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Cordial Labeling of Some Graphs

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Abstract
In this paper we prove that Shadow graph of Star $K_{1,n}$, Splitting graph of star $K_{1,n}$ and Degree Splitting graph of star $K_{1,n}$ are cordial graphs. Moreover we show that Jewel graph J_n and Jellyfish graph $J_{n,n}$ are cordial graphs.

Key words: Shadow Graph, Splitting of a graph, Degree splitting of a graph, Jewel graph and Jelly fish graph.

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1. Introduction

Throughout this paper, a graph G = (V, E) is a undirected, finite, connected, and simple graph with vertex set V and edge set E. For different notations and terminology we follow Gross and Yellen[2].

Definition 1. A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 2. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G''.

Definition 3. For a graph G the splitting graph S'(G) of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that N(v) = N(v').

Definition 4. Let G = (V(G), E(G)) be a graph with $V = S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_i \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V \setminus \cup S_i$. The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, w_3, \ldots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Definition 5. The jewel J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

Definition 6. For integers m, n ≥ 0 we consider the graph Jelly Fish J(m, n) with vertex set $V(J(m, n)) = \{u, v, x, y\} \cup \{x_1, x_2, \ldots, x_n\} \cup \{y_1, y_2, \ldots, y_n\}$ and the edge set
$E(J(m, n)) = \{(u, x), (u,y), (u, v), (v, x), (v, y)\} \cup \{(x_i, x) / 1 \leq i \leq n\} \cup \{(y_j, y)\ 1 \leq j \leq n\}$.

Cahit[3] introduced cordial labeling of graphs and derived various results on cordial graphs. Ghodasara and Rokad[4] proved that star of $K_{n,n}$, path union of $K_{n,n}$ and the graph obtained by joining

two copies of $K_{n,n}$ by a path of arbitrary length are cordial graphs. S. K. Vaidya and N. H. Shah[5] proved that splitting graphs of star $K_{1,n}$ is divisor cordial. S.K.Vaidya and Lekha Bijukumar[6] proved that shadow graphs of star $K_{1,n}$ is a mean graph.
A survey on different graph labeling techniques is given by Gallian[1].

2 Main Results

Theorem 1. $D_2(K_{1,n})$ is a cordial graph.

Proof. Consider two copies of $K_{1,n}$. Let v, v_1, v_2,..., v_n be the vertices of the first copy of $K_{1,n}$ and v', v'_1, v'_2 ,...,v'_n be the vertices of the second copy of $K_{1,n}$ where v and v' are the respective apex vertices. Let G be $D_2(K_{1,n})$.

The labeling $f:V(G) \rightarrow \{0,1\}$ is defined as follows.

For $n \equiv 0, 1, 2, 3 \pmod 4$
f(u) = 1,
$f(u_i)$ = 0; if i 0, 3(mod 4)
= 1; if i 1, 2(mod 4).
f(v) = 0,
$f(v_i)$ = 0; if i 1, 2(mod 4)
= 1; if i 0, 3(mod 4).

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 1.
Hence, $D_2(K_{1,n})$ is a cordial graph.

Let n = 4a + b, where $n \in N$.

Table 1: Table for *Theorem 1*

b	vertex conditions	edge conditions
0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$

Example 1. cordial labeling of the graph $D_2(K_{1,5})$ is shown in **Figure 1** as an illustration for the **Theorem 1**. It is the case related to $n \equiv 1 \pmod 4$.

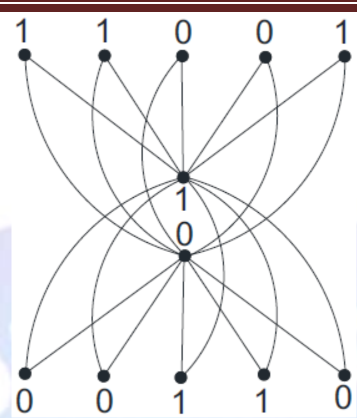

Figure 1

Theorem 2. $S'(K_{1,n})$ is cordial graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and u, u_1, $u_2, \ldots, u_n$ are added vertices corresponding to v, v_1, $v_2, \ldots, v_n$ to obtain $S'(K_{1,n})$. Let G be the graph $S'(K_{1,n})$ then $|V(G)| = 2n+2$ and $|E(G)| = 3n$.

The labeling $f:V(G) \to \{0,1\}$ is defined as per the following cases.

