Some Results on Intuitionistic Fuzzy Magic Labeling Graphs

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ABSTRACT

The objective of this work is to introduce the concept of magic labeling in intuitionistic fuzzy graph and to obtain some results related to this concept on some graphs like path, cycle and star graphs.

Mathematics Subject Classification Number: 05C72, 03E72.

Keywords Magic Labeling, Fuzzy Graph, Intuitionistic Fuzzy Graph.

INTRODUCTION

In 1736, Eular [9] introduced the concept of a graph. In 1965, Zadah [22] introduced the concept of fuzzy set which is an extension of crisp set. This concept developed a mathematical theory to deal uncertainty and impression. In 1975, Rosenfeld [18] introduced the concept of fuzzy graph. After that many authors worked on this new concept, fuzzy graph (see [7, 8, 21]). As an extension of fuzzy set, Atanassov [4, 5] introduced the idea of intuitionistic fuzzy set. In 1994, Sovan and Atanassov [2] introduced the concept of intutionistic fuzzy graph. Serveral authors have studied this new concept (see [14, 15, 16, 17]). Sadlack [19] introduced the notion of magic graph in 1964.Many other researchers have investigated different forms of magic graph (see [6, 21]. Gani and Subhashini [12] introduced the notion of fuzzy labeling graph. Recently, Gani and Subhashini[11,13] introduced fuzzy magic labeling graph. Motivated by fuzzy magic labeling graph, we introduce Intuitionistic fuzzy magic labeling graph and to obtain some results related this new concept on some graphs, like path, cycle, star graph.

2. PRELIMINARIES

Definition2.1. A graph G = (V, E) consists of a set of objects $V = \{u_1, u_2, u_3, \ldots\ldots\}$ called vertices, and another set $E=\{e_1, e_2, e_3, \ldots\ldots\}$, whose elements are called edges, such that each edge e_k, is identified with an unordered pair (u_i, u_j) of vertices. The vertices u_i, u_j associated with edge e_k are called the end vertices of e_k .

Definition2.2. Let E be the universal set. A fuzzy set A in E is represented by $A = \{(x, \mu_A(x)) \mid \mu_A(x) > 0, x \in E\}$, where the function $\mu_A : E \rightarrow [0,1]$ is the membership degree of element x in the fuzzy set A.

Definition2.3. Let a set E be fixed. An Intuitionistic Fuzzy set A in E is an object of the form $A = \{(x, \mu_A(x), \gamma_A(x)) \mid x \in E\}$, where the function $\mu_A : E \rightarrow [0, 1]$ and $\gamma_A : E \rightarrow [0, 1]$ determine the degree of membership and the degree of non - membership of the element $x \in E$, respectively and for every $x \in E$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition 2.4:- Let V be a non-empty set. A fuzzy graph is a pair of functions G: (σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. i.e. $\sigma : V \rightarrow [0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all u, v in V.

Definition 2.5:- A star in a fuzzy graph consist of two node set V and U with $|V| = 1$ and $|U| > 1$, such that $\mu(v, u_i) > 0$ and $\mu(v, u_i) = 0$, $1 \leq i \leq n$. It is denoted by $S_{1,n}$.

Definition 2.6:- A graph G: (σ, μ) is said to be fuzzy labeling graph, if σ: V → [0, l] and μ: V × V → [0, l] are bijective such that the membership value of nodes and edges are distinct and μ(x, y) ≤ σ(x) ∧ σ(y) for all x, y in V.

Definition 2.7:- A fuzzy labeling graph is said to be a fuzzy magic graph if μ_A(x) + μ_B(x, y) + μ_A(y) has a same magic value for all x, y in V.

Definition 2.8:- An Intuitionistic Fuzzy Graph (IFG) is of the form *G* = (*V*, *E*), where
(i) $V = \{v_1, v_2, \ldots v_n\}$ such that $\mu_A: V \to [0, 1]$ and $\gamma_A : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively
and $0 \leq \mu_A(v_i) + \gamma_A(v_i) \leq 1$, for every $v_i \in V$ ($i = 1, 2, \ldots, n$).
(ii) $E \in V \times V$, where $\mu_B : V \times V \to [0, 1]$ and $\gamma_B: V \times V \to [0, 1]$ are such that
$\mu_B(v_i v_j) \leq min\, [\mu_A(v_i), \mu_A(v_j)]$
$\gamma_B(v_i v_j) \leq max\, [\gamma_A(v_i), \gamma_A(v_j)]$
and $0 \leq \mu_B(v_i, v_j) + \gamma_B(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

