FUZZY DIGRAPH LABELING

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Abstract

In this paper we introduce some operations on fuzzy digraph .We also define fuzzy digraph labeling and some of its properties.

Keywords

Fuzzy digraph, Fuzzy digraph labeling.

1 Introduction

The first definition of fuzzy graph by Kaufmann in 1973 was based on Zadeh's fuzzy relations.Then Rosen field introduced fuzzy graphs to include fuzziness in relations. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs.In [5],Sunil Mathew and Sunitha M S introduced fuzzy analogues of several graph theoretic concepts such as subgraphs, paths ,connectedness etc.Mordeson and Peng introduced the operations on fuzzy graphs. In [1],John N Mordeson and Premchand S Nair presented many concepts and theoretical results of fuzzy graphs.

By graph labeling we mean an assignment of integers to the vertices or edges.In [2], A Nagoor Gani and D Rajalaxmi (a) Subahashini introduced fuzzy labeling and its properties.

In this paper, we introduce some operations of fuzzy digraphs and fuzzy digraph labeling.

2 Preliminaries

Definition 2.1

The union of two fuzzy graphs $G_1 =(V_1,m_1,\rho 1)$ and $G_2=(V_2,m_2,\rho_2)$ is the fuzzy graph $G= G_1 \cup G_2 = (m_1 \cup m_2, \rho 1 \cup \rho_2)$ defined by $(m_1 \cup m_2)(v) = \begin{cases} m_1(v) \ if \ v \in V_1 - V_2 \\ m_2(v) \ if \ v \in V_2 - V_1 \\ m_1(v) \vee m_2(v) \ if \ v \in V_1 \cap V_2 \end{cases}$

and $(\rho_1 \cup \rho_2)(u, v) = \begin{cases} \rho_1(u,v) \ if u,v \in E_1 - E_2 \\ \rho_2(u,v) \ if u,v \in E - E_1 \\ \rho_1(u,v) \vee \rho_2(u,v) if \ (u,v) \in E_1 \cap E_2 \end{cases}$

Definition 2.2

The join of two fuzzy graphs $G_1 =(V_1,m_1,\rho_1)$ and $G_2=(V_2,m_2,\rho_2)$ with $V_1 \cap V_2 = \phi$ is a fuzzy graph $G=(V_1 + V_2, m_1+ m_2, \rho_1+ \rho_2)$ defined by

$(m_1+ m_2)(v) = (m_1 \cup m_2)(v)$ for all $v \in V_1 \cup V_2$.

And $(\rho_1 + \rho_2)(u, v) = \begin{cases} (\rho 1 \ \cup \ \rho 2)(u,v) \ \ \text{if} \ \ (u,v) \in E_1 \cup E_2 \\ m_1(u) \wedge m_2(v) \ \ \ \ otherwise \end{cases}$

Definition 2.3
Let G =(V,m,ρ) be a fuzzy graph. The complement of G is the graph G^c=(V,m,1-ρ).

Definition 2.4
A fuzzy labeling is a function from the set of all vertices and edges G to [0,1]which assign each nodes m^ω(u) , m^ω(v) and edge ρ^ω(u,v) a membership degree such that
ρ^ω(u,v) < m^ω(u) ∧ m^ω(v).

3 Main Results

Definition 3.1
The union of two fuzzy digraphs DG_1 =(V_1,m_1,ρ_1) and DG_2=(V_2,m_2,ρ_2) is the fuzzy digraph DG= DG_1∪ DG_2=(V_1∪V_2 ,m_1∪ m_2, ρ1∪ ρ_2) defined by

$$(m_1 \cup m_2)(v) = \begin{cases} m_1(v) \ \ if\ v \in V_1 - V_2 \\ m_2(v)\ if\ v\ \in V_2 - V_1 \\ m_1(v) \vee m_2(v)\ if\ v \in V_1 \cap V_2 \end{cases}$$

$$\text{and } (\rho_1 \cup \rho_2)(u,v) = \begin{cases} \rho_1(u,v)\ \ if(u,v) \in E_1 - E_2 \\ \rho_2(u,v)\ \ if(u,v) \in E - E_1 \\ \rho_1(u,v) \vee \rho_2(u,v) if\ (u,v) \in E_1 \cap E_2 \end{cases}$$

where E_1 and E_2 are the set of ordered pair of vertices in DG_1 and DG_2 which symbolize the directed edges.

