

A FUZZY PRODUCTION INVENTORY MODEL WITH DETERIORATION RATE USING REGULAR WEIGHTED POINT TECHNIQUE

K.Dhanam¹

and

R.Dhanalakshmi²

¹Associate professor and ²Research scholar
Department of Mathematics,
Government Arts College for Women (Autonomous), Pudukkottai-622 001,
Tamilnadu, india.

Abstract:

This paper deals, a fuzzy production inventory model with two warehouse have been considered. In this model the deterioration rate and demand rate considered as increasing function of time and the production rate depends directly on demand rate. The lotus petal fuzzy number is defined and its properties are given. The parameters involved in this model are represented by lotus petal fuzzy number. The average total cost is defuzzified by the regular weighted point technique. The analytical expressions for temporary warehouse inventory level, permanent warehouse inventory level at time t_2 , deterioration level, maximum inventory level and the average total cost are derived for the proposed model by using non-linear programming technique. A numerical example is presented to illustrate the results.

Keywords: lotus petal fuzzy number, regular weighted point technique.

AMS Subject Classification (2010): 90B05.

1. Introduction:

Inventory problems are common in manufacturing, service and business operations in general. Some inventory models were formulated in a static environment where the demand is assumed to be constant and steady over a finite planning horizon. Many items of inventory such as electronic products, fashionable clothes, tasty food products etc., generate increasing sales after gaining consumer's acceptance. The sale for the other products may decline drastically due to the introduction of more competitive products or due to the change of consumer's preference. Therefore the demand of the product during its growth and decline phases can be taken as continuous time dependent function such as non-linear.

Most of the existing inventory models in the literature assume that items can be stored indefinitely to meet the future demands. However, certain types of commodities either deteriorate or become obsolete in the course of time and hence are unstable. Therefore, if the rate of deterioration is not sufficiently low, its impact on modelling of such an inventory model cannot be ignored. In this connections, inventory problems for deteriorating items have been studied extensively by many researchers [1],[4],[5] and [7] from time to time. Research in this area started with the work of Whitin [8], who considered fashion goods deteriorating at the end of prescribed storage period. Goyal and Giri [3] gave recent trends of

modelling in deteriorating item inventory. Samantha and Ajanta roy work based on realistic production lot-size inventory model for deteriorating items is given in [6].

In conventional inventory models, uncertainties are treated as randomness and are handled by probability theory. Furthermore, when addressing real world problems, frequently the parameters are imprecise numerical quantities. However, in certain situations, uncertainties are due to fuzziness and in such cases the fuzzy set theory introduced by Zadeh [9] is applicable. A. Faritha Asma and E.C. Henry Amirtharaj analyzed multi objective inventory model of deteriorating items with two constraints using fuzzy optimization technique [2].

In the existing inventory models, they used only rented warehouse and own warehouse. When using the rented warehouse the transportation cost, rental and maintenance cost are all high. Here, the model newly constructed with permanent warehouse and temporary warehouse.

As warehouse is a key factor in production inventory model, so these days researcher are paying more rent in stored the goods. The temporary warehouse used for reducing the rented amount and maintenance cost.

2. Assumptions and Notations:

The following assumptions and notations are used throughout this paper:

Assumptions:

1. Demand rate is time dependent which is taken as a increasing function that is $d = \alpha t^\beta$ $0 < \alpha < j$, $0 < \beta < k$, j and k are finite number.
2. Production rate is demand dependent that is $p = cd$, $0 < c < n$, n is finite number.
3. Deterioration rate is time dependent which is taken as a increasing function that is $\theta = \gamma t^\delta$ $0 < \gamma < 1$, $0 < \delta < 1$.
4. First, the goods are stored in permanent warehouse then the remaining goods are stored in temporary warehouse.
5. The goods of permanent warehouse are consumed only after consuming the goods kept in temporary warehouse.
6. The permanent warehouse has a fixed capacity of I_p units.
7. The inventory level depleted due to demand and deterioration.
8. Shortages are not allowed.

Notations:

$I_1(t)$ – the level of inventory at time t , $0 \leq t \leq t_1$.