3. INTUITIONISTIC FUZZY MAGIC LABELING GRAPH

Definition 3.1:- A graph G^* = (A, B) is said to be Intuitionistic fuzzy labeling graph $\mu_A: V \to [0, 1]$, $\gamma_A : V \to [0, 1]$, $\mu_B : V \times V \to [0, 1]$ and $\gamma_B: V \times V \to [0, 1]$ are bijective such that if μ_A(x), γ_A(x), μ_B(x, y), γ_B(x, y) ∈ [0,1] all are distinct for each nodes and edges, where μ_A is degree of membership and γ_A is degree of non-membership of nodes, similarly μ_B and γ_B are degree of membership and non-membership of edges.

Definition 3.2:- A Intuitionistic fuzzy labeling graph is said to be a intuitionistic fuzzy magic graph if the degree of membership value (μ_A(x) + μ_B(x, y) + μ_A(y)) remain equal ∀ x,y ∈ V and degree of non-membership value (γ_A(x) + γ_B(x, y) + γ_A(y)) remain equal ∀ x,y ∈ V. The magic membership value denoted m_μ(G) and the magic non-membership value denoted $m_\gamma(G^*)$. We denote an intuitionistic fuzzy magic graph by $M_0(G^*)$. Where $M_0(G^*) = (m_\mu(G^*), m_\gamma(G^*))$.

Theorem 3.3:- **For all n ≥ 1, the path P_n is an intuitionistic fuzzy magic graph.**
Proof:- Let P be any path with length n ≥ 1and $v_1, v_2, v_3,\ldots, v_n$ and $v_1v_2, v_2v_3, v_3v_4, \ldots, v_{n-1}v_n$ are nodes and edges of P. Let $\varepsilon_1, \varepsilon_2 \in [0,1]$ such that we choose ε_1=0.01 and ε_2 = 0.1 if n ≤ 4 and ε_1=0.001 and ε_2 = 0.01 if n ≥ 5. Where ε_1 and ε_2 choose for set of membership and non-membership degree in Intuitionistic fuzzy labeling.
Such intuitionistic fuzzy labeling is given as:
When length is odd:
$\mu_A(v_{2k-1})$ = (2n + 2 – k) ε_1 , $1 \leq k \leq \frac{n+1}{2}$
$\gamma_A(v_{2k-1})$ = (2n + 2 – k) ε_2 , $1 \leq k \leq \frac{n+1}{2}$
$\mu_A(v_{2k})$ = min $\{\mu_A(v_{2k-1}) \mid 1 \leq k \leq \frac{n+1}{2}\} - k\varepsilon_1$, $1 \leq k \leq \frac{n+1}{2}$
$\gamma_A(v_{2k})$ = min $\{\gamma_A(v_{2k-1}) \mid 1 \leq k \leq \frac{n+1}{2}\} - k\varepsilon_2$, $1 \leq k \leq \frac{n+1}{2}$
$\mu_B(v_{n-k+2}, v_{n+1-k})$ = max $\{\mu_A(v_k) \mid 1 \leq k \leq n+1\}$ – min $\{\mu_A(v_k) \mid 1 \leq k \leq n+1\}$ – (k–1)ε_1 ,
$1 \leq k \leq n$.
$\gamma_B(v_{n-k+2}, v_{n+1-k})$ = max $\{\gamma_A(v_k) \mid 1 \leq k \leq n+1\}$ – min $\{\gamma_A(v_k) \mid 1 \leq k \leq n+1\}$ – (k–1) ε_2 , $1 \leq k \leq n$.
Case (i) k is even.
Then k = 2a, where a∈ Z^+ and for each edge v_k, v_{k+1}
$m_\mu(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1})$
$= \mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1})$

$= \min \{\square_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} - a\varepsilon_1 + \max\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- \min \{\square_A(v_k) \mid 1 \le k \le n+1\} - (n - 2a)\varepsilon_1 + (2n - a + 1)\,\varepsilon_1$
$= \min \{\square_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} + \max\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- \min \{\square_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_1$
$m_\gamma(P_n) = \gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
$= \gamma_A(v_{2a}) + \gamma_B(v_{2a}, v_{2a+1}) + \gamma_A(v_{2a+1})$
$= \min \{\gamma_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} - a\varepsilon_2 + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - (n - 2a)\varepsilon_2 + (2n - a + 1)\,\varepsilon_2$
$= \min \{\gamma_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_2$

So that $M_0(P_n) = (m_\square(P_n), m_\gamma(P_n))$.

