# $\alpha_q$ open sets and $\beta q$ open sets in Quad Topological Space

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***Abstract***

***The main aim of this paper is to introduce new types of open sets namely $\alpha_q$ open sets and $\beta q$ open sets in quad topological spaces along with their several properties and characterization .As application to $\alpha_q$ open sets and $\beta q$ open sets we introduce $\alpha_q$*** *continuous functions* ***,*** *$\beta q$ continuous functions and obtainsome of their basic properties.*

***Keywords:*** $\alpha_q$ ***open sets,*** $\alpha_q$ continuous ***function and*** $\beta q$ ***open sets ,*** $\beta q$ continuous ***function,,***

AMS ***Mathematics Subject Classification (2010):*** 54A40

## Introduction

In 1965 Njastad [8 ] introduced generalization of open set in topological space called pre semi open sets ( $\alpha$ open sets). In 1990, M. Jelic [ 3] introduced the concept of $\alpha$ open sets in bitopological spaces. The concept of bitopological spaces was introduced by J .C. Kelly [4]. In 1986, D. Andrijevic [1] was introduced semi-pre open sets ( $\beta$ open sets) in topological spaces. F.H. Khedr, S.M. Al-Areefi and T. Noiri [ 5] generalized the notion of semi pre open sets to bitoplogical spaces and semi pre continuity in bitopological spaces. Tri topological space is a generalization of bitopological space.The study of tri-topological space was first initiated by Martin M. Kovar [6 ]. S. Palaniammal [9 ] studied tri topological space. N. F. Hameed and Moh. Yahya Abid [2] defined 123 open set in tri topological space.we [10] introduced $\alpha_T$ open set in tri topological space. We [11 ] introduce $\beta_T$ open set in tri topological space. D.V. Mukundan [7 ] introduced the concept of quad topology (4-tuple topology) and defined new types of open (closed) sets. we [ 12 ] introduce semi open set and pre open set in quad topological space. The purpose of the present paper is to introduce $\alpha_q$ and $\beta_q$ ***open sets and*** $\alpha_q$ ***and*** $\beta q$ continuity and their fundamental properties in quad topological space.

## 1. Preliminaries

**Definition 1.1[8]:** A subset A of a topological space $(X,\tau)$ is called pre-semi open set ($\alpha$ open sets) if $A \subseteq \operatorname{int}(cl(\operatorname{int} A))$.The complement of pre semi open set is called pre-semi closed set. The class of all pre-semi open sets of $X$ is denoted by $PSO(X,\tau)$.

**Definition 1.2[ 1]:** A subset A of a topological space $(X,\tau)$ is called semi-pre open set ($\beta$ open sets) if $A \subseteq cl(\operatorname{int}(clA))$.The complement of semi-pre open set is called semi-pre closed set. The class of all semi pre open sets of $X$ is denoted by $SPO(X,\tau)$.

**Definition 1.3[10]:** Let $(X,T_1,T_2,T_3)$ be a tri topological space. A subset $A$ of $X$ is called $\alpha_T$ open in $X$, if $A \subseteq ps\operatorname{int} psclps\operatorname{int} A$. The complement of $\alpha_T$ open set is called $\alpha_T$ closed set. The collection of all $\alpha_T$ open sets of $X$ is denoted by $PSO(X,T_1,T_2,T_3)$.

**Definition 1.4[11]:** Let $(X,T_1,T_2,T_3)$ be a tri topological space. A subset $A$ of $X$ is called $\beta_T$ open in $X$, if $A \subseteq spclsp\operatorname{int} spclA$. The complement of $\beta_T$ open set is called $\beta_T$ closed set. The collection of all $\beta_T$ open sets of $X$ is denoted by $SPO(X,T_1,T_2,T_3)$.

**Definition 1.5 [ 7] :**Let X be a nonempty set and $T_1,T_2,T_3$ and $T_3$are general topologies on X.Then a subset A of space X is said to be quad-open(q-open) set if $A \subset T_1 \cup T_2 \cup T_3 \cup T_4$ and its complement is said to be q-closed and set X with four topologies called quad topological spaces $(X,T_1,T_2,T_3,T_4)$ .q-open sets satisfy all the axioms of topology.

## 2. $\alpha_q$ open set in quad topological space

**Definition 2.1:** Let $(X,T_1,T_2,T_3,T_4)$ be a quad topological space. A subset $A$ of $X$ is called $\alpha_q$ open in $X$, if $A \subseteq p_q s_q \operatorname{int} p_q s_q cl p_q s_q \operatorname{int} A$. The complement of $\alpha_q$ open set is called $\alpha_q$ closed set. The collection of all $\alpha_q$ open sets of $X$ is denoted by $p_q s_q O(X,T_1,T_2,T_3)$.

**Example 2.2 :**Let $X=\{a,b,c,d\}$, $T_1=\{X,\phi,\{a\}\}$, $T_2=\{X,\phi,\{a,b,c\}\}$, $T_3=\{X,\phi,\{b,c,d\}\}$, $T_4=\{X,\phi,\{b,c\}\}$

q- open sets in quad topological spaces are union of all four topologies.

$\alpha_q$ open set of $X$ is denoted by $p_q s_q O(X)=\{X,\phi,\{a\},\{a,b,c\},\{b,c,d\},\{b,c\}\}$.

Let $A=\{a,b,c\}$;

$$\begin{aligned} p_q s_q \operatorname{int} p_q s_q cl p_q s_q \operatorname{int}\{a,b,c\} &= p_q s_q \operatorname{int} p_q s_q cl\{a,b,c\} \\ &= p_q s_q \operatorname{int} X \\ &= X \end{aligned}$$

$A=\{a,b,c\}$ is $\alpha_q$ open.

**Definition 2.3:** Let $(X,T_1,T_2,T_3,T_4)$ be a quad topological space. Let $A \subset X$. An element $x \in A$ is called $\alpha_q$ interior point of $A$, if there exist a $\alpha_q$ open set $U$ such that $x \in U \subset A$. The set of all $\alpha_q$ interior points of $A$ is called the $\alpha_q$ interior of $A$ and is denoted by $p_q s_q \operatorname{int}(A)$.

