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Some Results of General Class of Polynomial in Relation with Saigo Fractional Operators

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Abstract.
In the present paper, the authors derive certain interesting results of general class of polynomials in relation with Siago fractional operators of multivariable H-function. First, we establish two results that given the product of multivariable H-function and a general class of polynomials given in Saigo operators. On account of the general nature of the Saigo operator, multivariable H-function and a general class of polynomials a large number of new and known results involving Riemann-Liouville and Erde'ly-Kobe fractional differential operators and several special functions notably generalized Wright hypergeomeric function, Mittag-Leffler function, Whittaker function follow as special cases of our main findings.**2010 Mathematics Subject Classification**: 26A33, 33C45, 33C60, 33C70
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1. Introduction and Preliminaries

Fractional differential operator involving special functions, have found significant importance and applications in various sub-field of applicable mathematical analysis. Since last four decades, a number of workers like Love [12], McBride [13], Kalla [2, 3], Kalla and Sexena [4], Saigo [16], Kilbas and Sebastian [8], Saxena et al. [20], Kiryakova [10, 11] and Kilbas [6], etc. have studied in depth of properties, applications and different extensions of various hypergeometric operators of fractional differentiation. A detailed account of such operators along with their properties and applications can be found in the research monographs by Smako, Kilbas and Marichev [19], Miller and Ross [14], Kiryalova [10, 11], Kilbas, Srivastava and Trujillo [9] and Debnath and Bhatta [1].Generalization of the hypergeometric fractional differentials, including the Saigo operator [16, 17, 18], has been introduced by Samko et at. [19], and Kilbas and Saigo [7] as follows:The multivariable H-function, introduced by Srivastava and Panda ([23]), is an extension of the multivariable G-function. The multivariable H-function includes Fox's H-function, Meijer's G-function, the generalised Lauricella function of Srivastava and Daoust [22], Apell function, the Whittaker function as so on. The multivariable H-function is defined and represented in the following manner:

$$H[z_1,\dots,z_r] = H^{0,n:\{m_r,n_r\}}_{p,q:\{p_r,q_r\}}\left[\begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix}\left| \begin{matrix} (a_j,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p}:\left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\} \\ (b_j,\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q}:\left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\} \end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi i)^r} \int_{L_1} \cdots \int_{L_r} \psi(\xi_1, \ldots, \xi_r) \prod_{i=1}^{r} \left(\phi_i(\xi_i)\, z_i^{\xi_i} d\xi_i\right) \qquad (1.1)$$

where

$$\psi(\xi_1, \ldots\ldots, \xi_r) = \frac{\prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^i \xi_i)}{\prod_{j=m+1}^{p_i} \Gamma(a_j - \sum_{i=1}^{r} \alpha_j^i \xi_i) \prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} \beta_j^i \xi_i)} \qquad (1.2)$$

$$\phi_i \xi_i = \frac{\prod_{j=1}^{n_i} \Gamma(1 - c_j^i + \gamma_j^i \xi_i) \prod_{j=1}^{m_i} \Gamma(d_j^i - \delta_j^i \xi_i)}{\prod_{j=n+1}^{p_i} \Gamma(c_j^i - \gamma_j^i \xi_i) \prod_{j=m+1}^{q_i} \Gamma(1 - d_j^i + \delta_j^i \xi_i)} \quad (i \in \{1,2,3..\}) \qquad (1.3)$$

Here, $\{m_r, n_r\}$ stands for $m_1, n_1, \ldots, m_r, n_r$ and $\left\{\left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r}\right\}$ stands for the sequence of r ordered pairs $\left(c_j^{(1)}, \gamma_j^{(1)}\right)_{1,p_1}, \ldots, \left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r}$.

Also $S_n^m[x]$ occurring in the sequel denotes the general class of polynomials introduced by Srivastava [24]:

$$S_n^m[x] = \sum_{k=0}^{n/m} \frac{(-n)_{m,k}}{k!} A_{n,k}\, x^k \quad (m \in \mathbb{N}; n \in \mathbb{N}_0), \qquad (1.4)$$

where $\mathbb{N}$ is the set of positive integers, $\mathbb{N}_0 \coloneqq \mathbb{N} \cup \{0\}$, and the coefficient $A_{n,k}$ $(n, k \in \mathbb{N}_0)$ are arbitrary (real or complex) constants. It is noted that, by suitably specializing the coefficients $A_{n,k}$ $S_n^m[x]$ is seen to yield a number of known polynomials [24].

2. Fractional Calculus Operators

The generalized fractional differential operators were defined and investigated by Saigo [16] as follow:

$$(D_{0+}^{\alpha,\beta,\eta} f)(x) = \left(\frac{d}{dx}\right)^n (I_{0+}^{-\alpha+n,-\beta-n,\alpha+\eta-n} f)(x) \quad (Re(\alpha) \geq 0, \quad n = [Re(\alpha)] + 1) \qquad (2.1)$$

and

$$(D_{-}^{\alpha,\beta,\eta} f)(x) = \left(\frac{-d}{dx}\right)^n (I_{-}^{-\alpha+n,-\beta-n,\alpha+\eta} f)(x) \quad (Re(\alpha) \geq 0,\ n = [Re(\alpha)] + 1) \qquad (2.2)$$

where $\alpha, \beta, \eta \in \mathbb{C}, Re(\alpha) > 0$ and $^{\alpha,\beta,\eta}_{0+}$, $^{\alpha,\beta,\eta}_{-}$ known as generalized fractional differential operators introduced by Saigo [16].