$I_2(t)$ – the level of inventory in temporary warehouse at time t , $t_1 \leq t \leq t_2$.

$I_3(t)$ – the level of inventory in permanent warehouse at time t , $t_1 \leq t \leq t_3$.

I_m – maximum inventory level at time t_1 . (decision variable)

- I_P – fixed capacity for permanent warehouse at time t_1 .
- I_T – maximum inventory level of temporary warehouse at time t_1 .(decision variable)
- I_{P1} – inventory level of permanent warehouse at time t_2 . (decision variable)
- p – production rate.
- d – demand rate.
- θ - deterioration rate.
- θ_L - deterioration level. (decision variable)
- h_c – holding cost per unit per unit time.
- S_c – setup cost per cycle.
- T_{ws} – total space for temporary warehouse
- s – space per unit for temporary warehouse.
- T_{wc} – cost for 1 sqft of temporary warehouse.
- TC – average total cost per cycle.
- \tilde{S}_c - fuzzy setup cost per cycle.
- \tilde{h}_c - fuzzy holding cost per unit per unit time.
- \tilde{d}_c - fuzzy deteriorating cost per unit per unit time.
- \tilde{T}_{wc} - fuzzy cost for 1 square feet of temporary warehouse.
- \tilde{T}_{ws} - fuzzy total space for temporary warehouse.

3. Mathematical Formulation:

The proposed inventory model is formulated to minimize the average total cost, which includes setup cost, holding cost, deterioration cost and temporary warehouse cost.

The rate of change of the inventory during the following periods are governed by the following differential equations

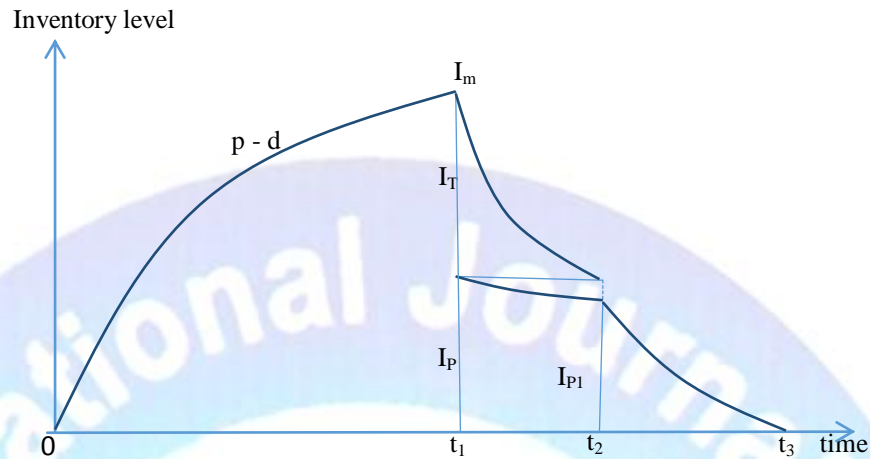


Figure 3.1

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = p - d \quad 0 \leq t \leq t_1 \quad \text{---- (1)}$$

with boundary condition $I_1(t) = 0$ at $t = 0$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \quad t_1 \leq t \leq t_2 \quad \text{---- (2)}$$

with boundary conditions $I_2(t) = 0$ at $t = t_2$ and $I_2(t) = I_T$ at $t = t_1$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = 0 \quad t_1 \leq t \leq t_2 \quad \text{---- (3)}$$

with boundary conditions $I_3(t) = I_P$ at $t = t_1$ and $I_3(t) = I_{P1}$ at $t = t_2$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -d \quad t_2 \leq t \leq t_3 \quad \text{---- (4)}$$

with boundary conditions $I_3(t) = 0$ at $t = t_3$

from (1)

$$I_1(t) = (c-1) \left(\frac{\alpha t^{\beta+1}}{\beta+1} + \frac{\alpha \gamma}{\delta+1} \frac{t^{\delta+\beta+2}}{\delta+\beta+2} + \alpha \left(\frac{\gamma}{\delta+1} \right)^2 \frac{t^{2(\delta+1)+\beta+1}}{2(\delta+1)+\beta+1} \right) e^{-\gamma \frac{t^{\delta+1}}{\delta+1}} \quad \text{---- (5)}$$