Definition 3.2
The join of two fuzzy digraphs DG_1 =(V_1,m_1,ρ_1) and DG_2=(V_2,m_2,ρ_2) with $V_1 \cap V_2 = \phi$ is a fuzzy digraph G=($V_1 + V_2$, m_1+ m_2, ρ_1+ ρ_2) is defined by
(m_1+ m_2)(v) = (m_1 ∪ m_2) (v) for all v∈V_1∪V_2.

$$\text{And } (\rho_1 + \rho_2)(u,v) = \begin{cases} (\rho 1\ \cup\ \rho 2)(u,v)\ \text{if}(u,v) \in E_1 \cup E_2 \\ m_1(u) \wedge m_2(v)\ otherwise \end{cases}$$

where E_1 and E_2 are the set of ordered pair of vertices in DG_1 and DG_2 which symbolize the directed edges.

Definition 3.3
If DG =(V,m,ρ) is a fuzzy digraph , then the complement is the fuzzy digraph DG' =(V,m,1-ρ) with direction of edges preserving.

Proposition 3.4
The underlying fuzzy graphs of any two complements are isomorphic.

Definition 3.5
If DG =(V,m,ρ) is a fuzzy digraph , then the fuzzy complement G' =(V,m,1-ρ) is the fuzzy digraph with the same vertex set and the edge set consisting of those ordered pair of vertices which are not in G and the membership grade of an edge (u,v) is 1 if neither (u,v) nor (v,u) is in DG.

Definition 3.6

A fuzzy digraph DG^{-1} is said to be the inverse of a fuzzy digraph if there exist a bijection from the set of all edges and vertices of DG and DG^{-1} such that f(v) = v and f((u,v))=(v,u).

Proposition 3.7

The underlying graphs of the fuzzy digraph and its inverse is same.

Definition 3.8

Let DG=(V,m,ρ) be a fuzzy digraph. By fuzzy digraph labeling we mean the existence of two functions m:V*→[0,1] and ρ: E →[0,1] both injective and ρ(a,b) ≤ m(a) ∧ m(b) ,where V* is an ordered set of vertices and E is the fuzzy edge set of directed edges.

A digraph DG with a fuzzy digraph labeling is called a fuzzy labeling digraph.

Definition 3.9

The fuzzy labeling digraph DH =(W,n,τ) is called a fuzzy labeling subgraph of DG =(V, m, ρ) if n(v) ≤ m(v) for all u∈ V and τ(u, v) ≤ ρ(u, v) for all u, v ∈ V.

Proposition 3.10

Union of two fuzzy labeling digraphs DG_1 =(V_1,m_1,ρ_1) and DG_2=(V_2,m_2,ρ_2) is again a fuzzy labeling digraph if $V_1 \cap V_2 = \phi$.

Proof

Let $V_1 \cap V_2 = \phi$.

$$\text{Then } (m_1 \cup m_2)(v) = \begin{cases} m_1(v) \;\; if\; v \in V_1 - V_2 \\ m_2(v)\; if\; v \in V_2 - V_1 \end{cases}$$

and

$$\text{And } (\rho_1 \cup \rho_2)(u,v) = \begin{cases} \rho_1(u,v) \;\; if (u,v) \in E_1 - E_2 \\ \rho_2(u,v) \;\; if (u,v) \in E - E_1 \end{cases}$$

Clearly both are injective.

$$\text{Also, } (\rho_1 \cup \rho_2)(u,v) \le \begin{cases} m_1(u) \wedge m_1(v)\; if (u,v) \in E_1 - E_2 \\ m_2(u) \wedge m_2(v)\; if (u,v) \in E_2 - E_1 \end{cases}$$

$$\le \begin{cases} m_1(u)\; if\; (u,v) \in E_1 - E_2 \\ m_2(v) if\; (u,v) \in E_2 - E_1 \end{cases}$$

$$\le m_1(u) \wedge m_2(v)$$

$$\le (m_1 \cup m_2)(u) \wedge (m_1 \cup m_2)(v)$$

Proposition 3.11

Join of two fuzzy labeling digraphs **D**G_1 =(V_1,m_1,ρ_1) and DG_2=(V_2,m_2,ρ_2) need not be a fuzzy labeling digraph.

Proof

Let $V_1 \cap V_2 = \phi$.