When $\beta = -\alpha$, the above operators (1.1) and (1.2) reduce to the following classical Riemann-Liouville fractional differential operators of order $\alpha \in \mathbb{C}$ $(Re\ (\alpha) \geq 0)$ [9, p. 80, Eqs. (2.2.3), (2.2.4)]:

$$(D_{0+}^{\alpha,-\alpha,\eta} f)(x) = (D_{0+}^{\alpha,} f)(x) \equiv \left(\frac{d}{dx}\right)^n \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f(t)dt}{(x-t)^{\alpha-n+1}}, (x > 0,\ \ n = [Re(\alpha)] + 1) \tag{2.3}$$

and

$$(D_{-}^{\alpha,-\alpha,\eta} f)(x) = (D_{-}^{\alpha,} f)(x) \equiv \left(\frac{-d}{dx}\right)^n \frac{1}{\Gamma(n-\alpha)} \int_x^\infty \frac{f(t)dt}{(x-t)^{\alpha-n+1}}, (x > 0,\ \ n = [Re(\alpha)] + 1) \tag{2.4}$$

Again, if $\beta = 0$, the operators (1.1) and (1.2) reduce to as the Erdelyi-Kober fractional differential operators defined below [9, p. 109, Eqs. (2.6.35) (2.2.36)]:

$$(D_{0+}^{\alpha,0,\eta} f)(x) = (D_{\eta,\alpha}^{\mathbf{0}+} f)(x) \equiv x^{-\eta} \left(\frac{d}{dx}\right)^n \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{t^{\alpha+\eta} f(t)dt}{(x-t)^{\alpha-n+1}}, (x > 0,\ \ n = [Re(\alpha)] + 1) \tag{2.5}$$

and

$$(D_{-}^{\alpha,0,\eta} f)(x) = (D_{\eta,\alpha}^{-} f)(x) \equiv x^{\alpha+\eta} \left(\frac{-d}{dx}\right)^n \frac{1}{\Gamma(n-\alpha)} \int_x^\infty \frac{t^{-\eta} f(t)dt}{(t-x)^{\alpha-n+1}}, (x > 0,\ \ n = [Re(\alpha)]) \tag{2.6}$$

The following lemma is required in the proof of our main results (see [8]):

Lemma 1.2: Let $\alpha, \beta, \eta \in \mathbb{C}$.

If $Re(\alpha) \geq 0, Re(\sigma) > -\min[0, Re(\alpha + \beta + \eta)]$, then

$$(D_{0+}^{\alpha,\beta,\eta} t^{\sigma-1})(x) = x^{\sigma+\beta+1} \frac{\Gamma(\sigma)\,\Gamma(\sigma+\alpha+\beta+\eta)}{\Gamma(\sigma+\beta)\,\Gamma(\sigma+\eta)} \qquad x > 0 \tag{2.7}$$

In particular, for $x > 0$

$$(D_{0+}^{\alpha,} t^{\sigma-1})(x) = x^{\sigma-\beta-1} \frac{\Gamma(\sigma)}{\Gamma(\sigma-\alpha)} \qquad (Re(\alpha) \geq 0,\ \ Re(\sigma) > 0) \tag{2.8}$$

$$(D_{\eta,\alpha}^{+} t^{\sigma-1})(x) = x^{\sigma-1} \frac{\Gamma(\sigma+\alpha+\beta+\eta)}{\Gamma(\sigma+\eta)}, \qquad (Re(\alpha) \geq 0,\ \ Re(\sigma) > -Re(\alpha+\eta)) \tag{2.9}$$

Lemma 1.3: Let $\alpha, \beta, \eta \in \mathbb{C}$.

If $Re(\alpha) \geq 0, Re(\sigma) < 1 + \min[\,Re(-\beta-\eta), Re(\alpha+\eta)]$, $n = [Re(\alpha)] + 1$ then

$$(D_{-}^{\alpha,\beta,\eta} t^{\sigma-1})(x) = x^{\sigma+\beta-1} \frac{\Gamma(1-\sigma)\,\Gamma(1-\sigma+\alpha+\eta)}{\Gamma(1-\sigma)\,\Gamma(1-\sigma+\eta-\beta)} \qquad x > 0 \tag{2.10}$$

In particular, for $x > 0$

$$(D_{-}^{\alpha,} t^{\sigma-1})(x) = x^{\sigma+\alpha-1} \frac{\Gamma(1-\sigma+\alpha)}{\Gamma(1-\sigma)}, \qquad (Re(\alpha) \geq 0,\ \ Re(\sigma) < 1 + Re(\alpha) - n) \tag{2.11}$$

$$(D^{-}_{\eta,\alpha} t^{\sigma-1})(x) = x^{\sigma-1} \frac{\Gamma(1-\sigma+\alpha+\eta)}{\Gamma(1-\sigma+\eta)}, \quad (Re(\alpha) \geq 0,\ Re(\sigma) < 1 + Re(\alpha+\eta) - n) \tag{2.12}$$

3. Main Results

In this section, we establish two theorems involving the products of multivariable H-function and general class of polynomials associated with the Siago fractional differential operators.

Theorem 3.1: Let $\alpha,\ \beta,\ \gamma,\ \mu, \eta,\ \delta_i,\ \nu_i,\ z_i,\ a,\ b,\ c_j \in \mathbb{C}$ with $\Re(\gamma) > 0$, and $\lambda_i > 0, \sigma_i > 0$ $(i \in \{1, \dots, r\};\ j \in \{1, \dots, s\})$. Then we have

$$\Bigg\{ (D^{\alpha,\beta,\gamma}_{o+} \Bigg((t^{\mu-1} (b-at)^{-\eta} \prod_{j=1}^{s} S^{m_j}_{n_j} [c_j\, t^{\lambda_j} (b-at)^{-\delta_j}] \times H[z_1 t^{\sigma_1} (b-at)^{-\nu_1}, \dots, z_r t^{\sigma_r} (b-at)^{-\nu_r}] \Bigg) \Bigg\} (x)$$