Case (ii) k is odd.

Then k = 2a + 1, where a∈ Z^+ and for each edge v_k, v_{k+1}.

$m_\square(P_n) = \square_A(v_k) + \square_B(v_k, v_{k+1}) + \square_A(v_{k+1})$
$= \square_A(v_{2a+1}) + \square_B(v_{2a+1}, v_{2a+2}) + \square_A(v_{2a+2})$
$= (2n - a + 1)\,\varepsilon_1 + \max \{\square_A(v_k) \mid 1 \le k \le n+1\} - \min\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- (n - 2a - 1)\,\varepsilon_1 + \min\{\square_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} - (a + 1)\,\varepsilon_1$
$= \min\{\square_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} + \max\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\square_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_1$.
$m_\gamma(P_n) = \gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
$= \gamma_A(v_{2a+1}) + \gamma_B(v_{2a+1}, v_{2a+2}) + \gamma_A(v_{2a+2})$
$= (2n - a + 1)\,\varepsilon_2 + \max \{\gamma_A(v_k) \mid 1 \le k \le n+1\} - \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- (n - 2a - 1)\varepsilon_2 + \min \{\gamma_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} - (a + 1)\,\varepsilon_2$
$= \min \{\gamma_A(v_{2k-1}) \mid 1 \le k \le \frac{n+1}{2}\} + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_2$.

So that $M_0(P_n) = (m_\square(P_n), m_\gamma(P_n))$.

When length is even:

$\square_A(v_{2k}) = (2n + 2 - k)\,\varepsilon_1$, $1 \le k \le \frac{n}{2}$
$\gamma_A(v_{2k}) = (2n + 2 - k)\,\varepsilon_2$, $1 \le k \le \frac{n}{2}$
$\square_A(v_{2k-1}) = \min\{\square_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - k\varepsilon_1$, $1 \le k \le \frac{n+2}{2}$
$\gamma_A(v_{2k-1}) = \min\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - k\varepsilon_2$, $1 \le k \le \frac{n+2}{2}$
$\square_B(v_{n-k+2}, v_{n-k+1}) = \max\{\square_A(v_k) \mid 1 \le k \le n+1\} - \min\{\square_A(v_k) \mid 1 \le k \le n+1\} - (k-1)\varepsilon_1$, $1 \le k \le n$.
$\gamma_B(v_{n-k+2}, v_{n+1-k}) = \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - (k-1)\,\varepsilon_2$, $1 \le k \le n$.

Case (i) k is even.

Then k = 2a, where a∈ Z^+ and for each edge v_k, v_{k+1}.

$m_\square(P_n) = \square_A(v_k) + \square_B(v_k, v_{k+1}) + \square_A(v_{k+1})$
$= \square_A(v_{2a}) + \square_B(v_{2a}, v_{2a+1}) + \square_A(v_{2a+1})$
$= (2n + 2 - a)\,\varepsilon_1 + \max\{\square_A(v_k) \mid , 1 \le k \le n+1\} - \min\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- (n - 2a)\varepsilon_1 + \min\{\square_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - (a + 1)\varepsilon_1$
$= \min\{\square_A(v_{2k}) \mid , 1 \le k \le \frac{n}{2}\} + \max\{\square_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\square_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_1$.

$m_\gamma(P_n) = \gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
$= \gamma_A(v_{2a}) + \gamma_B(v_{2a}, v_{2a+1}) + \gamma_A(v_{2a+1})$
$= (2n + 2 - a)\,\varepsilon_2 + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- (n - 2a)\varepsilon_2 + \min\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - (a + 1)\varepsilon_2$
$= \min\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_2$

So that $M_0(P_n) = (m_\mu(P_n), m_\gamma(P_n))$.

Case (ii) k is odd.

Then k = 2a + 1, where a∈ Z^+ and for each edge v_k, v_{k+1}

$m_\mu(P_n) = \mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1})$
$= \mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2})$
$= \min\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - (a + 1)\,\varepsilon_1 + \max\{\mu_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\mu_A(v_k) \mid 1 \le k \le n+1\} - (n - 2a - 1)\,\varepsilon_1 + (2n - a + 1)\,\varepsilon_1$
$= \min\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} + \max\{\mu_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\mu_A(v_k) \mid 1 \le k \le n+1\} + (n + 1)\,\varepsilon_1$.