from(2)

$$I_T = \left(\frac{1}{\beta+1} (t_2^{\beta+1} - t_1^{\beta+1}) + \frac{\gamma}{(\delta+1)(\delta+\beta+2)} (t_2^{\delta+\beta+2} - t_1^{\delta+\beta+2}) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} (t_2^{2(\delta+1)+\beta+1} - t_1^{2(\delta+1)+\beta+1}) \right) \alpha e^{-\gamma \frac{t_1^{\delta+1}}{\delta+1}} \quad \text{---- (6)}$$

from (3) $I_{P1} = I_P e^{\frac{\gamma}{\delta+1}(t_1^{\delta+1} - t_2^{\delta+1})} \quad \text{---- (7)}$

from (4)

$$I_3(t) = \left(\frac{1}{\beta+1} (t_3^{\beta+1} - t^{\beta+1}) + \frac{\gamma}{(\delta+1)(\delta+\beta+2)} (t_3^{\delta+\beta+2} - t^{\delta+\beta+2}) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} (t_3^{2(\delta+1)+\beta+1} - t^{2(\delta+1)+\beta+1}) \right) \alpha e^{-\gamma \frac{t^{\delta+1}}{\delta+1}} \quad \text{---- (8)}$$

The maximum inventory level per cycle is $I_m = I_T + I_P \quad \text{---- (9)}$

Deterioration level $(\theta_L) = \int_0^{t_3} \gamma t^\delta dt \Rightarrow \frac{\gamma t_3^{\delta+1}}{\delta+1} \quad \text{---- (10)}$

$$\text{Average total cost} = \frac{1}{t_3} \left[\text{setup cost} + \text{holding cost} + \text{deterioration cost} + \text{temporary warehouse cost} \right]$$

where,

Setup cost = S_c

$$\text{Holding cost} = h_c \left\{ \int_0^{t_1} I_1(t) dt + \int_{t_1}^{t_2} (I_2(t) + I_3(t)) dt + \int_{t_2}^{t_3} I_3(t) dt \right\}$$

$$\begin{aligned} \Rightarrow h_c \left\{ \alpha c \left[\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+2)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right. \right. \\ - \left. \left(\frac{\gamma}{(\delta+1)(\beta+1)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(\frac{t_1^{2\delta+\beta+4}}{(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \right. \\ \left. \frac{t_1^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \frac{t_1^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} + \left(\frac{\gamma}{\delta+1} \right)^4 \right. \\ \left. \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right] + \alpha \left[\frac{1}{\beta+2} (t_2^{\beta+2} + t_3^{\beta+2}) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+2} + t_3^{\delta+\beta+2}) \right. \\ + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+2} (t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}) - \left(\frac{\gamma}{(\delta+1)(\delta+2)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) - \left(\frac{\gamma}{\delta+1} \right)^2 \\ \left(\frac{t_2^{2\delta+\beta+4} + t_3^{2\delta+\beta+4}}{(\delta+2)(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{3(\delta+1)+\beta+2} + t_3^{3(\delta+1)+\beta+2}}{(\delta+2)(3(\delta+1)+\beta+2)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \left(\frac{t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}}{(2(\delta+1)+1)(2(\delta+1)+\beta+2)} \right) \\ \left. + \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{2(\delta+1)+\delta+\beta+3} + t_3^{2(\delta+1)+\delta+\beta+3}}{(2(\delta+1)+1)(2(\delta+1)+\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^4 \left(\frac{t_2^{4(\delta+1)+\beta+2} + t_3^{4(\delta+1)+\beta+2}}{(2(\delta+1)+1)(4(\delta+1)+\beta+2)} \right) \right] \\ + \alpha \left[\frac{-1}{(\beta+1)} \left(t_2^{\beta+1} t_1 + t_3^{\beta+1} t_2 - \frac{t_2^{\beta+2}}{\beta+2} \right) - \frac{\gamma}{(\delta+1)(\delta+\beta+2)} \left(t_2^{\delta+\beta+2} t_1 + t_3^{\delta+\beta+2} t_2 - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\ \left. \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} t_1 + t_3^{2(\delta+1)+\beta+1} t_2 - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) + \frac{\gamma}{(\delta+1)(\beta+1)} \right. \\ \left(t_2^{\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(t_2^{\delta+\beta+2} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\delta+\beta+2} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{2\delta+\beta+4}}{2\delta+\beta+4} \right) \\ + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \\ \left(t_2^{\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \times \\ \left(t_2^{\delta+\beta+2} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\delta+\beta+2} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^4 \frac{1}{2(\delta+1)+\beta+1} \\ \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right) \left. + I_p \left[t_2 + \frac{\gamma}{\delta+1} \left(t_1^{\delta+1} t_2 - \frac{t_2^{\delta+2}}{\delta+2} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \right. \right. \\ \left. \left(t_1^{2(\delta+1)} t_2 + \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 t_1^{\delta+1} \frac{t_2^{\delta+2}}{\delta+2} \right) - t_1 - \frac{\gamma}{\delta+2} t_1^{\delta+2} - \left(\frac{\gamma}{\delta+1} \right)^2 \left(t_1^{2(\delta+1)+1} + \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 \frac{t_1^{2\delta+3}}{\delta+2} \right) \right] \right\} \end{aligned}$$