**Theorem 2.4:** Let $A \subset X$ be a quad topological space. $p_q s_q \operatorname{int}(A)$ is equal to the union of all $\alpha_q$ open sets contained in $A$.

**Note 2.5:** 1. $p_q s_q \operatorname{int}(A) \subset A$.

2. $p_q s_q \operatorname{int}(A)$ is $\alpha_q$ open sets.

**Theorem 2.6:** $p_q s_q \operatorname{int}(A)$ is the largest $\alpha_q$ open sets contained in $A$.

**Theorem 2.7:** $A$ is $\alpha_q$ open if and only if $A = p_q s_q \operatorname{int}(A)$

**Theorem 2.8:** $p_q s_q \operatorname{int}(A \cup B) \supset p_q s_q \operatorname{int} A \cup p_q s_q \operatorname{int} B$.

**Definition 2.9:** Let $(X,T_1,T_2,T_3,T_4)$ be a quad topological space. Let $A \subset X$. The intersection of all $\alpha_q$ closed sets containing $A$ is called a $\alpha_q$ closure of $A$ and is denoted as $p_q s_q cl(A)$.

**Note 2.10 :** Since intersection of $\alpha_q$ closed sets is $\alpha_q$ closed set, $p_q s_q cl(A)$ is a $\alpha_q$ closed set.

**Note 2.11:** $p_q s_q cl(A)$ is the smallest $\alpha_q$ closed set containing $A$.

**Theorem 2.12:** $A$ is $\alpha_q$ closed set if and only if $A = p_q s_q cl(A)$.

**Theorem 2.13:** Let $A$ and $B$ be subsets of $(X,T_1,T_2,T_3,T_4)$ and $x \in X$

a) If $A \subset B$, then $p_q s_q cl(A) \subset p_q s_q cl(B)$.

b) $x \in p_q s_q cl(A)$ if and only if $A \cap U \neq \phi$ for every $\alpha_q$ open set $U$ containing $x$.

**Theorem 2.14:** Let $A$ be a subsets of $(X,T_1,T_2,T_3,T_4)$, if there exist an $\alpha_q$ open set $U$ such that $A \subset U \subset p_q s_q cl(A)$, then $A$ is $\alpha_q$ open.

**Theorem 2.15:** In a quad topological space $(X,T_1,T_2,T_3,T_4)$ , the union of any two $\alpha_q$ open sets is always an $\alpha_q$ open set.

**Proof:** Let $A$ and $B$ be any two $\alpha_q$ open sets in $X$.

Now $A \cup B \subseteq p_q s_q cl\left(p_q s_q \operatorname{int}(A)\right) \cup p_q s_q cl\left(p_q s_q \operatorname{int}(B)\right)$

$\Rightarrow A \cup B \subseteq p_q s_q cl\left(\operatorname{int}(A \cup B)\right)$. Hence $A \cup B$ is $\alpha_q$ open sets.

**Theorem 2.16:** Let $A$ and $B$ be subsets of $X$ such that $B \subseteq A \subseteq p_q s_q cl(B)$ .if $B$ is $\alpha_q$ open set then $A$ is also $\alpha_q$ open set.

## 3. $\alpha_q$ Continuity in quad topological space

**Definition 3.1:** A function f from a quad topological space $(X,T_1,T_2,T_3,T_4)$ into another quad topological space $(Y,W_1,W_2,W_3,W_4)$ is called $\alpha_q$ continuous if $f^{-1}(V)$ is $\alpha_q$ open set in $X$ for each quad open set $V$ in $Y$.

**Example 3.2:** Let, $X = \{a,b,c,d\}$ , $T_1 = \{X,\phi\}$ , $T_2 = \{X,\phi,\{a\},\{b,c\}\}$,
$T_3 = \{X,\phi,\{a,b,c\}\}, T_4 = \{X,\phi,\{a,d\}\}$

Open sets in quad topological spaces are union of all four topologies.
Then quad open sets of $X = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}, \{a,d\}\}$.
$p_q s_q O(X)$ sets of $X = \{X, \phi, \{a\}, \{b,c\}, \{a,b,c\}, \{a,d\}\}$.
Let $Y = \{1,2,3,4\}$, $W_1 = \{Y, \phi\}$, $W_2 = \{Y, \phi, \{1\}.\{2,3\}\}$, $W_3 = \{X, \phi, \{1,2,3\}\}, W_4 = \{X, \phi, \{1,4\}\}$
quad open sets of $Y = \{Y, \phi, \{1\}, \{2,3\}, \{1,2,3\}, \{1,4\}\}$.
$p_q s_q O(Y)$ sets of $Y = \{Y, \phi, \{1\}, \{2,3\}, \{1,2,3\}, \{1,4\}\}$.
Consider the function $f : X \to Y$ is defined as
$f^{-1}\{1\} = \{a\},\ f^{-1}\{2,3\} = \{b,c\},\ f^{-1}\{1,2,3\} = \{a,b,c\}\ f^{-1}(a,d) = \{1,4\},\ f^{-1}(\phi) = \phi,\ f^{-1}(Y) = X$.
Since the inverse image of each quad open set in $Y$ under $f$ is $\alpha_q$ open set in $X$. Hence $f$ is $\alpha_q$ continuous function.

**Theorem 3.3:** Let $f : (X, T_1, T_2, T_3, T_4) \to (Y, W_1, W_2, W_3, W_4)$ be a $\alpha_q$ continuous open function .If $A$ is an $\alpha_q$ open set of $X$, then $f(A)$ is $\alpha_q$ open in $Y$.

**Proof :** First ,let $A$ be $\alpha_q$ open set in $X$ .There exist an quad open set $U$ in $X$ such that $A \subset U \subset p_q s_q cl(A)$ .since f is $\alpha_q$ open function then $f(U)$ is quad open in $Y$. Since f is $\alpha_q$ continuous function, we have $f(A) \subset f(U) \subset f(p_q s_q cl(A)) \subset p_q s_q cl(f(A))$ .This show that $f(A)$ is $\alpha_q$ open in $Y$. Let $A$ be $\alpha_q$ open in $X$ .There exist an $\alpha_q$ open set $U$ such that $U \subset A \subset (p_q s_q cl(U))$. Since $f$ is $\alpha_q$ continuous function, we have $f(U) \subset f(A) \subset f(p_q s_q cl(U)) \subset p_q s_q cl(f(U))$ .by the proof of first part, $f(U)$ is $\alpha_q$ open in $X$ .Therefore , $f(A)$ is $\alpha_q$ open in $Y$.