$m_\gamma(P_n) = \gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
$= \gamma_A(v_{2a+1}) + \gamma_B(v_{2a+1}, v_{2a+2}) + \gamma_A(v_{2a+2})$
$= \min\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} - (a + 1)\,\varepsilon_2 + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$- \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - (n - 2a - 1)\varepsilon_2 + (2n - a + 1)\,\varepsilon_2$
$= \min\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n}{2}\} + \max\{\gamma_A(v_k) \mid 1 \le k \le n+1\} - \min\{\gamma_A(v_k) \mid 1 \le k \le n+1\}$
$+ (n + 1)\varepsilon_2$.

So that $M_0(P_n) = (m_\mu(P_n), m_\gamma(P_n))$.

The magic value $M_0(P_n)$ is same and unique in above cases. Thus P_n is intuitionistic fuzzy magic graph for all $n \ge 1$.

Example 3.4:- An example of an Intuitionistic fuzzy magic labeling Path graph P_4

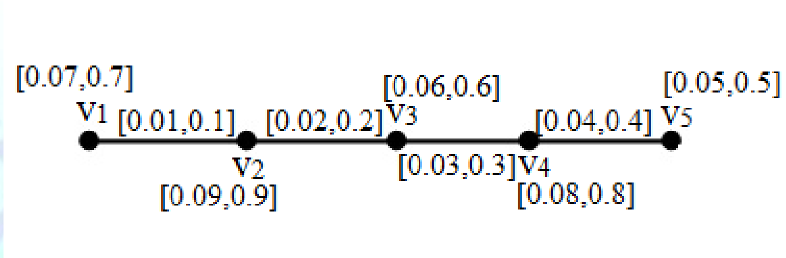

Figure 1

Every pair of vertices the magic value is same

$m_\mu(P_n) = \mu_A(x) + \mu_B(x, y) + \mu_A(y) = 0.07+0.01+0.09 = 0.17$

$m_\gamma(P_n) = \gamma_A(x) + \gamma_B(x, y) + \gamma_A(y) = 0.7+0.1+0.9 = 1.7$

$M_0(P_n) = (m_\mu(P_n), m_\gamma(P_n)) = (0.17, 1.7)$.

Theorem 3.5:- If n is odd, then the cycle C_n is an intuitionistic fuzzy magic graph.

Proof:- Let C_n be any cycle with odd number of nodes. $v_1, v_2, v_3, \ldots, v_n$ and $v_1v_2, v_2v_3, v_3v_4, \ldots, v_nv_1$ are having nodes and edges of C_n. Let $\varepsilon_1, \varepsilon_2 \in [0,1]$ such that we choose ε_1=0.01 and ε_2 = 0.1 if $n \le 3$ and ε_1=0.001 and ε_2 = 0.01 if $n \ge 4$. Where ε_1 and ε_2 choose for set of membership and non-membership degree in Intuitionistic fuzzy labeling.

The Intuitionistic fuzzy labeling for cycle is given as:

