_a(0)$.

Proof: Let (I, A) is a fuzzy soft ideal on R, then for each $a \in A$ the fuzzy subset equivalent to I_a of R satisfying in the third following conditions:

i) $I_a(x-y) \geq I_a(x) * I_a(y)$

ii) $I_a(x.y) \geq I_a(x)$, $\forall x, y \in R$

iii) If $I_a(xy) = I_a(0)$ then, $I_a(x) = I_a(0)$ or $I_a(y) = I_a(0)$, $\forall x, y \in R$

Let z is an arbitrary element of R then,

$$(\chi_R o I_a)(z) = \sup_{z=x.y} \{\min\{\chi_R(x), I_a(y)\}\} = \sup_{z=x.y} \{I_a(y)\} \leq I_a(x.y) = I_a(z)$$

In addition if z cannot be denoted as the form of z=x.y , where $x, y \in R$ therefore the following condition is satisfying:

$$(\chi_R o I_a)(z) = 0 \leq I_a(a)$$

and therefore we have:

$$\chi_R o I_a \leq I_a.$$

$Conversely$: Let (I, A) is a fuzzy soft subset on R such that for each $a \in A$ its equivalence fuzzy subset I_a of R satisfying in the following conditions:

$i)$ $I_a(x - y) \geq I_a(x) * I_a(y)$, $\forall x, y \in R$

ii) $\chi_R o I_a \leq I_a$.

Let $x, y \in R$ then: $I_a(x.y) \geq (\chi_R o I_a)(x.y)$

$$= \sup_{xy=p.q} \{\min\{\chi_R(p), I_a(q)\}\}$$

$\geq \min\{\chi_R(x), I_a(y)\} = I_a(y)$

This show that for each $\in A$, the I_a is a fuzzy prime ideal of R.

Therefore (I, A) is a fuzzy soft prime ideal of R and so the theorem is proved. □

3.7. Theorem: *Let (R, +,.) is a ring and E is a set of parameters and $A \subseteq E$, then (I, A) is a fuzzy prime soft ideal on R if and only if for every I_a $(a \in A)$, each level subset $(I_a)_t$, $t \in Im(I_a)$ is a prime ideal of R, I_a is a fuzzy subset of R equivalent to $a \in A$.*

Proof: Let (I, A) is a fuzzy soft prime ideal on R, then for each $a \in A$, the equivalence fuzzy subset I_a is a fuzzy prime ideal of R.

Now let $\in Im(I_a)$, $x, y \in (I_a)_t$, $r \in R$ are arbitrary, since I_a is a fuzzy prime ideal of R then:

$$I_a(x - y) \geq I_a(x) * I_a(y) \geq t \quad , \quad I_a(r.x) \geq I_a(x) \geq t.$$

Therefore:

$$x - y \in (I_a)_t \quad , \quad r.x \in (I_a)_t$$

And if $x.y \in (I_a)_t$ then, $x \in (I_a)_t$or $y \in (I_a)_t$.

So for every $\in Im(I_a)$, the $(I_a)_t$ is a prime ideal of R .

Conversely: Let for each $t \in Im(I_a)$, $(I_a)_t$ is a left ideal of R and for every $a \in A$ and$x, y \in R$ we have:

$I_a(x - y) < I_a(x) * I_a(y) = t_1$, (t_1 is an arbitrary parameter) ,

Then we have:

$x, y \in (I_a)_{t_1}$but$x - y \notin (I_a)_{t_1}$

This is contradict with prime ideal of $(I_a)_{t_1}$ in R, so

$$I_a(x - y) \geq I_a(x) * I_a(y)$$

In addition let $I_a(xy) < I_a(y) = t_2$ (t_2 is arbitrary), this implied that $y \in (I_a)_{t_2}$but . $y \notin (I_a)_{t_2}$.

This is contradict with prime ideal of $(I_a)_{t_2}$, so

$$I_a(x.y) \geq I_a(y)$$

Hence for every $\in A$, I_a is a fuzzy prime ideal of R . □

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