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Equitable Domination in Fuzzy Soft graphs

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Abstract:

*Let $G_{A,V}$ be a fuzzy soft graph. $V=\{x_1,x_2,\cdots x_n\}$ (non-empty set), $E=\{e_1,e_2,\cdots e_n\}$ (parameters set) and let x_i and x_j be two vertices of $G_{A,V}$. A subset D of V is called a **fuzzy soft equitable dominating set** if every $x_j \in V-D,$ there exist a vertex $x_i \in D$ such that $x_i x_j \in E(G)$ and $|\deg(x_i)-\deg(x_j)|\le 1$ where $\deg(x_i)$ and $\deg(x_j)$ denote the degrees of vertices of x_i and x_j respectively and $\mu_e(x_i,x_j)\le \rho_e(x_i)\wedge \rho_e(x_j)$ for each parameter $e\in A$ and $\forall i, j=1,2,3\cdots n$.*

In this paper we introduce the concept of fuzzy soft equitable dominating set, minimal fuzzy soft equitable dominating set, strong(weak) fuzzy soft equitable dominating set, fuzzy soft equitable independent set.

Key words:- Fuzzy soft equitable dominating set, fuzzy soft equitable neighborhood, fuzzy soft equitable independent set.

1.Introduction

The concept of soft set theory was first introduced by D Molodtsov to solve imprecise problems in different subjects like Economics Engineering and Environment. There are many theories can be considered as mathematical tools to deal with uncertainties and all those theories have their own inherent difficulties. Out of these theories soft set theory is free from inherent difficulties and which is a combination of fuzzy set and soft set. In fact, the notion of fuzzy soft set is more generalized than that of fuzzy set and soft set The concept of equitable domination in fuzzy graphs was introduced by K M Dharmalingam and M Rani[4] and the notion of domination in fuzzy graphs was developed by A somasundaram and S somasundaram[8]. In this paper we introduce the new concepts in equitable domination in fuzzy soft graphs.

2. Preliminaries

Definition 2.1[8] A fuzzy graph $G=(\rho,\mu)$ is a set with two functions $\rho:V\to[0,1]$ and $\mu:V\times V\to[0,1]$ such that $\mu(x,y)\le\rho(x)\wedge\rho(y)\ \forall x,y\in V.$

Definition 2.2[8] The order p and q of a fuzzy graph are defined to be $p=\sum_{x\in V}\rho(x)$ and $q=\sum_{xy\in E}\mu(x,y).$

Definition 2.3 [4] Let G be a fuzzy graph. Let x and y be the two values of G. A Subset D of V is called a fuzzy equitable dominating set if for every $y \in V - D$ there exist a vertex $x \in D$ such that $xy \in E(G)$ and $|\deg(x) - \deg(y)| \leq 1$ & $\mu(x, y) \leq \rho(x) \wedge \rho(y)$.

The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{ef}.

Let U be an initial universal set and E be a set of parameters. Let I^U denotes the collection of all fuzzy subset of U and $A \subseteq E$.

Definition 2.4[7] Let $A \subseteq E$. Then the mapping $F_A : E \to I^U$ defined by $F_A(e) = \mu^e_{F_A}$ (a fuzzy subset of U) is called fuzzy soft set over (U, E) where $\mu^e_{F_A} = \varphi \; if \; e \notin A$ & $\mu^e_{F_A} \neq \varphi \; if \; e \in A$ and φ denotes the null fuzzy set.

The set of all fuzzy soft sets over (U, E) is denoted by $FS(U, E)$.

Definition 2.5[7] Let $V = \{x_1, x_2, x_3, \cdots x_n\}$ (non empty set) E (parameters) and $A \subseteq E$. Also let

(i) $\rho : A \to F(V)$, collection of all fuzzy subsets in V and each element e of A is mapped to $\rho(e) = \rho_e$ (say) and $\rho_e : V \to [0, 1]$, each element x_i is mapped to $\rho_e(x_i)$ and we call (A, ρ), a fuzzy soft vertex.

(ii) $\mu : A \to F(V \times V)$, collection of all fuzzy subsets in $V \times V$, which mapped each element e to $\mu(e) = \mu_e$ (say) and $\mu_e : V \times V \to [0,1]$, which mapped each element (x_i, x_j) to and we call (A, μ) as a fuzzy soft edge.

Then $((A, \rho), (A, \mu))$, is called fuzzy soft graph if and only if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j) \; \forall e \in A$ and $\forall i, j = 1,2,3, \cdots n$, this fuzzy soft graph is denoted by $G_{A,V}$.

3.Definition and main results

Definition 3.1 Let $G_{A,V}$ be a fuzzy soft graph. $V = \{x_1, x_2, \cdots x_n\}$ (non-empty set), $E = \{e_1, e_2, \cdots e_n\}$ (parameters set) and let x_i and x_j be two vertices of $G_{A,V}$. A subset D of V is called a ***fuzzy soft equitable dominating set*** if every $x_j \in V - D$, there exist a vertex $x_i \in D$ such that $x_i x_j \in E(G)$ and $|\deg(x_i) - \deg(x_j)| \leq 1$ where $\deg(x_i)$ and $\deg(x_j)$ denote the degrees of vertices of x_i and x_j respectively and $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $\forall i, j = 1,2,3 \cdots n$.