$\mu_A(v_{2k}) = (2n + 1 - k)\,\varepsilon_1$, $1 \le k \le \frac{n-1}{2}$

$\gamma_A(v_{2k})$ = (2n + 1 – k) ε_2 , $1 \le k \le \frac{n-1}{2}$
$\mu_A(v_{2k-1})$ = min $\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – kε_1 , $1 \le k \le \frac{n+1}{2}$
$\gamma_A(v_{2k-1})$ = min $\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – kε_2 , $1 \le k \le \frac{n+1}{2}$
$\mu_B(v_1, v_n) = \frac{1}{2}$ max $\{\mu_A(v_k) \mid 1 \le k \le n\}$
$\gamma_B(v_1, v_n) = \frac{1}{2}$ max $\{\gamma_A(v_k) \mid 1 \le k \le n\}$
$\mu_B(v_{n-k+1}, v_{n-k}) = \mu_B(v_1, v_n)$ – kε_1 , $1 \le k \le n-1$
$\gamma_B(v_{n-k+1}, v_{n-k}) = \gamma_B(v_1, v_n)$ – kε_2 , $1 \le k \le n-1$.
Case (i) k is even.
Then k = 2a, where a∈ Z^+ and for each edge v_k, v_{k+1}.
$m_\mu(C_n)$ = $\mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1})$
= $\mu_A(v_{2a}) + \mu_B(v_{2a}, v_{2a+1}) + \mu_A(v_{2a+1})$
= (2n + 1 – a) ε_1 + $\frac{1}{2}$ max $\{\mu_A(v_k) \mid 1 \le k \le n\}$ – (n – 2a)ε_1
+ min $\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – (a + 1)ε_1
= $\frac{1}{2}$ max $\{\mu_A(v_k) \mid 1 \le k \le n\}$ + min $\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ + nε_1
$m_\gamma(C_n)$ = $\gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
= $\gamma_A(v_{2a}) + \gamma_B(v_{2a}, v_{2a+1}) + \gamma_A(v_{2a+1})$
= (2n + 1 – a) ε_2 + $\frac{1}{2}$ max $\{\gamma_A(v_k) \mid 1 \le k \le n\}$ – (n – 2a)ε_2
+ min $\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – (a + 1)ε_2
= $\frac{1}{2}$ max $\{\gamma_A(v_k) \mid 1 \le k \le n\}$ + min $\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ + nε_2.
So that $M_0(C_n) = (m_\mu(C_n), m_\gamma(C_n))$.
Case (ii) k is odd.
Then k = 2a + 1, where a∈ Z^+ and for each edge v_k, v_{k+1}
$m_\mu(C_n)$ = $\mu_A(v_k) + \mu_B(v_k, v_{k+1}) + \mu_A(v_{k+1})$
= $\mu_A(v_{2a+1}) + \mu_B(v_{2a+1}, v_{2a+2}) + \mu_A(v_{2a+2})$
= min $\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – (a + 1)ε_1 + $\frac{1}{2}$ max $\{\mu_A(v_k) \mid 1 \le k \le n\}$
– (n – 2a - 1)ε_1 + (2n – a)ε_1
= $\frac{1}{2}$ max $\{\mu_A(v_k) \mid 1 \le k \le n\}$ + min $\{\mu_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ + nε_1
$m_\gamma(C_n)$ = $\gamma_A(v_k) + \gamma_B(v_k, v_{k+1}) + \gamma_A(v_{k+1})$
= $\gamma_A(v_{2a}) + \gamma_B(v_{2a}, v_{2a+1}) + \gamma_A(v_{2a+1})$
= min $\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ – (a + 1)ε_2 + $\frac{1}{2}$ max $\{\gamma_A(v_k) \mid 1 \le k \le n\}$
– (n – 2a – 1)ε_2 + (2n – a)ε_2
= $\frac{1}{2}$ max $\{\gamma_A(v_k) \mid 1 \le k \le n\}$ + min $\{\gamma_A(v_{2k}) \mid 1 \le k \le \frac{n-1}{2}\}$ + nε_2.
Hence $M_0(C_n) = (m_\mu(C_n), m_\gamma(C_n))$.
The magic value $M_0(C_n)$ is same and unique in above cases. Thus C_n is an Intuitionistic fuzzy magic graph.
Example 3.6:- An example of an Intuitionistic fuzzy magic labeling cycle with five nodes and five edges in figure 2.