$$= b^{-\eta}\, x^{\mu+\beta-1} \sum_{k_1}^{(n_1/m_1)}, \dots, \sum_{k_s}^{(n_s/m_s)} \frac{(-n_1)_{m_1 k_1} \dots (-n_s)_{m_s k_s}}{k_1! \dots k_s!} A^1{}_{m_1 k_1} \dots A^s{}_{m_s k_s} c_1{}^{k_1} \dots c_s{}^{k_s} \times b^{-\sum_{j=1}^{s} \delta_j k_j} x^{\sum_{j=1}^{s} \lambda_j k_j}$$

$$\times H^{0,n+3:\{m_r,n_r\};1,0}_{p+3,q+3:\{p_r,q_r\};0,1} \left[\begin{matrix} z_1 \frac{x^{\sigma_1}}{b^{\nu_1}} \\ \vdots \\ z_r \frac{x^{\sigma_r}}{b^{\nu_r}} \\ -\frac{a}{b} x \end{matrix} \middle| \begin{matrix} (a_j, A_j^{(1)}, \dots, A_j^{(r)})_{1,p}, E : \left\{ \left(c_j^{(r)}, \gamma_j^{(r)} \right)_{1,p_r} \right\}; \text{---} \\ (b_j, B_j^{(1)}, \dots, B_j^{(r)})_{1,q}, E' : \left\{ \left(d_j^{(r)}, \delta_j^{(r)} \right)_{1,q_r} \right\}; (0,1) \end{matrix} \right] \tag{3.1}$$

where, for simplicity, E and E' are denoted by the following arrays:

$$E := \left(1 - \eta - \sum_{j=1}^{s} \delta_j k_j ; \nu_1, \dots, \nu_r, 1 \right), \left(1 - \mu - \sum_{j=1}^{s} \lambda_j k_j ; \sigma_1, \dots, \sigma_r, 1 \right), \left(1 - \mu - \alpha - \beta - \gamma - \sum_{j=1}^{s} \lambda_j k_j ; \sigma_1, \dots, \sigma_r, 1 \right);$$

and

$$E' := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;\nu_1,\dots,\nu_r,0\right),\left(1-\mu-\beta-\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right),$$
$$\left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right).$$

The sufficient conditions of validity of (2.1) are

(i) $|argz_i| < \frac{\pi}{2}\,\Omega_i, \Omega_i > 0\ (i = 1,\dots,r)$, where

$$\Omega_i = -\sum_{j=n+1}^{p} A_j^{(i)} - \sum_{j=1}^{q} B_j^{(i)} + \sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{m_i}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}\delta_j^{(i)} > 0, \forall\, i \in (1,\dots,r) \tag{3.2}$$

(ii) $\Re(\mu) > 0$

$$\Re(\mu) + \sum_{i=1}^{r}\sigma_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > \max\{0,\Re(\beta-\gamma)\}$$

and

$$\Re(\eta) + \sum_{i=1}^{r}\nu_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > \max\{0,\Re(\beta-\gamma)\}. \tag{3.3}$$

(iii) $\left|\frac{a}{b}x\right| < 1.$

Proof: Let $\mathcal{L}$ be the left-hand side of (2.1). Using (1.4) and (1.1), by using the generalized binomial theorem, expanding the term $(b-at)^{-\zeta}$ as follows:

$$(b-at)^{-\zeta} = b^{-\zeta}\sum_{s=0}^{\infty}\frac{(\zeta)_s}{s!}\left(\frac{ax}{b}\right)^s \qquad \left(\frac{ax}{b} < 1\right), \tag{3.4}$$

and interchanging the summations and the integrals, after a little simplification, we obtain

$$\mathcal{L} = \sum_{k_1}^{(n_1/m_1)},\dots,\sum_{k_s}^{(n_s/m_s)}\frac{(-n_1)_{m_1k_1}\dots(-n_s)_{m_sk_s}}{k_1!\dots k_s!}A^1{}_{m_1k_1}\dots A^s{}_{m_sk_s}\,c_1{}^{k_1}\dots c_s{}^{k_s}(b)^{-\sum_{j=1}^{s}\delta_j k_j}$$
$$\times\frac{1}{(2\pi i)^{r+1}}\int_{L_1}\dots\int_{L_r}\psi(\xi_1,\dots,\xi_r)\prod_{i=1}^{r}\left(\phi_i(\xi_i)\,z_i{}^{\xi_i}\right)(b)^{\sum_{j=1}^{s}(-\nu_j\xi_j)}\,d\xi_1\dots d\xi_r$$

$$\times \int_{L_{r+1}} \frac{\Gamma\left(1-\eta-\sum_{j=1}^{s}\delta_j k_j+\sum_{i=1}^{rt}\nu_i\xi_i+\ \xi_{r+1}\right)}{\Gamma\left(1-\eta-\sum_{j=1}^{s}\delta_j k_j+\sum_{i=1}^{rt}\nu_i\xi_i\right)\Gamma(1+\xi_{r+1})}\left(\frac{-a}{b}\right)^{\xi_{r+1}} d\xi_{r+1}$$

$$\times\left(D_{o+}^{\alpha,\beta,\gamma}\, t^{\mu+\sum_{j=1}^{s}\delta_j k_j+\sum_{i=1}^{r}\nu_i\xi_i+\ \xi_{r+1}-1}\right)(x). \tag{3.5}$$

Then applying (2.7) to the last integral in (3.5) and interpreting the involved Mellin-Barnes contour integrals in terms of the multivariable H-function of $r+1$ variables, we ready obtains the right-hand side of (3.1).

If we put $\beta=-\alpha$ in theorem (3.1), we arrive at the following new and interesting corollary concerning Riemann-Liouville fractional differential operators defined by (2.3) and using (2.8).