Case 1: $n \equiv 0 \pmod 4$
$f(u) = 1$,
$f(u_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.
$f(v) = 0$,
$f(v_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.

Case 2: $n \equiv 1 \pmod 6$
$f(u) = 1$,
$f(u_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.
$f(v) = 0$,
$f(v_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$.

Case 3: $n \equiv 2, 3 \pmod 6$
$f(u) = 1$,
$f(u_i) = 0$; if $i \equiv 0, 1 \pmod 4$
$= 1$; if $i \equiv 2, 3 \pmod 4$.

$f(v) = 0$,
$f(v_i) = 0$; if $i \equiv 2, 3 \pmod 4$
$= 1$; if $i \equiv 0, 1 \pmod 4$.

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 2.
Hence, $S'(K_{1,n})$ is cordial graph.
Let $n = 4a + b$, where $n \in N$.

Table 2: Table for *Theorem 2*

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Example 2. cordial labeling of the graph $S'(K_{1,7})$ is shown in **Figure 2** as an illustration for the **Theorem 2**. It is the case related to $n \equiv 3 \pmod 4$.

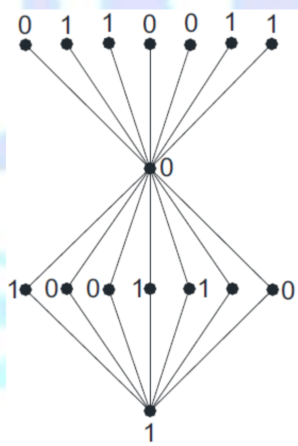

Figure 2

Theorem 3. $DS(K_{1,n})$ is cordial graph.

Proof. Consider $K_{1,n}$ with $V(K_{1,n}) = \{u, v_i : 1 \leq i \leq n\}$, where v_i are pendant vertices. Here $V(K_{1,n}) = V_1 \cup V_2$, where $V_1 = u$ and $V_2 = \{v_i : 1 \leq i \leq n\}$. Now in order to obtain $DS(K_{1;n})$ from G, we add w_1 and w_2 corresponding to V_1 and V_2. Then $|V(DS(K_{1,n}))| = n + 3$ and $|E(DS(K_{1,n}))| = \{V_1w_1, V_2w_2\} \cup \{uv_i, uw_2, w_1v_i :$ 1
$\leq i \leq n\}$. So, $|E(DS(B_{n;n}))| = 2n + 1$.

The labeling $f:V(G) \rightarrow \{0,1\}$ is defined as per the following cases.

Case 1: $n \equiv 0 \pmod 4$
$f(u) = 1$,

$f(w_1) = 1$,
$f(w_2) = 0$.
$f(u_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$.

Case 2: $n \equiv 1, 2 \pmod 4$ $f(u) = 1$,
$f(w_1) = 1$, $f(w_2) = 0$.
$f(u_i) = 0$; if $i \equiv 0, 1 \pmod 4$
$= 1$; if $i \equiv 2, 3 \pmod 4$.

Case 3: $n \equiv 3 \pmod 4$
$f(u) = 1$,
$f(w_1) = 0$,
$f(w_2) = 0$.
$f(u_i) = 0$; if $i \equiv 0, 1 \pmod 4$
$= 1$; if $i \equiv 2, 3 \pmod 4$.

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 3.
Hence, $DS(K_{1,n})$ is cordial graph.

Let $n = 4a + b$, where $n \in N$.

Table 3: Table for *Theorem 3*

b	vertex conditions	edge conditions
0,2	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1) + 1$
1	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Example 3. cordial labeling of the graph $DS(K_{1,5})$ is shown in **Figure 3** as an illustration for the **Theorem 3**. It is the case related to $n \equiv 1 \pmod 4$.

Theorem 4. Jewel graph J_n is cordial.

Proof. Let $G = J_n$. Let $V(G) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and $E(G) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.
Then G is of order $|V(J_n)| = n + 4$ and $|E(J_n)| = 2n + 5$.
The labeling $f:V(G) \rightarrow \{0,1\}$ is defined as per the following cases.

Case 1: $n \equiv 0, 1 \pmod 4$
$f(u) = 1$,
$f(v) = 0$,
$f(x) = 0$,
$f(y) = 1$,
$f(u_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$.

Case 2: $n \equiv 2, 3 (\mathrm{mod}\ 4)$
$f(u) = 1$,
$f(v) = 0$,
$f(x) = 0$,
$f(y) = 1$,
$f(u_i) = 0$; if $i \equiv 2, 3(\mathrm{mod}\ 4)$
$= 1$; if $i \equiv 0, 1(\mathrm{mod}\ 4)$.