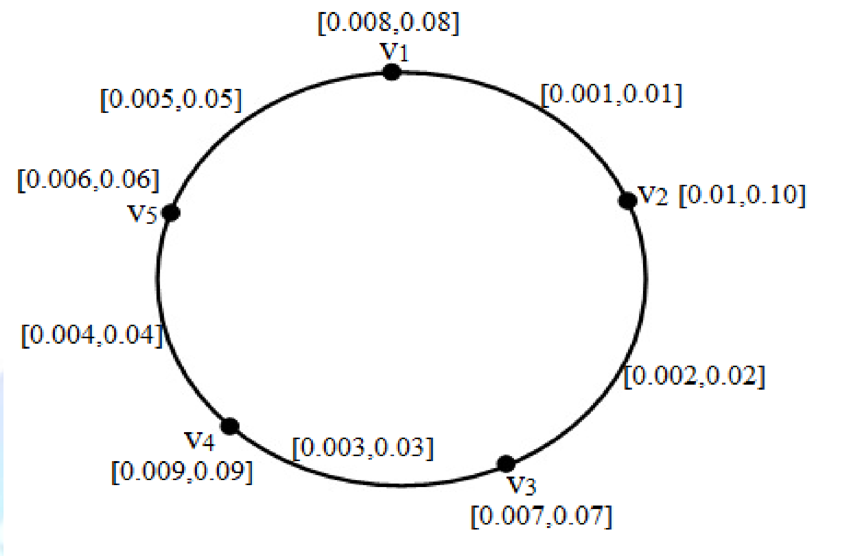

Figure 2

Every pair of vertices the magic value is same

$m_{\mu}(C_n)$ = μ_A(x) + μ_B(x, y) + μ_A(y) = 0.008+0.001+0.010 = 0.019

$m_{\gamma}(C_n)$ = γ_A(x) + γ_B(x, y) + γ_A(y) = 0.08+0.01+0.10 = 0.19

$M_0(C_n)$ = ($m_{\mu}(C_n)$, $m_{\gamma}(C_n)$) = (0.019,0.19).

Theorem 3.7:- For any n ≥ 2, Star graph $S_{1,n}$ is an Intuitionistic fuzzy magic graph.

Proof:- Let $S_{1,n}$ be a star graph having v, $u_1, u_1, u_2, u_3,\ldots, u_n$ as nodes and vu_1, vu_2, vu_3,…, vu_n as edges. Let $\varepsilon_1, \varepsilon_2 \in [0,1]$ such that we choose ε_1=0.01 and ε_2 = 0.1 if n ≤ 4 and ε_1=0.001 and ε_2 = 0.01 if n ≥ 5. Where ε_1 and ε_2 choose for set of membership and non-membership degree in Intuitionistic fuzzy labeling.

Such an Intuitionistic fuzzy labeling is given as:-

$\mu_A(u_k)$ = [2(n + 1) – k]ε_1 , 1 ≤ k ≤ n

$\gamma_A(u_k)$ = [2(n + 1) – k]ε_2 , 1 ≤ k ≤ n

μ_A(v) = min {$\mu_A(u_k)$ | 1 ≤ k ≤ n} – ε_1

γ_A(v) = min {$\gamma_A(u_k)$ | 1 ≤ k ≤ n} – ε_2

μ_B(v, u_{n-k}) = max {$\mu_A(v_k)$,μ_A(v) | 1 ≤ k ≤ n}– min {$\mu_A(v_k)$,μ_A(v) |1 ≤ k ≤ n}– kε_1 ,
0 ≤ k ≤ n – 1

γ_B(v, u_{n-k}) = max {$\gamma_A(v_k)$,γ_A(v) | 1 ≤ k ≤ n}– min {$\gamma_A(v_k)$,γ_A(v) |1 ≤ k ≤ n}– kε_2 ,
0 ≤ k ≤ n – 1

Case (i) k is even.

Then k = 2a, where a∈ Z^+ and for each edge v, u_k.

$m_{\mu}(S_{1,n})$ = μ_A(v) + μ_B(v,u_k) + $\mu_A(u_k)$
= μ_A(v) + μ_B(v,u_{2a}) + $\mu_A(u_{2a})$
= min {$\mu_A(u_k)$ | 1 ≤ k ≤ n} – ε_1+ max {$\mu_A(v_k)$,μ_A(v) | 1 ≤ k ≤ n}
– min {$\mu_A(v_k)$,μ_A(v) |1 ≤ k ≤ n}– (n – 2a)ε_1 + [2(n + 1) – 2a]ε_1
= min {$\mu_A(u_k)$ | 1 ≤ k ≤ n} + max {$\mu_A(v_k)$,μ_A(v) | 1 ≤ k ≤ n}
– min {$\mu_A(v_k)$,μ_A(v) |1 ≤ k ≤ n} + (n + 1)ε_1

$m_{\gamma}(S_{1,n})$ = γ_A(v) + γ_B(v,u_k) + $\gamma_A(u_k)$
= γ_A(v) + γ_B(v,u_{2a}) + $\gamma_A(u_{2a})$
= min {$\gamma_A(u_k)$ | 1 ≤ k ≤ n} – ε_2+ max {$\gamma_A(v_k)$,γ_A(v) | 1 ≤ k ≤ n}
– min {$\gamma_A(v_k)$,γ_A(v)| 1 ≤ k ≤ n}– (n – 2a)ε_2 + [2(n + 1) – 2a]ε_2

$= \min\{\gamma_A(u_k) \mid 1 \le k \le n\} + \max\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\}$
$- \min\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\} + (n+1)\varepsilon_2$.