Corollary 3.2:

$$\left\{(D_{o+}^{\alpha,}\left(\left(t^{\mu-1}(b-at)^{-\eta}\prod_{j=1}^{s}S_{n_j}^{m_j}\left[c_j\, t^{\lambda_j}(b-at)^{-\delta_j}\right]\right.\right.\right.$$

$$\left.\left.\left.\times\, H[z_1 t^{\sigma_1}(b-at)^{-\nu_1},\ldots,z_r t^{\sigma_r}(b-at)^{-\nu_r}]\right)\right)\right\}(x)$$

$$= b^{-\eta}\, x^{\mu-\alpha-1}\sum_{k_1}^{(n_1/m_1)},\ldots,\sum_{k_s}^{(n_s/m_s)}\frac{(-n_1)_{m_1k_1}\ldots(-n_s)_{m_sk_s}}{k_1!\ldots k_s!}A^1{}_{m_1k_1}\ldots A^s{}_{m_sk_s}$$

$$c_1{}^{k_1}\ldots c_s{}^{k_s} b^{-\sum_{j=1}^{s}\delta_j k_j} x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\, H_{p+2,q+2:\{p_r,q_r\};0,1}^{0,n+2:\{m_r,n_r\};1,0}\left[\begin{matrix} z_1\frac{x^{\sigma_1}}{b^{\nu_1}}\\ \cdot\\ \cdot\\ \cdot\\ z_r\frac{x^{\sigma_r}}{b^{\nu_r}}\\ -\frac{a}{b}x\end{matrix}\left|\begin{matrix}(a_j,A_j^{(1)},\ldots,A_j^{(r)})_{1,p},E_1{}^{*}:\left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\};\ \text{—}\\ (b_j,B_j^{(1)},\ldots,B_j^{(r)})_{1,q},E_1{}^{**}:\left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\};(0,1)\end{matrix}\right.\right] \tag{3.6}$$

where

$$E_1^* := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; \nu_1,\dots,\nu_r, 1\right), \left(1-\mu-\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1,\dots,\sigma_r, 1\right),$$

and

$$E_1^{**} := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; \nu_1,\dots,\nu_r, 0\right), \left(1-\mu+\alpha-\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1,\dots,\sigma_r, 1\right).$$

Again, if we $\beta = 0$ in theorem (3.1), we get the following result which is also believed to be new and pertains to Erde'lyi-Kober fractional differential operators defined by (2.5) and using (2.9).

Corollary 3.3:

$$\left\{(D^{+}_{\gamma,\boldsymbol{\alpha}}\left((t^{\mu-1}(b-at)^{-\eta}\prod_{j=1}^{s} S_{n_j}^{m_j}[c_j\, t^{\lambda_j}(b-at)^{-\delta_j}]\right.\right.$$

$$\left.\left.\times\, H[z_1 t^{\sigma_1}(b-at)^{-\nu_1},\dots,z_r t^{\sigma_r}(b-at)^{-\nu_r}]\right)\right\}(x)$$

$$= b^{-\eta}\, x^{\mu-1}\sum_{k_1}^{(n_1/m_1)},\dots,\sum_{k_s}^{(n_s/m_s)}\frac{(-n_1)_{m_1k_1}\dots(-n_s)_{m_sk_s}}{k_1!\dots k_s!}\,A^1{}_{m_1k_1}\dots A^s{}_{m_sk_s}$$

$$c_1{}^{k_1}\dots c_s{}^{k_s}\, b^{-\sum_{j=1}^{s}\delta_j k_j}\, x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\, H^{0,n+2:\{m_r,n_r\};1,0}_{p+2,q+2:\{p_r,q_r\};0,1}\left[\begin{matrix} z_1\frac{x^{\sigma_1}}{b^{\nu_1}}\\ \vdots\\ z_r\frac{x^{\sigma_r}}{b^{\nu_r}}\\ -\frac{a}{b}x\end{matrix}\left|\begin{matrix}(a_j, A_j^{(1)},\dots,A_j^{(r)})_{1,p}, E_2^* : \left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\};\, —\\ (b_j, B_j^{(1)},\dots,B_j^{(r)})_{1,q}, E_2^{**} : \left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\};(0,1)\end{matrix}\right.\right] \tag{3.7}$$

where

$$E_2^* := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; \nu_1,\dots,\nu_r, 1\right), \left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1,\dots,\sigma_r, 1\right),$$

and

$$E_2^{**} := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; \nu_1,\ldots,\nu_r, 0\right), \left(1-\mu+\alpha-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1,\ldots,\sigma_r, 1\right).$$

The sufficient conditions of validity of (3.7) are:

(i) $\Re(\alpha) > 0$ and

$$\Re(\mu)+\sum_{i=1}^{r}\sigma_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > -\Re(\gamma)$$

$$\Re(\eta)+\sum_{i=1}^{r}\nu_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > -\Re(\gamma)$$

and the condition (i), (ii) and (iv) in theorem (3.1) are also satisfied.

Theorem 3.4: Let $\alpha,\ \beta,\ \gamma,\ \mu,\eta,\ \delta_i,\ \nu_i,\ z_i,\ a,\ b,\ c_j \in \mathbb{C}$ with $\Re(\gamma) > 0$, and $\lambda_i > 0, \sigma_i > 0$ $(i \in \{1,\ldots,r\};\ j \in \{1,\ldots,s\})$. Then we have

$$\left\{\left(D_{-}^{\alpha,\beta,\gamma}\left(\left(t^{\mu-1}(b-at)^{-\eta}\prod_{j=1}^{s}S_{n_j}^{m_j}\left[c_j t^{\lambda_j}(b-at)^{-\delta_j}\right]\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\times H\left[z_1 t^{\sigma_1}(b-at)^{-\nu_1},\ldots,z_r t^{\sigma_r}(b-at)^{-\nu_r}\right]\right)\right)\right\}(x)$$