$$\begin{aligned} \text{Deterioration cost} &= d_c \left\{ \int_0^{t_1} (p-d) dt - I_m + I_T - \int_{t_1}^{t_2} d dt + I_P - I_{P1} + I_{P1} - \int_{t_2}^{t_3} d dt \right\} \\ &= d_c \left\{ \alpha c \frac{t_1^{\beta+1}}{\beta+1} - \alpha \frac{t_3^{\beta+1}}{\beta+1} \right\} \end{aligned}$$

Temporary warehouse cost = $T_{wc} \times T_{ws}$

Min $TC(t_1, t_2, t_3)$

$$\begin{aligned}
 &= \frac{1}{t_3} \left[S_c + h_c \left\{ \alpha c \left[\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+2)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} \right. \right. \right. \\
 &\quad \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} - \left(\frac{\gamma}{(\delta+1)(\beta+1)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(\frac{t_1^{2\delta+\beta+4}}{(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \\
 &\quad \frac{t_1^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \frac{t_1^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} + \left(\frac{\gamma}{\delta+1} \right)^4 \\
 &\quad \left. \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right] + \alpha \left[\frac{1}{\beta+2} (t_2^{\beta+2} + t_3^{\beta+2}) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) \right. \\
 &\quad + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+2} (t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}) - \left(\frac{\gamma}{(\delta+1)(\delta+2)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) - \left(\frac{\gamma}{\delta+1} \right)^2 \\
 &\quad \left(\frac{t_2^{2\delta+\beta+4} + t_3^{2\delta+\beta+4}}{(\delta+2)(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{3(\delta+1)+\beta+2} + t_3^{3(\delta+1)+\beta+2}}{(\delta+2)(3(\delta+1)+\beta+2)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \left(\frac{t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}}{(2(\delta+1)+1)(2(\delta+1)+\beta+2)} \right) \\
 &\quad + \left. \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{2(\delta+1)+\delta+\beta+3} + t_3^{2(\delta+1)+\delta+\beta+3}}{(2(\delta+1)+1)(2(\delta+1)+\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^4 \left(\frac{t_2^{4(\delta+1)+\beta+2} + t_3^{4(\delta+1)+\beta+2}}{(2(\delta+1)+1)(4(\delta+1)+\beta+2)} \right) \right] \\
 &\quad + \alpha \left[\frac{-1}{(\beta+1)} \left(t_2^{\beta+1} t_1 + t_3^{\beta+1} t_2 - \frac{t_2^{\beta+2}}{\beta+2} \right) - \frac{\gamma}{(\delta+1)(\delta+\beta+2)} \left(t_2^{\delta+\beta+2} t_1 + t_3^{\delta+\beta+2} t_2 - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\
 &\quad \left. \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} t_1 + t_3^{2(\delta+1)+\beta+1} t_2 - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) + \frac{\gamma}{(\delta+1)(\beta+1)} \right. \\
 &\quad \left(t_2^{\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(t_2^{\delta+\beta+2} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\delta+\beta+2} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{2\delta+\beta+4}}{2\delta+\beta+4} \right) \\
 &\quad + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \\
 &\quad \left(t_2^{\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \\
 &\quad \left(t_2^{\delta+\beta+2} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\delta+\beta+2} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^4 \frac{1}{2(\delta+1)+\beta+1} \\
 &\quad \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right) \left. \right] + I_p \left[t_2 + \frac{\gamma}{\delta+1} \left(t_1^{\delta+1} t_2 - \frac{t_2^{\delta+2}}{\delta+2} \right) \right. \\
 &\quad + \left(\frac{\gamma}{\delta+1} \right)^2 \left(t_1^{2(\delta+1)} t_2 + \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 t_1^{\delta+1} \frac{t_2^{\delta+2}}{\delta+2} \right) - t_1 - \frac{\gamma}{\delta+2} t_1^{\delta+2} - \left(\frac{\gamma}{\delta+1} \right)^2 \\
 &\quad \left. \left(t_1^{2(\delta+1)+1} + \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 \frac{t_1^{2\delta+3}}{\delta+2} \right) \right] + d_c \left\{ \alpha c \frac{t_1^{\beta+1}}{\beta+1} - \alpha \frac{t_3^{\beta+1}}{\beta+1} \right\} + T_{wc} T_{ws} \Bigg]
 \end{aligned}$$