The minimum cardinality of a fuzzy soft equitable dominating set is denoted by γ^{efs}.

Definition 3.2 a vertex $x_i \in V$ is said to be ***degree equitable fuzzy soft graph*** with a vertex $x_i \in V$ if $|\deg(x_i) - \deg(x_j)| \leq 1$ and $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $\forall i, j = 1,2,3 \cdots n$.

Remark If D is a fuzzy soft equitable dominating set then any super set of D is a fuzzy soft equitable dominating set.

Definition 3.3 A fuzzy soft equitable dominating set D is said to be a ***minimal fuzzy soft equitable dominating set*** if no proper sub set of D is a fuzzy soft equitable dominating set.

Remark If a vertex $x_i \in V$ be such that $|\deg(x_i) - \deg(x_j)| \geq 2$ and if $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ $\forall x_j \in V - \{x_i\}$ then x_i is in every equitable dominating set. Such points are called equitable isolates, and I_e denote the set of all isolates.

Definition 3.4 Let $x_i \in V$. The ***fuzzy soft equitable neighbourhood*** of x_i denoted by $N^{efs}(x_i)$ is defined as $N^{efs}(x_i) = \{x_j \in V \mid x_j \in N(x_i), |\deg(x_i) - \deg(x_j)| \leq 1$ and $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)\}$ $\forall e \in A \,\&\, \forall i, j = 1,2,3, \cdots n$.

Theorem 3.5 A dominating set D is a minimal fuzzy soft equitable dominating set if and only if for each $x_i \in D,\ i = 1,2,3, \cdots n$ one of the following two conditions holds

(i) $N^{efs}(x_i) \cap D = \phi.$

(ii) There is a vertex $x_t \in V - D$ such that $N^{efs}(x_t) \cap D = \{x_i\}$.

Proof:-

Let D be a minimal fuzzy soft equitable dominating set and $x_i \in D$. Then $D_{\{x_i\}} = D - \{x_i\}$ is not a fuzzy soft equitable dominating set and hence $\exists x_j \in V - D_{\{x_i\}}$ such that x_j is not dominated by any element of $D_{\{x_i\}} = D - \{x_i\}$ If $x_j = x_i$ then $N^{efs}(x_i) \cap D = \phi.$ and if $x_j \neq x_i$, since $x_i \in D$, there exist a vertex $x_t \in V - D$ such that $N^{efs}(x_t) \cap D = \{x_i\}$.

Conversely suppose for every $x_i \in D$, one of the statements (i) or (ii) holds. Suppose D is not a minimal fuzzy soft equitable dominating set. Then there exist $x_i \in D$ such that $D - \{x_i\}$ is a fuzzy soft equitable dominating set. Since every fuzzy soft equitable dominating set is a degree equitable dominating set, there exists $x_j \in D - \{x_i\}$ such that x_j is equitably dominates x_i.

That is $x_j \in N^{efs}(x_i)$ and $|\deg(x_i) - \deg(x_j)| \leq 1.$

There fore x_i does not satisfy (i). Then x_i must satisfy (ii). Then there exist $x_j \in V - D$ such that $N^{efs}(x_j) \cap D = \{x_i\}$ and $|\deg(x_i) - \deg(x_j)| \leq 1$. ,since $D - \{x_i\}$ is a degree equitable dominating set, there exist $x_w \in D - \{x_i\}$ such that x_w is adjacent to x_j and is degree x_w equitable with x_j.

There fore $x_w \in N^{efs}(x_j) \cap D,$ $|\deg(x_w) - \deg(x_j)| \leq 1$ and $x_w \neq x_i$, a contradiction to $N^{efs}(x_j) \cap D = \{x_i\}$ $\therefore D$ is a minimal fuzzy soft equitable dominating set.

Remark The cardinality of $N^{efs}(x_i)$ is called fuzzy soft equitable degree of x_i and it is denoted by $d_G{}^{efs}(x_i)$.

Definition 3.6 The maximum and minimum ***fuzzy soft equitable degree*** of a vertex in G are denoted by $\Delta^{efs}(G) \;\&\; \delta^{efs}(G)$.

That is $\Delta^{efs}(G) = \max_{x_i \in V} |N^{efs}(x_i)|$ and $\delta^{efs}(G) = \min_{x_i \in V} |N^{efs}(x_i)|$.

Definition 3.7 Let G be a graph. Then $D \subseteq V$ is said to be a ***strong (weak) fuzzy soft equitable dominating set*** of G if every vertex $x_j \in V - D$ is strongly (weakly) dominated by some vertex u in D. we denote a strong (weak) fuzzy soft equitable dominating set by sefsd – set (wefsd-set).

The minimum cardinality of a sefsd-set (wefsd-set) is called the strong (weak) fuzzy soft equitable domination number of G and is denoted by $\gamma^{sefs}(G) \;\&\; \gamma^{wefs}(G)$.