**Theorem 3.4:** Let $(X, T_1, T_2, T_3, T_4)$ and $(Y, W_1, W_2, W_3, W_4)$ be two quad topological space .Then $f : X \to Y$ is $\alpha_q$ continuous function if and only if $f^{-1}(V)$ is $\alpha_q$ closed in $X$ whenever $V$ is quad closed in $Y$.

**Theorem 3.5:** Let $(X, T_1, T_2, T_3, T_4)$ and $(Y, W_1, W_2, W_3, W_4)$ be two quad topological spaces. A function $f : X \to Y$ is $\alpha_q$ continuous if and only if the inverse image of every $\alpha_q$ open set in $Y$ is quad open in $X$.

**Theorem 3.6:** Let $f : (X, T_1, T_2, T_3, T_4) \to (Y, W_1, W_2, W_3, W_4)$ be a $\alpha_q$ continuous open function .If $V$ is an $\alpha_q$ open set of $Y$, then $f^{-1}(V)$ is $\alpha_q$ open in $X$.

**Proof :** First ,let $V$ be $\alpha_q$ open set of $Y$ .There exist an $\alpha_q$ set $W$ in $Y$ .such that $V \subset W \subset p_q s_q cl(V)$ .Since $f$ is quad open set ,we have $f^{-1}(V) \subset f^{-1}(W) \subset f^{-1}(p_q s_q cl(V)) \subset p_q s_q cl(f^{-1}(V))$ .since $f$ is $\alpha_q$ continuous, $f^{-1}(W)$ is $\alpha_q$ open set in $X$ .By theorem 2.14 , $f^{-1}(V)$ is $\alpha_q$ open set in $X$ .The proof of the second part is shown by using the fact of first part.

**Theorem 3.7:** The following are equivalent for a function $f : (X, T_1, T_2, T_3, T_4) \to (Y, W_1, W_2, W_3, W_4)$:

a) $f$ is $\alpha_q$ continuous function ;
b) the inverse image of each $\alpha_q$ closed set of $Y$ is $\alpha_q$ closed in $X$ ;

c) For each $x \in X$ and each quad open set $V$ in $W$ containing $f(x)$ ,there exist an $\alpha_q$ open set $U$ of $X$ containing $x$ such that $f(U) \subset V$ ;

d) $p_q s_q cl(f^{-1}(B)) \subset f^{-1}(p_q s_q cl(B))$ for every subset $B$ of $Y$ .

e) $f(p_q s_q cl(A)) \subset p_q s_q cl(f(A))$ for every subset $A$ of $X$ .

**Theorem 3.8:** If $f:(X,P_1,P_2,P_3,P_4) \to (Y,W_1,W_2,W_3,W_4)$ and $g:(Y,W_1,W_2,W_3,W_4) \to (Z,\eta_1,\eta_2,\eta_3,\eta_4)$ be two $\alpha_q$ continuous function then $gof:(X,P_1,P_2,P_3,P_4) \to (Z,\eta_1,\eta_2,\eta_3,\eta_4)$ may not be $\alpha_q$ continuous function.

**Theorem 3.9:** Let $f:(X,T_1,T_2,T_3,T_4) \to (Y,W_1,W_2,W_3,W_4)$ be bijective .Then the following conditions are equivalent:

i) $f$ is $\alpha_q$ open continuous function.

ii) $f$ is $\alpha_q$ closed continuous function and

iii) $f^{-1}$ is $\alpha_q$ continuous function.

**Proof:**(i)$\to$(ii) Suppose $B$ is a quad closed set in $X$ .Then $X-B$ is an quad open set in $X$ .Now by (i) $f(X-B)$ is a $\alpha_q$ open set in $Y$ .Now since $f$ is bijective so $f(X-B) = Y - f(B)$ .Hence $f(B)$ is a $\alpha_q$ closed set in $Y$ .Therefore $f$ is a $\alpha_q$ closed continuous function.

(ii)$\to$(iii) Let $f$ is an $\alpha_q$ closed map and $B$ be $\alpha_q$ closed set of $X$ .Since $f^{-1}$ is bijective so $(f^{-1})^{-1}(B)$ which is an $\alpha_q$ closed set in $Y$ . Hence $f^{-1}$ is $\alpha_q$ continuous function.

(iii)$\to$(i) Let $A$ be a quad open set in $X$ .Since $f^{-1}$ is a $\alpha_q$ continuous function so $(f^{-1})^{-1}(A) = f(A)$ is a $\alpha_q$ open set in $Y$ . Hence $f$ is $\alpha_q$ open continuous function.

**Theorem 3.10:** Let $X$ and $Y$ are two quad topological spaces. Then $f:(X,P_1,P_2,P_3,P_4) \to (Y,W_1,W_2,W_3,W_4)$ is $\alpha_q$ continuous function if one of the followings holds:

a) $f^{-1}(p_q s_q \operatorname{int}(B)) \subseteq p_q s_q \operatorname{int}(f^{-1}(B))$ , for every quad open set $B$ in $Y$ .

b) $p_q s_q cl(f^{-1}(B)) \subseteq f^{-1}(p_q s_q cl(B))$ , for every quad open set $B$ in $Y$ .

**Proof:** Let $B$ be any quad open set in $Y$ and if condition (i) is satisfied then $f^{-1}(p_q s_q \operatorname{int}(B)) \subseteq p_q s_q \operatorname{int}(f^{-1}(B))$.

We get $f^{-1}(B) \subseteq p_q s_q \operatorname{int}(f^{-1}(B))$ .Therefore $f^{-1}(B)$ is a quad pre semi open set in $X$ .Hence $f$ is $\alpha_q$ continuous function. Similarly we can prove (ii).