$$= b^{-\eta}\,x^{\mu+\beta-1}\sum_{k_1}^{(n_1/m_1)},\ldots,\sum_{k_s}^{(n_s/m_s)}\frac{(-n_1)_{m_1k_1}\cdots(-n_s)_{m_sk_s}}{k_1!\ldots k_s!}\,A^1{}_{m_1k_1}\ldots A^s{}_{m_sk_s}$$
$$c_1{}^{k_1}\ldots c_s{}^{k_s}\,b^{-\sum_{j=1}^{s}\delta_j k_j}\,x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times H_{p+3,q+3:\{p_r,q_r\};0,1}^{0,n+3:\{m_r,n_r\};1,0}\left[\begin{matrix} z_1\frac{x^{\sigma_1}}{b^{\nu_1}} \\ \vdots \\ z_r\frac{x^{\sigma_r}}{b^{\nu_r}} \\ -\frac{a}{b}x \end{matrix}\middle|\begin{matrix}(a_j, A_j^{(1)},\ldots,A_j^{(r)})_{1,p}, F : \left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\};\ \text{—} \\ (b_j, B_j^{(1)},\ldots,B_j^{(r)})_{1,q}, F' : \left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\};(0,1)\end{matrix}\right] \tag{3.8}$$

where, for simplicity, F and F' are denoted by the following arrays:

$$F := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;\nu_1,\dots,\nu_r,1\right),\left(\mu+\beta+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right),$$
$$\left(\mu-\alpha-\gamma+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right);$$

and

$$F' := \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;\nu_1,\dots,\nu_r,0\right),\left(\mu+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right),$$
$$\left(\mu+\beta-\gamma+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right).$$

The sufficient conditions of validity of (3.8) are

(i) $\Re(\alpha) > 0$ and

$$\Re(\mu)+\sum_{i=1}^{r}\sigma_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > -\Re(\gamma)$$
$$\Re(\eta)+\sum_{i=1}^{r}\nu_i \min_{1\le j\le m_i}\frac{\Re\left(d_j^{(i)}\right)}{\delta_j^{(i)}} > -\Re(\gamma)$$

and the condition (i), (ii) and (iv) in theorem (3.1) are also satisfied.

Proof:

We easily obtain the theorem (3.4) after a small simplification on making use of similar lines as adopted in theorem (3.1) and using Lemma (1.3).

If we put $\beta = -\alpha$ in theorem (3.4), we arrive at the following new and interesting corollary concerning Riemann-Liouville fractional differential operators defined by (2.4) and using (2.11).

Corollary 3.5:

$$\Bigg\{ \Big(D_{-}^{\alpha,} \Bigg((t^{\mu-1}\,(b-at)^{-\eta} \prod_{j=1}^{s} S_{n_j}^{m_j}[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}]$$

$$\times\, H[z_1 t^{\sigma_1}\,(b-at)^{-\nu_1},\dots,z_r t^{\sigma_r}\,(b-at)^{-\nu_r}] \Bigg)\Bigg\}\,(x)$$

$$=\; b^{-\eta}\,x^{\mu-\alpha-1} \sum_{k_1}^{(n_1/m_1)},\dots,\sum_{k_s}^{(n_s/m_s)} \frac{(-n_1)_{m_1k_1}\dots(-n_s)_{m_sk_s}}{k_1!\dots k_s!}\,A^1{}_{m_1k_1}\dots A^s{}_{m_sk_s}$$

$$c_1{}^{k_1}\dots c_s{}^{k_s}\,b^{-\sum_{j=1}^{s}\delta_j k_j}\,x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\, H^{0,n+2:\{m_r,n_r\};1,0}_{p+2,q+2:\{p_r,q_r\};0,1}\left[\begin{matrix} z_1\frac{x^{\sigma_1}}{b^{\nu_1}} \\ \vdots \\ z_r\frac{x^{\sigma_r}}{b^{\nu_r}} \\ -\frac{a}{b}x \end{matrix}\left|\begin{matrix}(a_j,A_j^{(1)},\dots,A_j^{(r)})_{1,p},F_1{}^{*}:\left\{\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\right\};\,\text{—} \\ (b_j,B_j^{(1)},\dots,B_j^{(r)})_{1,q},F_1{}^{**}:\left\{\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\right\};(0,1)\end{matrix}\right.\right] \tag{3.9}$$

where

$$F_1{}^{*} \coloneqq \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;\nu_1,\dots,\nu_r,1\right),\left(\beta+\mu+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right),$$

and

$$F_1{}^{**} \coloneqq \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;\nu_1,\dots,\nu_r,0\right),\left(\mu+\sum_{j=1}^{s}\lambda_j k_j\,;\sigma_1,\dots,\sigma_r,1\right).$$

Again, if we $\beta = 0$ in theorem (3.4), we get the following result which is also believed to be new and pertains to Erde'lyi-Kober fractional differential operators defined by (2.6) and using (2.12).

Corollary 3.6:

$$\Bigg\{ \Big(D_{\gamma,\boldsymbol{\alpha}}^{+} \Bigg((t^{\mu-1}\,(b-at)^{-\eta} \prod_{j=1}^{s} S_{n_j}^{m_j}[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}]$$

$$\times\, H[z_1 t^{\sigma_1}\,(b-at)^{-\nu_1},\dots,z_r t^{\sigma_r}\,(b-at)^{-\nu_r}] \Bigg)\Bigg\}\,(x)$$

$$= b^{-\eta}\, x^{\mu-1} \sum_{k_1}^{(n_1/m_1)}, \ldots, \sum_{k_s}^{(n_s/m_s)} \frac{(-n_1)_{m_1k_1} \ldots (-n_s)_{m_sk_s}}{k_1! \ldots k_s!} A^1{}_{m_1k_1} \ldots A^s{}_{m_sk_s}$$