Then(m_1+ m_2)(v) = ($m_1 \cup m_2$) (v) for all v∈$V_1 \cup V_2$.

$$\text{And } (\rho_1 + \rho_2)(u,v) = \begin{cases} (\rho 1 \; \cup \; \rho 2)(u,v) \text{ if } \;\; (u,v) \in E_1 \cup E_2 \\ \quad m_1(u) \wedge m_2(v) \qquad otherwise \end{cases}$$

$$\le (m_1 \cup m_2)(u) \wedge (m_1 \cup m_2)(v)$$

$$\le (m_1 + m_2)(u) \wedge (m_1 + m_2)(v)$$

But $\rho_1 + \rho_2$ is not injective.

Proposition 3.12

Complement of a fuzzy labeling digraph need not be a a fuzzy labeling digraph.

Proof

Let G=(V, m, ρ) be a fuzzy labeling graph and G'=(V,m,1-ρ) be its complement.
Since m and ρ are injective , 1- ρ is also injective.
ρ(u,v) ≤ m(u) ∧ m(v)
(1-ρ)(u,v) ≰ m(u) ∧ m(v), for consider an edge (u,v) with m(u)= 0.5 m(v)=0.6 and
ρ(u,v)= 0.4
Then (1-ρ)(u,v) =0.6 > 0.5=m(u) ∧ m(v).

Proposition 3.13

The number of fuzzy complements of a fuzzy digraph DG =(V,m,ρ) is 2(nC_2-e) ,where e = number of edges in G.

Proof

Maximum number of edges in the underlying graph is nC_2.
The number of edges in the complement of its underlying graph is nC_2-e.
For each edge there are two directions possible and corresponding to each direction there is a graph which is a complement of G.
So The number of complements of a fuzzy digraph is 2(nC_2-e) .

Proposition 3.14

Fuzzy Complement of a fuzzy labeling digraph need not be a a fuzzy labeling digraph.

Proof

Let G=(V,m,ρ) be a fuzzy labeling graph and G'=(V,m,1-ρ) be its complement.
Since m and ρ are injective , 1- ρ is also injective.
ρ(u,v) ≤ m(u) ∧ m(v)
(1-ρ)(u,v) ≰ m(u) ∧ m(v)

Proposition 3.15

The inverse of a fuzzy labeling digraph is also a fuzzy labeling digraph.

Proof

Since the inverse differs from the given graph only in direction of edges, all other properties are satisfied and so a fuzzy labeling digraph.

Proposition 3.16

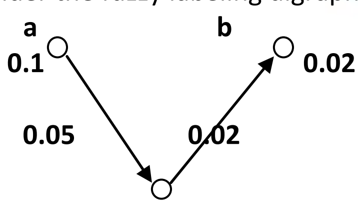
The sum of a fuzzy labeling digraph and its inverse is the trivial graph.

Proposition 3.17

Fuzzy Complement of a fuzzy labeling digraph is not unique.

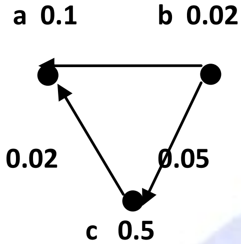
Proof

Consider the fuzzy labeling digraph

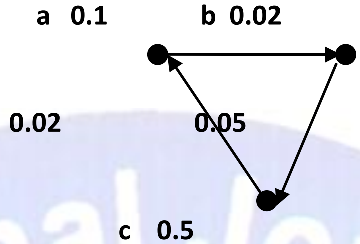


c 0.5

The fuzzy complements are



and



Conclusion

In this paper we introduced some operations on fuzzy digraphs including union, join, complement and fuzzy complement. We also introduced fuzzy digraph labeling and some of its properties.

References

[1] John N Mordeson and Premchand S Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag[2000].

[2] A Nagoor Gani and D Rajalaxmi (a) Subahashini, *Properties of Fuzzy Labeling Graph*, Applied Mathematical Sciences,Vol.6,2012, no.70,3461-3466.

[3]K Radha and N Kumaravel, *The Degree of an Edge in Union and Join of Fuzzy Graphs,*International Journal of Fuzzy Mathematical Archive,Vol.4, No.1,2014,8-19.

[4] G Nirmala and S Prabavathi, *Characteristics of Fuzzy Digraph With Generalized Modus Tollens Using Mathematical Models,* International Journal of Fuzzy Mathematics and systems,ISSN 2248-9940 Volume 5, Number 1 (2015),pp.87-97.

[5] Sunil Mathew and M. S Sunitha, *Fuzzy graphs Basics, Concepts and Applications*, LAP LAMBERT Academic Publishing.