**Theorem 3.11:** A function $f:(X,P_1,P_2,P_3,P_4) \to (Y,W_1,W_2,W_3,W_4)$ is called $\alpha_q$ open continuous function if and only if $f(p_q s_q \operatorname{int}(A)) \subseteq p_q s_q \operatorname{int}(f(A))$ , for every $\alpha_q$ open set $A$ in $X$ .

**Proof:** Suppose that $f$ is a $\alpha_q$ open continuous function.

Since $p_q s_q \operatorname{int}(A) \subseteq A$ so $f(p_q s_q \operatorname{int}(A)) \subseteq f(A)$.

By hypothesis $f(p_q s_q \operatorname{int}(A))$ is an $\alpha_q$ open set and $p_q s_q \operatorname{int}(f(A))$ is largest $\alpha_q$ open set contained in $f(A)$ so $f(p_q s_q \operatorname{int}(A)) \subseteq p_q s_q \operatorname{int}(f(A))$.

Vol.09 Issue-01, (January - June, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

Conversely, suppose $A$ is an $\alpha_q$ open set in $X$ .So $f(p_q s_q \operatorname{int}(A)) \subseteq p_q s_q \operatorname{int}(f(A))$.

Now since $A = p_q s_q \operatorname{int}(A)$ so $f(A) \subseteq p_q s_q \operatorname{int}(f(A))$.Therefore $f(A)$ is a $\alpha_q$ open set in $Y$ and $f$ is $\alpha_q$ open continuous function.

**Theorem 3.12:** A function $f:(X,P_1,P_2,P_3,P_4) \to (Y,W_1,W_2,W_3,W_4)$ is called $\alpha_q$ closed continuous function if and only if $p_q s_q cl(f(A)) \subseteq f(p_q s_q cl(A))$, for every $\alpha_q$ closed set $A$ in $X$ .

**Proof:** Suppose that $f$ is a $\alpha_q$ closed continuous function. Since $A \subseteq p_q s_q cl(A)$ so $f(A) \subseteq f(pscl(A))$.By hypothesis, $f(p_q s_q cl(A))$ is a $\alpha_q$ closed set and $p_q s_q cl(f(A))$ is smallest $\alpha_q$ closed set containing $f(A)$ so $p_q s_q cl(f(A)) \subseteq f(p_q s_q cl(A))$.

Conversely, suppose $A$ is an $\alpha_q$ closed set in $X$ .So $p_q s_q cl(f(A)) \subseteq f(p_q s_q cl(A))$.

Since $A = p_q s_q cl(A)$ so $p_q s_q cl(f(A)) \subseteq f(A)$.Therefore $f(A)$ is a $\alpha_q$ closed set in $Y$ and $f$ is $\alpha_q$ closed continuous function.

**Theorem 3.13:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological space. Then, $f:X \to Y$ is $\alpha_q$ continuous function if and only if $f(p_q s_q cl(A)) \subset p_q s_q cl(f(A))\ \forall A \subset X$ .

**Proof:** Suppose $f:X \to Y$ is $\alpha_q$ continuous function. Since $p_q s_q cl[f(A)]$ is $\alpha_q$ closed in $Y$. Then by theorem (3.4) $f^{-1}[p_q s_q cl(f(A))]$ is $\alpha_q$ closed in $X$ ,

$p_q s_q cl(f^{-1}(p_q s_q clf(A))) = f^{-1}(p_q s_q clf(A))$ ----(1).

Now : $f(A) \subset p_q s_q cl(f(A)),\ A \subset f^{-1}(f(A)) \subset f^{-1}(p_q s_q clf(A))$.

Then $p_q s_q cl(A) \subset p_q s_q cl(f^{-1}(p_q s_q clf(A))) = f^{-1}(p_q s_q clf(A))$ by (1).

Then $f(p_q s_q cl(A)) \subset p_q s_q cl(f(A))$.

Conversely, Let $f(p_q s_q cl(A)) \subset p_q s_q cl(f(A))\ \forall A \subset X$ .

Let $F$ be $\alpha_q$ closed set in $Y$, so that $p_q s_q cl(F) = F$. Now $f^{-1}(F) \subset X$ , by hypothesis,

$f(p_q s_q cl(f^{-1}(F))) \subset p_q s_q cl(f(f^{-1}(F)) \subset p_q s_q cl(F) = F$.

Therefore $p_q s_q cl(f^{-1}(F) \subset f^{-1}(F)$. But $f^{-1}(F) \subset p_q s_q cl(f^{-1}(F)$ .

Hence $p_q s_q cl(f^{-1}(F) = f^{-1}(F)$ and so $f^{-1}(F)$ is $\alpha_q$ closed in $X$ .

Hence by theorem (3.4) $f$ is $\alpha_q$ continuous function.

**Theorem 3.14:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological spaces. Then, $f:X \to Y$ is $\alpha_q$ continuous function if and only if $p_q s_q cl(f^{-1}(B)) \subset f^{-1}(p_q s_q cl(B))\ \forall B \subset Y$.

**Proof:** Suppose $f:X \to Y$ is $\alpha_q$ continuous. Since $p_q s_q cl(B)$ is $\alpha_q$ closed in $Y$, then by theorem (3.4) $f^{-1}(p_q s_q cl(B))$ is $\alpha_q$ closed in $X$ and therefore , $p_q s_q cl(f^{-1}(p_q s_q cl(B))) = f^{-1}(p_q s_q cl(B))$.........(2)

Now, $B \subset p_q s_q cl(B)$,then $f^{-1}(B) \subset f^{-1}(p_q s_q cl(B))$,then

$p_q s_q cl(f^{-1}(B)) \subset p_q s_q cl(f^{-1}(p_q s_q cl(B))) = f^{-1}(p_q s_q clB)$ by (2)

Vol.09 Issue-01, (January - June, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

Conversely: Let the condition hold and let $F$ be any quad closed set in $Y$ so that $p_q s_q cl(F) = F$. By hypothesis, $p_q s_q cl(f^{-1}(F)) \subset f^{-1}(p_q s_q cl(F)) = f^{-1}(F)$ .But $f^{-1}(F) \subset p_q s_q cl(f^{-1}(F))$ always.Hence $p_q s_q cl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is $\alpha_q$ closed in $X$. It follows from theorem (3.4) that $f$ is $\alpha_q$ continuous function.