----- (11)

subject to $s I_T \leq T_{ws}$

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the equations, the values of t_1^* , t_2^* and t_3^* are obtained. Substituting t_1^* , t_2^* and t_3^*

in (6), (7), (9), (10) and (11) the optimum values I_T^* , I_{pl}^* , θ_L^* , I_m^* and TC^* are obtained.

4. Lotus petal fuzzy number and its Properties:

Definition: Lotus petal fuzzy number

A Lotus petal fuzzy number \tilde{A} described as a normalized convex fuzzy subset on the real line \mathbb{R} whose membership function $\mu_{\tilde{A}}(x)$ is defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2} & \text{at } x = a, c \\ \frac{1}{2} \left[1 + \sqrt{\frac{x-a}{b-a}} \right] & \text{at } a \leq x \leq b \\ \frac{1}{2} \left[1 + \sqrt{\frac{c-x}{c-b}} \right] & \text{at } b \leq x \leq c \\ \frac{1}{2} \left[\left(1 - \frac{c-x}{c-b} \right)^2 \right] & \text{at } c \geq x \geq b \\ \frac{1}{2} \left[\left(1 - \frac{x-a}{b-a} \right)^2 \right] & \text{at } b \geq x \geq a \\ 0 \& 1 & \text{at } x = b \end{cases}$$

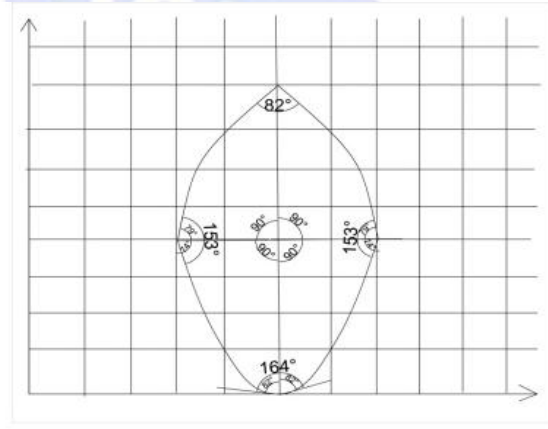


Figure 4.1

This type of fuzzy number be denoted as $\tilde{A} = [a, b, c]$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\tilde{A}}(x)$ is a continuous mapping from \mathbb{R} to the closed interval $[0,1]$
2. $\mu_{\tilde{A}}(x)$ is a convex function.
3. $\mu_{\tilde{A}}(x) = 0 \& 1$ at $x=b$.
4. $\mu_{\tilde{A}}(x) = \frac{1}{2}$ at $x= a \& c$.
5. $\mu_{\tilde{A}}(x)$ is strictly decreasing as well as increasing and continuous on $[a,b]$ and $[b,c]$.