$$c_1{}^{k_1} \ldots c_s{}^{k_s}\, b^{-\sum_{j=1}^{s}\delta_j k_j}\, x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times H^{0,n+2:\{m_r,n_r\};1,0}_{p+2,q+2:\{p_r,q_r\};0,1}\left[\begin{matrix} z_1\frac{x^{\sigma_1}}{b^{v_1}} \\ \vdots \\ z_r\frac{x^{\sigma_r}}{b^{v_r}} \\ -\frac{a}{b}x \end{matrix}\middle| \begin{matrix} (a_j, A_j^{(1)}, \ldots, A_j^{(r)})_{1,p}, F_2{}^* : \left\{\left(c_j^{(r)}, \gamma_j^{(r)}\right)_{1,p_r}\right\}; \text{---} \\ (b_j, B_j^{(1)}, \ldots, B_j^{(r)})_{1,q}, F_2{}^{**} : \left\{\left(d_j^{(r)}, \delta_j^{(r)}\right)_{1,q_r}\right\}; (0,1) \end{matrix}\right] \tag{2.10}$$

where

$$F_2{}^* \coloneqq \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; v_1, \ldots, v_r, 1\right), \left(\mu-\alpha-\gamma+\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1, \ldots, \sigma_r, 1\right),$$

and

$$F_2{}^{**} \coloneqq \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,; v_1, \ldots, v_r, 0\right), \left(\mu+\beta-\gamma+\sum_{j=1}^{s}\lambda_j k_j\,; \sigma_1, \ldots, \sigma_r, 1\right).$$

The conditions of validity of the above results follow easily from the conditions given with theorem (2.4).

4. Special Cases and Applications

The generalized fractional differential operator Theorem (3.1) and (3.4) established her are unified in nature and act as key formulae. Thus the product of the general class of polynomials involved in Theorem (3.1) and (3.4) reduce to a large spectrum of polynomials listed by Srivastava and Singh [26], and so from Theorem (3.1) and (3.4) we can further obtain various fractional differential results involving a number of simpler polynomials. Again, the multivariable H-function occurring in these Theorems can be suitably specialized to a remarkably wide variety of useful functions which are expressible in terms of the generalized Wright hypergeometric function, the generalized Mittag-Leffler function and Bessel function of one variable.

(i) If we reduce the multivariable H-function involved in (3.1) to the product of r different Whittaka function Srivastava et al. [25, p. 18, equation (2.6.7)] and taking $\sigma_1 = 1,\ v_1 \to 0$, we get the following new and interesting result:

$$\left\{\left(D_{o+}^{\alpha,\beta,\gamma}\left(\left(t^{\mu-1}\,(b-at)^{-\eta}\prod_{j=1}^{s} S_{n_j}^{m_j}\left[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}\right]\times\prod_{i=1}^{r} e^{\left(\frac{z_i t}{2}\right)} W_{\sigma_i,\mu_i}\,(z_i t)\right)\right\}(x)\right.$$

$$= b^{-\eta}\,x^{\mu+\beta-1}\sum_{k_1}^{\left({n_1}/{m_1}\right)},\ldots,\sum_{k_s}^{\left({n_s}/{m_s}\right)}\frac{(-n_1)_{m_1k_1}\ldots(-n_s)_{m_sk_s}}{k_1!\ldots k_s!}A^1{}_{m_1k_1}\ldots A^s{}_{m_sk_s}c_1{}^{k_1}\ldots c_s{}^{k_s}$$

$$\times\, b^{-\sum_{j=1}^{s}\delta_j k_j}x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\, H^{0,3:2,0;\ldots;2,0;1,0}_{3,3:1,2;\ldots;1,2;0,1}\left[\begin{array}{c} z_1x\\ \vdots\\ z_rx\\ -\frac{a}{b}x\end{array}\left|\begin{array}{l}\left(1-\eta-\sum_{j=1}^{s}\delta_jk_j\,;\underbrace{1,\ldots 1}_{r},\,1\right),\left(1-\mu-\sum_{j=1}^{s}\lambda_jk_j\,;\underbrace{1,\ldots 1}_{r},\,1\right),\\ \left(1-\eta-\sum_{j=1}^{s}\delta_jk_j\,;\underbrace{1,\ldots 1}_{r},\,0\right),\left(1-\mu-\beta-\sum_{j=1}^{s}\lambda_jk_j\,;\underbrace{1,\ldots 1}_{r},\,1\right),\end{array}\right.\right.$$

$$\left.\left.\begin{array}{l}\left(1-\mu-\alpha-\beta-\gamma-\sum_{j=1}^{s}\lambda_jk_j\,;\underbrace{1,\ldots 1}_{r},1\right):(1-\lambda_1,1);\ldots;\,(1-\lambda_r,1);-\\ \left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_jk_j\,;\underbrace{1,\ldots 1}_{r},1\right):\left(\frac{1}{2}\pm\mu_1,1\right);\ldots;\left(\frac{1}{2}\pm\mu_r,1\right);(0,1)\end{array}\right]\right.$$

(4.1)

The conditions of validity of the (4.1) result easily follows from (3.1).

(ii) If we reduce the multivariable H-function in to the product of H-function of two variable in Theorem (3.1) and then reduce Fox's H-function to the exponential function by taking $\sigma_1 = 1,\ \nu_1 \to 0$, we get the following result after a simple simplification which is believed to be new:

$$\left\{\left(D_{o+}^{\alpha,\beta,\gamma}\left(\left(t^{\mu-1}\,(b-at)^{-\eta}\prod_{j=1}^{s} S_{n_j}^{m_j}\left[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}\right]\times H\left[z_2t^{\sigma_2}\,(b-at)^{-\nu_2}\left|\begin{array}{c}(c_j,\gamma_j)_{1,p_2}\\(d_j,\delta_j)_{1,q_2}\end{array}\right.\right]\right)\right\}(x)\right.$$