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 4.
Hence, Jewel graph J_n is cordial.

Let n = 4a + b, where n 2 N.

Table 4: Table for *Theorem 4*

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1	$v_f(0) + 1 = v_f(1)$	$e_f(0) + 1 = e_f(1)$
3	$v_f(0) = v_f(1) + 1$	$e_f(0) + 1 = e_f(1)$

Example 4. cordial labeling of J_4 is shown in **Figure 4** as an illustration for the **Theorem 4**. It is the case related to
$n \equiv 0(\mathrm{mod}\ 4)$.

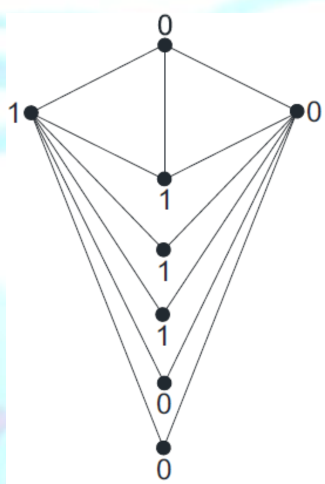

Figure 4

Theorem 5. Jelly fish $J_{n,n}$ is cordial.

Proof. Let $V(J_{n,n}) = \{u, v, x, y, u_i, v_i : 1 \leq i \leq n\}$ and $E(J_{n,n}) = \{ux, uy, vx, vy, xy, uu_i, vv_i : 1 \leq i \leq n\}$.Then G is of order

$|V(J_n)| = 2n + 4$ and $|E(J_n)| = 2n + 5$.

The labeling $f:V(G) \rightarrow \{0,1\}$ is defined as per the following cases.

Case 1: $n \equiv 0 \pmod 4$

$f(u) = 1$,
$f(v) = 0$,
$f(x) = 0$,
$f(y) = 1$,
$f(u_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$
$f(v_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$

Case 2: $n \equiv 1 \pmod 4$

$f(u) = 1$,
$f(v) = 0$,
$f(x) = 1$,
$f(y) = 1$,
$f(u_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.
$f(v_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.

Case 3: $n \equiv 2 \pmod 4$

$f(u) = 1$,
$f(v) = 0$,
$f(x) = 0$,
$f(y) = 1$,
$f(u_i) = 0$; if $i \equiv 0, 1 \pmod 4$
$= 1$; if $i \equiv 2, 3 \pmod 4$.
$f(v_i) = 0$; if $i \equiv 0, 1 \pmod 4$
$= 1$; if $i \equiv 2, 3 \pmod 4$.

Case 4: $n \equiv 3 \pmod 4$

$f(u) = 1$,
$f(v) = 0$,
$f(x) = 0$,
$f(y) = 1$,

$f(u_i) = 0$; if $i \equiv 1, 2 \pmod 4$
$= 1$; if $i \equiv 0, 3 \pmod 4$.
$f(v_i) = 0$; if $i \equiv 0, 3 \pmod 4$
$= 1$; if $i \equiv 1, 2 \pmod 4$.

The labeling pattern defined in above cases satisfies the condition $|v_f(0) – v_f(1)| \leq 1$ and $|e_f(0) – e_f(1)| \leq 1$ in each case which is shown in Table 5.

Hence, Jelly fish $J_{n,n}$ is cordial.

Let n = 4a + b, where n ∈ N.

Table 5: Table for *Theorem 5*

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$

Example 5. cordial labeling of Jelly fish $J_{5,5}$ is shown in **Figure 5** as an illustration for the **Theorem 5**. It is the case related to n ≡ 1(mod 4).

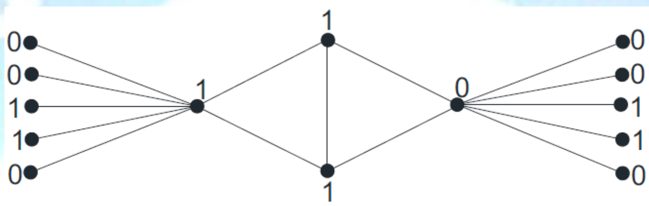

Figure 5

3 Conclusion

In this paper we investigated Five new cordial graphs. The results proved in this paper are novel. Illustrations are provided at the end of each theorem for better understanding of the labeling pattern defined in each theorem.

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