Theorem 3.8 Let G be a fuzzy soft graph of order p then

(i) $\gamma^{efs}(G) \leq \gamma^{sefs}(G) \leq p - \Delta^{efs}(G)$

(ii) $\gamma^{efs}(G) \leq \gamma^{wefs}(G) \leq p - \delta^{efs}(G)$.

Proof: Every strong fuzzy soft equitable dominating set is a fuzzy soft equitable dominating set of G $\gamma^{efs}(G) \leq \gamma^{sefs}(G)$ and every weak fuzzy soft equitable dominating set is a fuzzy soft equitable dominating set, $\gamma^{efs}(G) \leq \gamma^{wefs}(G)$.

Let $x_i, x_j \in V$. If $d_G{}^{efs}(x_i) = \Delta^{efs}(G)$ and $d_G{}^{efs}(x_j) = \delta^{efs}(G)$, then clearly $V - N^{efs}(x_i)$ is a strong fuzzy soft equitable dominating set and $V - N^{efs}(x_j)$ is a weak fuzzy soft equitable dominating set, $\gamma^{sefs}(G) \leq \left|V - N^{efs}(x_i)\right|$ and $\gamma^{wefs}(G) \leq \left|V - N^{efs}(x_j)\right|$. That is $\gamma^{sefs}(G) \leq p - \Delta^{efs}(G)$ and $\gamma^{wefs}(G) \leq p - \delta^{efs}(G)$.

Theorem 3.9 Let G be a fuzzy soft graph with isolated vertices. Let D be a minimal fuzzy soft equitable dominating set of G .Then $V \mid D$ is a fuzzy soft equitable dominating set of G

Proof: Let x_i be any vertex in D. since G has no isolated vertices, there is a vertex $x_j \in N(x_i)$. By theorem 3.5 there is $x_j \in V \mid D$. Thus every element of D is dominated by some element of $V \mid D$.

4. Equitable domination in Soft Fuzzy Graphs

Definition 4.1 A set S of vertices of a soft fuzzy graph is said to be soft fuzzy equitable independent set, if for any $x_i \in S, x_j \notin N^e(x_i)$ for all $x_j \in S - \{x_i\}$ and $\mu_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$ for each parameter $e \in A$ and $\forall i, j = 1,2,3 \cdots n$ & $\forall\ x_i, x_j \in S.$.

Theorem 4.2 If D is a soft fuzzy equitable independent dominating set of a graph G then D is both minimal soft fuzzy equitable dominating set and a maximal soft fuzzy equitable independent set.

Proof : Suppose that D is a soft fuzzy equitable independent dominating set of a graph G. Then $D_{\{x_i\}} = D - \{x_i\}$ is not a soft fuzzy equitable dominating set for every $x_i \in D$ and $D \cup \{x_j\}$ is not a soft fuzzy equitable independent set for every $x_j \notin D$. So D is a minimal soft fuzzy equitable dominating set and a maximal soft fuzzy equitable independent set.

Remark The maximum cardinality of a soft fuzzy equitable independent set is denoted by β^{efs}.

Theorem 4.3 Let G be a soft fuzzy graph. Then $\gamma^{efs}(G) \leq \beta^{efs}(G)$.

Proof : Let S be a soft fuzzy equitable independent set of vertices in G such that $|S| = \beta^{efs}(G)$. Then no large soft fuzzy equitable independent set is contained in G. Thus every vertex x_i in $V - S$ is adjacent to at least one vertex of S. Therefore S is a soft fuzzy equitable dominating set. Thus $\gamma^{efs}(G) \leq |S|$. But $|S| = \beta^{efs}(G)$. Hence $\gamma^{efs}(G) \leq \beta^{efs}(G)$.

Theorem 4.4 A soft fuzzy equitable independent set S is maximal soft fuzzy equitable independent set if and only if it is a soft fuzzy equitable dominating set.

Proof : Suppose that a soft fuzzy equitable independent set S is maximal soft fuzzy equitable independent set. Then for every vertex $x_i \in V - S$, the set $S \cup \{x_i\}$ is not soft fuzzy equitable independent set, that is for every vertex $x_i \in V - S$, there is a vertex x_j such that x_i is adjacent to x_j . Thus S is a soft fuzzy equitable dominating set. So S is both soft fuzzy equitable independent and soft fuzzy equitable dominating set.

Conversely suppose that a set S is both soft fuzzy equitable independent and soft fuzzy equitable dominating set. We have to prove that it is a maximal soft fuzzy equitable independent set. Suppose on the contrary that S is not maximal soft fuzzy equitable independent set. Then there exist a vertex x_i in $V - S$ such that $S \cup \{x_i\}$ is a soft fuzzy equitable independent set. Thus no vertex in S is adjacent to x_i . Hence S is not a soft fuzzy equitable dominating set, which is a contradiction. Therefore S is maximal soft fuzzy equitable independent set.

5.Conclusion

The soft fuzzy equitable domination and soft fuzzy equitable independent are very useful for solving wide range of problems. In this paper we have introduced the concept of soft fuzzy equitable domination and soft fuzzy equitable independence.

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