$$= b^{-\eta}\,x^{\mu+\beta-1}\sum_{k_1}^{\left({n_1}/{m_1}\right)},\ldots,\sum_{k_s}^{\left({n_s}/{m_s}\right)}\frac{(-n_1)_{m_1k_1}\ldots(-n_s)_{m_sk_s}}{k_1!\ldots k_s!}A^1{}_{m_1k_1}\ldots A^s{}_{m_sk_s}c_1{}^{k_1}\ldots c_s{}^{k_s}$$

$$\times\, b^{-\sum_{j=1}^{s}\delta_j k_j}x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\ H^{0,3:1,0;m_2,n_2,1,0}_{3,3:0,1;p_2,q_2,0,1}\left[\begin{matrix} z_1 x \\ z_2\,\frac{x^{\sigma_2}}{b^{\nu_2}} \\ -\frac{a}{b}x \end{matrix}\left|\begin{matrix}\left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,1\right),\left(1-\mu-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right), \\ \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,0\right),\left(1-\mu-\beta-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(1-\mu-\alpha-\beta-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):-;\,(c_j,\gamma_j)_{1,p_2};- \\ \left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):(0,1);\,(d_j,\delta_j)_{1,q_2};(0,1)\end{matrix}\right] \quad (4.2)$$

The conditions of validity of the (4.2) result easily follows from (3.1).

If we put $\beta=-\alpha;\ \eta,\nu_2=0$ and $S^{m_j}_{n_j}=1$ and make suitable adjustment of the parameters in the equation (4.2), we arrive at the known result Kilbas and Saigo [7, p. 55, equation (2.7.22).

(iii) If we reduce the H-function of one variable to the generalized Whittaka hypergeometic function Srivastava et al. [25, p. 18, equation (2.6.7)], in the result given by (4.2), we get the following new and interesting result after little simplification:

$$\left\{(D^{\alpha,\beta,\gamma}_{o+}\left((t^{\mu-1}\,(b-at)^{-\eta}\prod_{j=1}^{s}S^{m_j}_{n_j}\left[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}\right]\right.\right.\right.$$

$$\left.\left.\left.\times\ p_2\Psi q_2\left[-z_2 t^{\sigma_2}\,(b-at)^{-\nu_2}\left|\begin{matrix}(1-c_j,\gamma_j)_{1,P_2} \\ (0,1),(1-d_j,\delta_j)_{1,Q_2}\end{matrix}\right.\right]\right)\right)\right\}(x)$$

$$=\ b^{-\eta}\,x^{\mu+\beta-1}\sum_{k_1}^{(n_1/m_1)},\ldots,\sum_{k_s}^{(n_s/m_s)}\frac{(-n_1)_{m_1k_1}\ldots(-n_s)_{m_sk_s}}{k_1!\ldots k_s!}A^1{}_{m_1k_1}\ldots A^s{}_{m_sk_s}c_1{}^{k_1}\ldots c_s{}^{k_s}$$

$$\times\ b^{-\sum_{j=1}^{s}\delta_j k_j}x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\ H^{0,3:1,0;1,P_2,1,0}_{3,3:0,1;P_2,Q_2,0,1}\left[\begin{matrix} z_1 x \\ -z_2\frac{x^{\sigma_2}}{b^{\nu_2}} \\ -\frac{a}{b}x \end{matrix}\left|\begin{matrix}\left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,1\right),\left(1-\mu-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right), \\ \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,0\right),\left(1-\mu-\beta-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right),\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(1-\mu-\alpha-\beta-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):-;\ (c_j,\gamma_j)_{1,p_2};- \\ \left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):(0,1);\left(d_j,\delta_j\right)_{1,q_2};(0,1)\end{matrix}\right] \tag{4.3}$$

The condition of validity of the above result easily follows from (3.1).

If we put $\beta=-\alpha;\ \eta,\nu_2=0$ and $S^{m_j}_{n_j}=1, z_2$ and make suitable adjustment of the parameters in the equation (4.3), we arrive at the known result Kilbas and Saigo [7, p. 119, equation (14)].

(iv) If we take $z_2,\sigma_2=1$ and $\nu_2=0$ in the equation (4.2) and reducing the H-function of one variable occurring therein to, generalized Mittag-Laffler function Prabhakar et al. [15], we easily get after little simplification the following new and interesting result:

$$\left\{(D^{\alpha,\beta,\gamma}_{o+}\left((t^{\mu-1}\,(b-at)^{-\eta}\prod_{j=1}^{s}S^{m_j}_{n_j}\left[c_j\,t^{\lambda_j}\,(b-at)^{-\delta_j}\right]\right.\right.$$

$$\left.\left.\times\ p_2\Psi q_2\left[-z_2 t^{\sigma_2}\,(b-at)^{-\nu_2}\left|\begin{matrix}\left(1-c_j,\gamma_j\right)_{1,P_2} \\ (0,1),\left(1-d_j,\delta_j\right)_{1,Q_2}\end{matrix}\right.\right]\right)\right\}(x)$$

$$=\ b^{-\eta}\,x^{\mu+\beta-1}\sum_{k_1}^{\left({}^{n_1}/_{m_1}\right)},\dots,\sum_{k_s}^{\left({}^{n_s}/_{m_s}\right)}\frac{(-n_1)_{m_1k_1}\dots(-n_s)_{m_sk_s}}{k_1!\dots k_s!}\,A^1{}_{m_1k_1}\dots A^s{}_{m_sk_s}c_1{}^{k_1}\dots c_s{}^{k_s}$$

$$\times\ b^{-\sum_{j=1}^{s}\delta_j k_j}x^{\sum_{j=1}^{s}\lambda_j k_j}$$

$$\times\ H^{0,3:1,0;1,P_2,1,0}_{3,3:0,1;P_2,Q_2,0,1}\left[\begin{matrix} z_1 x \\ -z_2\frac{x^{\sigma_2}}{b^{\nu_2}} \\ -\frac{a}{b}x \end{matrix}\left|\begin{matrix}\left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,1\right),\left(1-\mu-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right), \\ \left(1-\eta-\sum_{j=1}^{s}\delta_j k_j\,;1,\nu_2,0\right),\left(1-\mu-\beta-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right),\end{matrix}\right.\right.$$

$$\left.\begin{matrix} \left(1-\mu-\alpha-\beta-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):-;\ \left(c_j,\gamma_j\right)_{1,p_2};- \\ \left(1-\mu-\gamma-\sum_{j=1}^{s}\lambda_j k_j\,;1,\sigma_2,1\right):(0,1);\left(d_j,\delta_j\right)_{1,q_2};(0,1) \end{matrix}\right] \tag{4.4}$$

The condition of validity of the above result easily follows from (3.1).