**Theorem 3.15** Let $(X, T_1, T_2, T_3, T_4)$ and $(Y, W_1, W_2, W_3, W_4)$ be two quad topological spaces. Then, $f : X \to Y$ is $\alpha_q$ continuous open function if and only if $f^{-1}(p_q s_q \operatorname{int}(B)) \subset p_q s_q \operatorname{int}(f^{-1}(B)) \quad \forall B \subset Y$.

**Proof:** Let $f : X \to Y$ be a quad continuous. Since $p_q s_q \operatorname{int}(B)$ is $\alpha_q$ open in $Y$, then by theorem (3.3) $f^{-1}(p_q s_q \operatorname{int}(B))$ is $\alpha_q$ open in $X$ and therefore , $p_q s_q \operatorname{int}(f^{-1}(p_q s_q \operatorname{int}(B))) = f^{-1}(p_q s_q \operatorname{int}(B))$.........(3)

Now, $p_q s_q \operatorname{int}(B) \subset B$ ,then $f^{-1}(p_q s_q \operatorname{int}(B)) \subset f^{-1}(B)$ ,then $p_q s_q \operatorname{int}(f^{-1}(p_q s_q \operatorname{int}(B))) \subset p_q s_q \operatorname{int}(f^{-1}(B))$ by (3)

**Conversely:** Let the condition hold and let $G$ be any $\alpha_q$ open set in $Y$ so that $p_q s_q \operatorname{int}(G) = G$. By hypothesis, $f^{-1}(p_q s_q \operatorname{int}(G)) \subset p_q s_q \operatorname{int} f^{-1}(G)$ .Since $f^{-1}(p_q s_q \operatorname{int}(G)) = f^{-1}(G)$ then $f^{-1}(G) \subset p_q s_q \operatorname{int}(f^{-1}(G))$ But $p_q s_q \operatorname{int}(f^{-1}(G)) \subset f^{-1}(G)$ always and so $p_q s_q \operatorname{int}(f^{-1}(G)) = f^{-1}(G)$ .Therefore $f^{-1}(G)$ is $\alpha_q$ open in $X$. Consequently by theorem (3.3) $f$ is $\alpha_q$ continuous function.

## 4. $\beta_q$ open sets in quad topological space

**Definition 4.1:** Let $(X, T_1, T_2, T_3, T_4)$ be a quad topological space. A subset $A$ of $X$ is called $\beta_q$ open in $X$, if $A \subseteq s_q p_q cl s_q p_q \operatorname{int} s_q p_q cl A$. The complement of $\beta_q$ open set is called $\beta_q$ closed set. The collection of all $\beta_q$ open sets of $X$ is denoted by $S_q P_q O(X, T_1, T_2)$.

**Example 4.2:** Let $X = \{a, b, c, d\}$, $T_1 = \{X, \phi, \{a\}, \{a, b, d\}\}$, $T_2 = \{X, \phi, \{a, d\}\}$, $T_3 = \{X, \phi, \{b\}\}$
$T_4 = \{X, \phi, \{a, b\}\}$

Quad open sets in quad topological spaces are union of all four topologies.

$\beta_q$ Open set of $X$ is denoted by $s_q p_q O(X) = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}\}$.

Let $A = \{a\}$;

$$\begin{aligned} s_q p_q cl s_q p_q \operatorname{int} s_q p_q cl\{a\} &= s_q p_q cl s_q p_q \operatorname{int} X \\ &= s_q p_q cl X \\ &= X \end{aligned}$$

$A = \{a\}$ is $\beta_q$ open.

**Definition 4.3:** Let $(X, T_1, T_2, T_3, T_4)$ be a quad topological space. Let $A \subset X$. An element $x \in A$ is called $\beta_q$ interior point of $A$, if there exist a $\beta_q$ open set $V$ such that $x \in V \subset A$. The set of all $\beta_q$ interior points of $A$ is called the $\beta_q$ interior of $A$ and is denoted by $s_q p_q \operatorname{int}(A)$.

**Theorem 4.4:** Let $A \subset X$ be a quad topological space. $s_q p_q \operatorname{int}(A)$ is equal to the union of all $\beta_q$ open sets contained in $A$.

**Note 4.5:** 1. $s_q p_q \operatorname{int}(A) \subset A$.

2. $s_q p_q \operatorname{int}(A)$ is $\beta_q$ open sets.

**Theorem 4.6:** $s_q p_q \operatorname{int}(A)$ is the largest $\beta_q$ open sets contained in $A$.

**Theorem 4.7:** $A$ is $\beta_q$ open if and only if $A = s_q p_q \operatorname{int}(A)$

**Theorem 4.8:** $s_q p_q \operatorname{int}(A \cup B) \supset s_q p_q \operatorname{int} A \cup s_q p_q \operatorname{int} B$.

**Definition 4.9:** Let $(X, T_1, T_2, T_3, T_4)$ be a quad topological space. Let $A \subset X$. The intersection of all $\beta_q$ closed sets containing $A$ is called a $\beta_q$ closure of $A$ and is denoted as $s_q p_q cl(A)$.

**Note 4.10 :** Since intersection of $\beta_q$ closed sets is $\beta_q$ closed set, $s_q p_q cl(A)$ is a $\beta_q$ closed set.

**Note 4.11:** $s_q p_q cl(A)$ is the smallest $\beta_q$ closed set containing $A$.

**Theorem 4.12:** $A$ is $\beta_q$ closed set if and only if $A = s_q p_q cl(A)$.

**Theorem 4.13:** Let $A$ and $B$ be subsets of $(X, T_1, T_2, T_3, T_4)$ and $x \in X$

a) If $A \subset B$, then $s_q p_q cl(A) \subset s_q p_q cl(B)$.

b) $x \in s_q p_q cl(A)$ if and only if $A \cap U \neq \phi$ for every $\beta_q$ open set $U$ containing $x$.

**Theorem 4.14:** Let $A$ be a subsets of $(X, T_1, T_2, T_3, T_4)$, if there exist an $\beta_q$ open set $U$ such that $A \subset U \subset s_q p_q cl(A)$, then $A$ is $\beta_q$ open.