So that $M_0(S_{1,n}) = (m_\mu(S_{1,n}), m_\gamma(S_{1,n}))$.

Case (i) k is odd.

Then k = 2a + 1, where a∈ Z^+ and for each edge v, u_k.

$m_\mu(S_{1,n}) = \mu_A(v) + \mu_B(v, u_k) + \mu_A(u_k)$
$= \mu_A(v) + \mu_B(v, u_{2a+1}) + \mu_A(u_{2a+1})$
$= \min\{\mu_A(u_k) \mid 1 \le k \le n\} - \varepsilon_1 + \max\{\mu_A(v_k), \mu_A(v) \mid 1 \le k \le n\}$
$- \min\{\mu_A(v_k), \mu_A(v) \mid 1 \le k \le n\} - (n - 2a - 1)\varepsilon_1 + [2(n+1) - 2a + 1]\varepsilon_1$
$= \min\{\mu_A(u_k) \mid 1 \le k \le n\} + \max\{\mu_A(v_k), \mu_A(v) \mid 1 \le k \le n\}$
$- \min\{\mu_A(v_k), \mu_A(v) \mid 1 \le k \le n\} + (n+1)\varepsilon_1$

$m_\gamma(S_{1,n}) = \gamma_A(v) + \gamma_B(v, u_k) + \gamma_A(u_k)$
$= \gamma_A(v) + \gamma_B(v, u_{2a+1}) + \gamma_A(u_{2a+1})$
$= \min\{\gamma_A(u_k) \mid 1 \le k \le n\} - \varepsilon_2 + \max\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\}$
$- \min\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\} - (n - 2a - 1)\varepsilon_2 + [2(n+1) - 2a - 1]\varepsilon_2$
$= \min\{\gamma_A(u_k) \mid 1 \le k \le n\} + \max\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\}$
$- \min\{\gamma_A(v_k), \gamma_A(v) \mid 1 \le k \le n\} + (n+1)\varepsilon_2$.

So that $M_0(S_{1,n}) = (m_\mu(S_{1,n}), m_\gamma(S_{1,n}))$.

The magic value is same and unique in above cases. Thus star graphs are Intuitionistic fuzzy magic graph.

Example 3.7:- An example of an Intuitionistic fuzzy magic labeling Star graph $S_{1,4}$

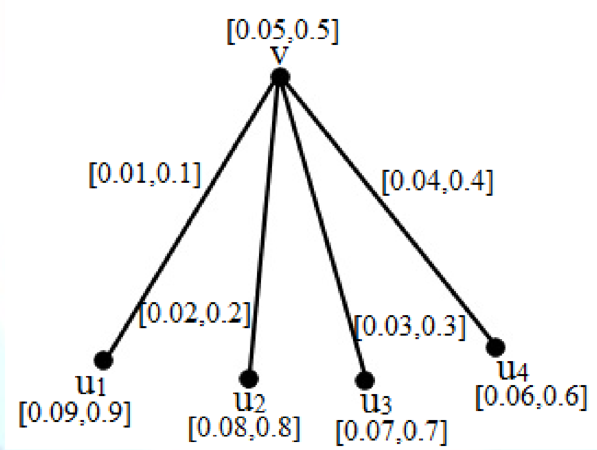

Figure 3

Every pair of vertices the magic value is same

$m_\mu(S_{1,n}) = \mu_A(x) + \mu_B(x, y) + \mu_A(y) = 0.05+0.01+0.09 = 0.15$

$m_\gamma(S_{1,n}) = \gamma_A(x) + \gamma_B(x, y) + \gamma_A(y) = 0.5+0.1+0.9 = 1.5$

$M_0(S_{1,n}) = (m_\mu(S_{1,n}), m_\gamma(S_{1,n})) = (0.15, 1.5)$.

4. CONCLUSION

In this work, we introduced the Intuitionsitic fuzzy labeling graph and Intuitionsitic fuzzy magic labeling graphs. it has been growing fast and has numerous applications in various fields. Further, research on Intuitionsitic fuzzy labeling graph has been witnessing an exponential growth; both within mathematics and its applications in science and Technology. In future we can extend this concept to Neutrosophic graphs and also many properties could be derived.

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