If we put $\beta = -\alpha;\ \eta, \nu_2 = 0$ and $S_{n_j}^{m_j} = 1$ and make suitable adjustment of the parameters in the equation (4.4), we arrive at the known result Sexena et al. [20, p. 118, equation (2.1)].

(v) If we take $\beta = -\alpha$ and $\eta, \nu_2 = 0$ and $S_{n_j}^{m_j} = 1,\ z_2 = \frac{1}{4}, \sigma_2 = 2$ and reduce the H-function to the Bessel function of first kind in the equation (4.2), we also get know result Kilbas and Sebartain [8, p. 330, equation 31-35].

A number of several special cases of theorem (3.1) and theorem (3.4) can also be obtained but we do not mention them here on account of lack of space.

5. Conclusion

In the present paper, we have obtained the result namely theorem (3.1) and theorem (3.4) of the generalized fractional derivative operators given by Saigo. The theorems have been developed in terms of the product of multivariable H-function and a general class of polynomials in a compact and elegant with the help of Saigo operator. Most of the results obtained have been put in a compact form, avoiding the occurrence of infinite series and thus making them useful in applications.

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References

[1] Dehnath, L. and Bhatta, D. (2006). Integral transforms and their applications, chapman and Hall/CRC Press, Boca Raton FL.
[2] Kalla, S. L. (1969). Integral operations involving Fox's H-function I, Acta Maxicana Cienc Tecn 3, 117-122.
[3] Kalla, S. L. (1969). Integral operations involving Fox's H-function II, Acta Maxicana Cienc Tecn 7, 72-79.

[4] Kalla, S. L. and Saxena, R. K. (1969). Integral operators involving hypergeometric function, Math. Z. 108, 231-234.

[5] Kalla, S. L. and Saxena, R. K. (1969). Integral operators involving hypergeometric functions II, Univ. Nac. Tucuman, Rev. Sev., A24, 31-36.

[6] Kilbas, A. A. (2005). Fractional calculus of the generalized Wright function, Fract. Calc. Appl. Anal. 8 (2), 113-126.

[7] Kilbas, A. A. and Saigo, M. (2004). H-function, theory and applications, Chapman and Hall/CRC Press, Boca Raton, FL.

[8] Kilbas, A. A. and Sebastain, N. (2008). Generalized Differentiation of Bessel function of first kind, Mathematica Balkanica 22, 323-246.

[9] Kilba, A. A., Srivastava, H. M. and Trujilla, J. J. (2006). Theory and applications of Fractional Differential Equations, 204 (North-Holland Mathematics), Elsevier, 540.

[10] Kiryakavo, V. (1994). Generalized fractional calculus and applications, Longman Scientific and Tech., Essex.

[11] Kiryakavo, V. (2008). A brief story about the operators of the generalized fractional calculus. Fract. Calc. Appl. Anal. 11(2), 203-220.

[12] Love, E. R. (1967). Some integral equations involving hypergeometric function, Proc. Edin. Math. Soc. 15(3), 169-198.

[13] McBride, A. C. (1982). Fractional power of a class of ordinary differential operators, Proc. London Math. Soc. (11) 45, 519-546.

[14] Miller, K. S. and Ross, B. (1993). An introduction to the Fractional Calculus and Differential Equations, A. Willey Interscience. Publication, John Willy and Sons Inc., New York.

[15] Prabhakar, T. R. (1971). A singular integral equation with a generalized Mitag-Leffler function in the kernel, Yokohama Math. J. 19, 7-15.

[16] Saigo, M. (1977/78). A remark on integral operators involving the gauss hypergeometric functions, Math. Rep. College of General Edu. Kyushu University, bf 11, 135-143.

[17] Saigo, M. (1979). A certain boundary value problem for the Euler-Darboux Equation I, Math. Japonica, 24(4), 377-385.

[18] Saigo, M. (1980). A certain boundary value problem for the Euler-Darboux Equation II, Math. Japonica, 25(2), 211-220.

[19] Sanko, S., Kilbass, A. A. and Marichev, O. (1993). Fractional Integrals and Derivatives, Theory and applications, Gardon and Breach Sci. Publ., New York.

[20] Saxena, R. K., Ram, J. and Suthar, D. L. (2009). Fractional calculus of generalized Mittag-Leffler function, J. India Acad. Math. 31(1), 165-172.

[21] Srivastava, H. M. (1972). A contour integral involving Fox's H-function, Indian. J. Math. 14, 1-6.

[22] Srivastava, H. M. and Daoust, M. C. (1972). A note on the convergence of Kampe de Feriet double hypergeometric series. Math. Nachr., 53, 151-159, 1.

[23] Srivastava, H. M. and Panda, R. (1975/76). Some expansion theorems and generating relations for the H-function of several complex variables I, II, Comment. Math. Univ. St. Paul. 24/25, 119-137/167-191, 1.

[24] Srivastava, H. M. and Panda, P. (1976). Some bilateral generating functions for a class of generalized hypergeometric polynomials, J. Reine Angew. Math. 283/284, 265-274.

[25] Srivastava, H. M., Gupta, K. C. and Goyal, S. P. (1982). The H-function of one and two variables with application, South Asian Publishers, New Delhi, Madras.