**Theorem 4.15:** In a quad topological space $(X, T_1, T_2, T_3, T_4)$, the union of any two $\beta_q$ open sets is always an $\beta_q$ open set.

**Theorem 4.16:** Let $A$ and $B$ be subsets of $X$ such that $B \subseteq A \subseteq s_q p_q cl(B)$ .if $B$ is $\beta_q$ open set then $A$ is also $\beta_q$ open set.

## 5. $\beta_T$ Continuity in quad topological space

**Definition 5.1:** A function f from a quad topological space $(X, T_1, T_2, T_3, T_4)$ into another quad topological space $(Y, W_1, W_2, W_3, W_4)$ is called $\beta_q$ continuous if $f^{-1}(V)$ is $\beta_q$ open set in $X$ for each quad open set $V$ in $Y$.

**Example 5.2:** Let, $X = \{a,b,c,d\}$, $T_1 = \{X, \phi\}$, $T_2 = \{X, \phi, \{d\}\}$, $T_3 = \{X, \phi, \{c,d\}\}$, $T_4 = \{X, \phi, \{c\}\}$

Open sets in quad topological spaces are union of four topologies.

Then quad open sets of $X = \{X, \phi, \{d\}, \{c\}, \{c,d\}\}$.

$s_q p_q O(X)$ sets of $X = \{X, \phi, \{d\}, \{c\}, \{c,d\}\}$.

Let $Y = \{1,2,3,4\}$, $W_1 = \{Y, \phi\}$, $W_2 = \{Y, \phi, \{1\}\}$, $W_3 = \{Y, \phi, \{2\}\}$, $W_4 = \{Y, \phi, \{1,2\}\}$

quad open sets of $Y = \{Y, \phi, \{1\}, \{2\}, \{1,2\}\}$.

$s_q p_q O(Y)$ sets of $Y = \{Y, \phi, \{1\}, \{2\}, \{1,2\}\}$.

Consider the function $f : X \to Y$ is defined as

$f^{-1}\{1\}=\{c\},\ f^{-1}\{2\}=\{d\},\ f^{-1}\{1,2\}=\{c,d\}$ , $f^{-1}(\phi)=\phi,\ \ f^{-1}(Y)=X$ .
Since the inverse image of each quad open set in $Y$ under $f$ is $\beta_q$ open set in $X$ . Hence $f$ is $\beta_q$ continuous function.

**Theorem 5.3:** Let $f:(X,T_1,T_2,T_3,T_4)\to(Y,W_1,W_2,W_3,W_4)$ be a $\beta_q$ continuous open function .If $A$ is an $\alpha_q$ open set of $X$ , then $f(A)$ is $\beta_q$ open in $Y$ .

**Proof :** First ,let $A$ be $\beta_q$ open set in $X$ .There exist an quad open set $U$ in $X$ such that $A\subset U\subset s_q p_q cl(A)$ .since f is $\beta_q$ open function then $f(U)$ is quad open in $Y$ . Since f is $\beta_q$ continuous function, we have $f(A)\subset f(U)\subset f(s_q p_q cl(A))\subset s_q p_q cl(f(A))$ .This show that $f(A)$ is $\beta_q$ open in $Y$ . Let $A$ be $\beta_q$ open in $X$ .There exist an $\beta_q$ open set $U$ such that $U\subset A\subset(p_q s_q cl(U))$. Since $f$ is $\beta_q$ continuous function, we have $f(U)\subset f(A)\subset f(s_q p_q cl(U))\subset s_q p_q cl(f(U))$ .by the proof of first part, $f(U)$ is $\beta_q$ open in $X$ .Therefore , $f(A)$ is $\beta_q$ open in $Y$ .

**Theorem 5.4:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological space .Then $f:X\to Y$ is $\beta_q$ continuous function if and only if $f^{-1}(V)$ is $\beta_q$ closed in $X$ whenever $V$ is quad closed in $Y$ .

**Theorem 5.5:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological spaces. A function $f:X\to Y$ is $\beta_q$ continuous if and only if the inverse image of every $\beta_q$ open set in $Y$ is quad open in $X$.

**Theorem 5.6:** Let $f:(X,T_1,T_2,T_3,T_4)\to(Y,W_1,W_2,W_3,W_4)$ be a $\beta_q$ continuous open function .If $V$ is an $\beta_q$ open set of $Y$ , then $f^{-1}(V)$ is $\beta_q$ open in $X$ .

**Proof :** First ,let $V$ be $\beta_q$ open set of $Y$ .There exist an $\beta_q$ set $W$ in $Y$ .such that $V\subset W\subset s_q p_q cl(V)$ .Since $f$ is quad open set ,we have $f^{-1}(V)\subset f^{-1}(W)\subset f^{-1}(s_q p_q cl(V))\subset s_q p_q cl(f^{-1}(V))$ .since $f$ is $\beta_q$ continuous, $f^{-1}(W)$ is $\beta_q$ open set in $X$ .By theorem 4.14, $f^{-1}(V)$ is $\beta_q$ open set in $X$ .The proof of the second part is shown by using the fact of first part.

**Theorem 5.7:** The following are equivalent for a function $f:(X,T_1,T_2,T_3,T_4)\to(Y,W_1,W_2,W_3,W_4)$:

f) $f$ is $\beta_q$ continuous function ;
g) The inverse image of each $\beta_q$ closed set of $Y$ is $\beta_q$ closed in $X$ ;
h) For each $x\in X$ and each quad open set $V$ in $W$ containing $f(x)$ ,there exist an $\beta_q$ open set $U$ of $X$ containing $x$ such that $f(U)\subset V$ ;
i) $s_q p_q cl(f^{-1}(B))\subset f^{-1}(s_q p_q cl(B))$ for every subset $B$ of $Y$ .
j) $f(s_q p_q cl(A))\subset s_q p_q cl(f(A))$ for every subset $A$ of $X$ .