Properties:

1. Left and right opposite angles are equal.
2. The horizontal and vertical diagonal bisect each other and meet at 90° .
3. The lower angle is twice that of the upper angle.

5. Regular weighted point of lotus petal fuzzy number:

For the Lotus petal fuzzy number $\tilde{A}=[a, b, c]$, the α -cut is $\tilde{A}_\alpha = [L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)]$ and the regular weighted point for \tilde{A} is given by,

$$r_w(\tilde{A}) = \frac{\int_0^1 \frac{L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)}{2} f(\alpha) d\alpha}{\int_0^1 f(\alpha) d\alpha} = \int_0^1 [L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)] f(\alpha) d\alpha$$

where,

$$f(\alpha) = \begin{cases} \omega(1-2\alpha) & \text{when } \alpha \in \left[0, \frac{1}{2}\right], \\ (1+\omega)(2\alpha-1) & \text{when } \alpha \in \left[\frac{1}{2}, 1\right] \end{cases}, 0 < \omega < 1.$$

The regular weighted point of a lotus petal fuzzy number is of the form

$$r_w(\tilde{A}) = \omega \left(\frac{31a + 58b + 31c}{120} \right) + \left(\frac{a + 2b + c}{8} \right)$$

6. Inventory Model in Fuzzy Environment:

The proposed inventory model in fuzzy environment is

$$\begin{aligned} & \text{Min } TC(t_1, t_2, t_3) \\ & = \frac{1}{t_3} \left[\tilde{S}_c + \tilde{h}_c \left\{ \alpha c \left[\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+2)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right. \right. \right. \\ & \quad \left. \left. \left. - \left(\frac{\gamma}{(\delta+1)(\beta+1)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(\frac{t_1^{2\delta+\beta+4}}{(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \right. \right. \right. \\ & \quad \left. \left. \left. + \frac{t_1^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \frac{t_1^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} + \left(\frac{\gamma}{\delta+1} \right)^4 \right. \right. \right. \\ & \quad \left. \left. \left. \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right] + \alpha \left[\frac{1}{\beta+2} (t_2^{\beta+2} + t_3^{\beta+2}) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) \right. \right. \right. \\ & \quad \left. \left. \left. + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+2} (t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}) - \left(\frac{\gamma}{(\delta+1)(\delta+2)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \right. \right. \\ & \quad \left. \left. \left. \left(\frac{t_2^{2\delta+\beta+4} + t_3^{2\delta+\beta+4}}{(\delta+2)(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{3(\delta+1)+\beta+2} + t_3^{3(\delta+1)+\beta+2}}{(\delta+2)(3(\delta+1)+\beta+2)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \left(\frac{t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}}{(2(\delta+1)+1)(2(\delta+1)+\beta+2)} \right) \right. \right. \right. \\ & \quad \left. \left. \left. + \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{2(\delta+1)+\delta+\beta+3} + t_3^{2(\delta+1)+\delta+\beta+3}}{(2(\delta+1)+1)(2(\delta+1)+\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^4 \left(\frac{t_2^{4(\delta+1)+\beta+2} + t_3^{4(\delta+1)+\beta+2}}{(2(\delta+1)+1)(4(\delta+1)+\beta+2)} \right) \right] \right. \right. \\ & \quad \left. \left. + \alpha \left[\frac{-1}{(\beta+1)} \left(t_2^{\beta+1} t_1 + t_3^{\beta+1} t_2 - \frac{t_2^{\beta+2}}{\beta+2} \right) - \frac{\gamma}{(\delta+1)(\delta+\beta+2)} \left(t_2^{\delta+\beta+2} t_1 + t_3^{\delta+\beta+2} t_2 - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} t_1 + t_3^{2(\delta+1)+\beta+1} t_2 - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) + \frac{\gamma}{(\delta+1)(\beta+1)} \\
 & \left(t_2^{\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(t_2^{\delta+\beta+2} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\delta+\beta+2} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{2\delta+\beta+4}}{2\delta+\beta+4} \right) \\
 & + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \\
 & \left(t_2^{\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \\
 & \left(t_2^{\delta+\beta+2} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\delta+\beta+2} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^4 \frac{1}{2(\delta+1)+\beta+1} \\
 & \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right) \Big] + I_P \left[t_2 + \frac{\gamma}{\delta+1} \left(t_1^{\delta+1} t_2 - \frac{t_2^{\delta+2}}{\delta+2} \right) \right. \\
 & \left. + \left(\frac{\gamma}{\delta+1} \right)^2 \left(t_1^{2(\delta+1)} t_2 + \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 t_1^{\delta+1} \frac{t_2^{\delta+2}}{\delta+2} \right) - t_1 - \frac{\gamma}{\delta+2} t_1^{\delta+2} - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\
 & \left. \left(t_1^{2(\delta+1)+1} + \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 \frac{t_1^{2\delta+3}}{\delta+2} \right) \right] + \tilde{d}_c \left\{ \alpha c \frac{t_1^{\beta+1}}{\beta+1} - \alpha \frac{t_3^{\beta+1}}{\beta+1} \right\} + \tilde{T}_{wc} \tilde{T}_{ws} \Big] \quad \text{----- (12)}
 \end{aligned}$$