Vol.09 Issue-01, (January - June, 2017) ISSN: 2394-9309 (E) / 0975-7139 (P)
Aryabhatta Journal of Mathematics and Informatics (Impact Factor- 5.856)

**Theorem 5. 8:** If $f:(X,P_1,P_2,P_3,P_4)\to(Y,W_1,W_2,W_3,W_4)$ and $g:(Y,W_1,W_2,W_3,W_4)\to(Z,\eta_1,\eta_2,\eta_3,\eta_4)$ be two $\beta_q$ continuous function then $gof:(X,P_1,P_2,P_3,P_4)\to(Z,\eta_1,\eta_2,\eta_3,\eta_4)$ may not be $\beta_q$ continuous function .

**Theorem 5.9:** Let $f:(X,P_1,P_2,P_3,P_4)\to(Y,W_1,W_2,W_3,W_4)$ be bijective .Then the following conditions are equivalent:

i) $f$ is $\beta_q$ open continuous function.

ii) $f$ is $\beta_q$ closed continuous function and

iii) $f^{-1}$ is $\beta_q$ continuous function.

**Proof:**(i)$\to$(ii) Suppose $B$ is a quad closed set in $X$ .Then $X-B$ is an quad open set in $X$ .Now by (i) $f(X-B)$ is a $\beta_q$ open set in $Y$ .Now since $f$ is bijective so $f(X-B)=Y-f(B)$.Hence $f(B)$ is a $\beta_q$ closed set in $Y$ .Therefore $f$ is a $\beta_q$ closed continuous function.

(ii)$\to$(iii) Let $f$ is an $\beta_q$ closed map and $B$ be $\beta_q$ closed set of $X$ .Since $f^{-1}$ is bijective so $(f^{-1})^{-1}(B)$ which is an $\beta_q$ closed set in $Y$ . Hence $f^{-1}$ is $\beta_q$ continuous function.

(iii)$\to$(i) Let $A$ be a quad open set in $X$ .Since $f^{-1}$ is a $\beta_q$ continuous function so $(f^{-1})^{-1}(A)=f(A)$ is a $\beta_q$ open set in $Y$ . Hence $f$ is $\beta_q$ open continuous function.

**Theorem 5.10:** Let $X$ and $Y$ are two quad topological spaces. Then $f:(X,T_1,T_2,T_3,T_4)\to\left(Y,W_1,W_2,W_3,W_4\right)$ is $\beta_q$ continuous function if one of the followings holds:

c) $f^{-1}(s_q p_q \operatorname{int}(B))\subseteq s_q p_q \operatorname{int}(f^{-1}(B))$, for every quad open set $B$ in $Y$ .

d) $s_q p_q cl(f^{-1}(B))\subseteq f^{-1}(s_q p_q cl(B))$, for every quad open set $B$ in $Y$ .

**Proof:** Let $B$ be any quad open set in $Y$ and if condition (i) is satisfied then $f^{-1}(s_q p_q \operatorname{int}(B))\subseteq s_q p_q \operatorname{int}(f^{-1}(B))$.

We get $f^{-1}(B)\subseteq s_q p_q \operatorname{int}(f^{-1}(B))$.Therefore $f^{-1}(B)$ is a $\beta_q$ open set in $X$ .Hence $f$ is $\beta_q$ continuous function. Similarly we can prove (ii).

**Theorem 5.11:** A function $f:(X,T_1,T_2,T_3,T_4)\to\left(Y,W_1,W_2,W_3,W_4\right)$ is called $\beta_q$ open continuous function if and only if $f(s_q p_q \operatorname{int}(A))\subseteq s_q p_q \operatorname{int}(f(A))$, for every quad open set $A$ in $X$ .

**Proof:** Suppose that $f$ is a $\beta_q$ open continuous function.

since $s_q p_q \operatorname{int}(A)\subseteq A$ so $f(s_q p_q \operatorname{int}(A))\subseteq f(A)$.

By hypothesis $f(s_q p_q \operatorname{int}(A))$ is an $\beta_q$ open set and $s_q p_q \operatorname{int}(f(A))$ is largest $\beta_q$ open set contained in $f(A)$ so $f(s_q p_q \operatorname{int}(A))\subseteq s_q p_q \operatorname{int}(f(A))$.

Conversely, suppose $A$ is an quad open set in $X$ .So $f(s_q p_q \operatorname{int}(A))\subseteq s_q p_q \operatorname{int}(f(A))$ .

Now since $A=s_q p_q \operatorname{int}(A)$ so $f(A)\subseteq s_q p_q \operatorname{int}(f(A))$ .Therefore $f(A)$ is a $\beta_q$ open set in $Y$ and $f$ is $\beta_q$ open continuous function.

**Theorem 5.12:** A function $f:(X,T_1,T_2,T_3,T_4)\rightarrow(Y,W_1,W_2,W_3,W_4)$ is called $\beta_q$ closed continuous function if and only if $s_q p_q cl(f(A))\subseteq f(s_q p_q cl(A))$, for every quad closed set $A$ in $X$.

**Proof:** Suppose that $f$ is a $\beta_q$ closed continuous function. since $A\subseteq s_q p_q cl(A)$ so $f(A)\subseteq f(s_q p_q cl(A))$ .By hypothesis, $f(s_q p_q cl(A))$ is a $\beta_q$ closed set and $s_q p_q cl(f(A))$ is smallest $\beta_q$ closed set containing $f(A)$ so $s_q p_q cl(f(A))\subseteq f(s_q p_q cl(A))$.

Conversely, suppose $A$ is an quad closed set in $X$ .So $s_q p_q cl(f(A))\subseteq f(s_q p_q cl(A))$.

Since $A=s_q p_q cl(A)$ so $s_q p_q cl(f(A))\subseteq f(A)$ .Therefore $f(A)$ is a $\beta_q$ closed set in $Y$ and $f$ is $\beta_q$ closed continuous function.

**Theorem 5.13:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological space .Then, $f:X\rightarrow Y$ is $\beta_q$ continuous function if and only if $f^{-1}(V)$ is $\beta_q$ closed in $X$ whenever $V$ is quad closed in $Y$

**Theorem 5.14:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological space . Then, $f:X\rightarrow Y$ is $\beta_q$ continuous function if and only if $f(s_q p_q cl(A))\subset s_q p_q cl(f(A))\ \forall A\subset X$.

**Proof:** Suppose $f:X\rightarrow Y$ is $\beta_q$ continuous function. Since $s_q p_q cl[f(A)]$ is $\beta_q$ closed in $Y$. Then by theorem (5.4) $f^{-1}[s_q p_q cl(f(A))]$ is $\beta_q$ closed in $X$,

$s_q p_q cl(f^{-1}(s_q p_q clf(A)))=f^{-1}(s_q p_q clf(A))$ ----(1).

Now: $f(A)\subset s_q p_q cl(f(A)),\ A\subset f^{-1}(f(A))\subset f^{-1}(s_q p_q clf(A))$.

Then $s_q p_q cl(A)\subset s_q p_q cl(f^{-1}(s_q p_q clf(A)))=f^{-1}(s_q p_q clf(A))$ by (1).

Then $f(s_q p_q cl(A))\subset s_q p_q cl(f(A))$.

Conversely, Let $f(s_q p_q cl(A))\subset s_q p_q cl(f(A))\ \forall A\subset X$.

Let $F$ be $\beta_q$ closed set in $Y$ ,so that $s_q p_q cl(F)=F$ .Now $f^{-1}(F)\subset X$, by hypothesis,

$f(s_q p_q cl(f^{-1}(F)))\subset s_q p_q cl(f(f^{-1}(F))\subset s_q p_q cl(F)=F$.

Therefore $s_q p_q cl(f^{-1}(F)\subset f^{-1}(F)$. But $f^{-1}(F)\subset s_q p_q cl(f^{-1}(F)$ always.

Hence $s_q p_q cl(f^{-1}(F)=f^{-1}(F)$ and so $f^{-1}(F)$ is $\beta_q$ closed in $X$.

Hence by theorem (5.14) $f$ is $\beta_q$ continuous function.

**Theorem 5.15:** Let $(X,T_1,T_2,T_3,T_4)$ and $(Y,W_1,W_2,W_3,W_4)$ be two quad topological spaces. Then, $f:X\rightarrow Y$ is $\beta_q$ continuous function if and only if $s_q p_q cl(f^{-1}(B))\subset f^{-1}(s_q p_q cl(B))\forall B\subset Y$.

**Proof:** Suppose $f:X\rightarrow Y$ is $\beta_q$ continuous. Since $s_q p_q cl(B)$ is $\beta_q$ closed in $Y$, then by theorem (5.4) $f^{-1}(s_q p_q cl(B))$ is $\beta_q$ closed in $X$ and therefore , $s_q p_q cl(f^{-1}(s_q p_q cl(B)))=f^{-1}(s_q p_q cl(B))$.........(2)

Now, $B\subset s_q p_q cl(B)$ then $f^{-1}(B)\subset f^{-1}(s_q p_q cl(B))$,

then $s_q p_q cl(f^{-1}(B))\subset s_q p_q cl(f^{-1}(s_q p_q cl(B)))=f^{-1}(s_q p_q clB)$ by (2)

Conversely: Let the condition hold and let $F$ be any quad closed set in $Y$ so that $spcl(F)=F$. By hypothesis, $s_q p_q cl(f^{-1}(F)) \subset f^{-1}(s_q p_q cl(F)) = f^{-1}(F)$ .But $f^{-1}(F) \subset s_q p_q cl(f^{-1}(F))$ always. Hence $s_q p_q cl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is $\beta_q$ closed in $X$ . It follows from theorem (5.6) that $f$ is $\beta_q$ continuous function.

**Theorem 5.16:** Let $(X, T_1, T_2, T_3, T_4)$ and $(Y, W_1, W_2, W_3, W_4)$ be two quad topological spaces. Then, $f: X \to Y$ is $\beta_q$ open continuous function if and only if $f^{-1}(s_q p_q \operatorname{int}(B)) \subset s_q p_q \operatorname{int}(f^{-1}(B)) \quad \forall B \subset Y$.

**Proof:** Let $f: X \to Y$ be a quad continuous. Since $s_q p_q \operatorname{int}(B)$ is $\beta_q$ open in $Y$, then by theorem (5.3) $f^{-1}(s_q p_q \operatorname{int}(B))$ is $\beta_q$ open in $X$ and therefore, $s_q p_q \operatorname{int}(f^{-1}(s_q p_q \operatorname{int}(B))) = f^{-1}(s_q p_q \operatorname{int}(B))$.........(3)
Now, $s_q p_q \operatorname{int}(B) \subset B$, then $f^{-1}(s_q p_q \operatorname{int}(B)) \subset f^{-1}(B)$ ,then $s_q p_q \operatorname{int}(f^{-1}(s_q p_q \operatorname{int}(B))) \subset s_q p_q \operatorname{int}(f^{-1}(B))$ by (3)

**Conversely:** Let the condition hold and let $G$ be any $\beta_q$ open set in $Y$ so that $s_q p_q \operatorname{int}(G) = G$.
Byhypothesis, $f^{-1}(s_q p_q \operatorname{int}(G)) \subset s_q p_q \operatorname{int} f^{-1}(G)$ .Since $f^{-1}(s_q p_q \operatorname{int}(G)) = f^{-1}(G)$ then
$f^{-1}(G) \subset s_q p_q \operatorname{int}(f^{-1}(G))$ But $s_q p_q \operatorname{int}(f^{-1}(G)) \subset f^{-1}(G)$ always and so $s_q p_q \operatorname{int}(f^{-1}(G)) = f^{-1}(G)$ .Therefore $f^{-1}(G)$ is $\beta_q$ open in $X$ . Consequently by theorem (5.2) $f$ is $\beta_q$ continuous function.

**CONCLUSION:** In this paper we studied new forms of $\alpha_q$ open sets and $\beta_q$ open set in quad topological space. We also studied $\alpha_q$ continuous function and $\beta_q$ continuous function in quad topological space. It is established that composition of any two $\alpha_q$ open sets is again a $\alpha_q$ open set in quad topological space and composition of any two $\beta_q$ open sets is again a $\beta_q$ open set in quad topological space. In $\alpha_q$ continuity, inverse image of every quad open is $\alpha_q$ open set and in $\beta_q$ continuity, inverse image of every quad open is $\beta_q$ open set.

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