subject to $s I_T \leq \tilde{T}_{ws}$

where \sim represents for fuzzification of the parameters.

$\tilde{S}_c = (S_{c1}, S_{c2}, S_{c3})$, $\tilde{h}_c = (h_{c1}, h_{c2}, h_{c3})$, $\tilde{d}_c = (d_{c1}, d_{c2}, d_{c3})$, $\tilde{T}_{wc} = (T_{wc1}, T_{wc2}, T_{wc3})$ and $\tilde{T}_{ws} = (T_{ws1}, T_{ws2}, T_{ws3})$.

Now using the regular weighted point of lotus petal fuzzy number, the above model is defuzzified as follows:

$$\text{Min } r_w(TC(t_1, t_2, t_3))$$

$$\begin{aligned}
 &= \frac{1}{t_3} \left[r_w(S_c) + r_w(h_c) \right] \left\{ \alpha c \left[\left(\frac{t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+2)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right. \right. \\
 &\quad \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} - \left(\frac{\gamma}{(\delta+1)(\beta+1)} \right) \left(\frac{t_1^{\delta+\beta+3}}{(\delta+\beta+3)} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(\frac{t_1^{2\delta+\beta+4}}{(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \\
 &\quad \frac{t_1^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \frac{t_1^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \frac{t_1^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} + \left(\frac{\gamma}{\delta+1} \right)^4 \\
 &\quad \left. \frac{1}{2(\delta+1)+\beta+1} \frac{t_1^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right] + \alpha \left[\frac{1}{\beta+2} (t_2^{\beta+2} + t_3^{\beta+2}) + \left(\frac{\gamma}{(\delta+1)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) \right. \\
 &\quad + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{2(\delta+1)+\beta+2} (t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}) - \left(\frac{\gamma}{(\delta+1)(\delta+2)(\delta+\beta+3)} \right) (t_2^{\delta+\beta+3} + t_3^{\delta+\beta+3}) - \left(\frac{\gamma}{\delta+1} \right)^2 \\
 &\quad \left(\frac{t_2^{2\delta+\beta+4} + t_3^{2\delta+\beta+4}}{(\delta+2)(2\delta+\beta+4)} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{3(\delta+1)+\beta+2} + t_3^{3(\delta+1)+\beta+2}}{(\delta+2)(3(\delta+1)+\beta+2)} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \left(\frac{t_2^{2(\delta+1)+\beta+2} + t_3^{2(\delta+1)+\beta+2}}{(2(\delta+1)+1)(2(\delta+1)+\beta+2)} \right) \\
 &\quad \left. + \left(\frac{\gamma}{\delta+1} \right)^3 \left(\frac{t_2^{2(\delta+1)+\delta+\beta+3} + t_3^{2(\delta+1)+\delta+\beta+3}}{(2(\delta+1)+1)(2(\delta+1)+\delta+\beta+3)} \right) + \left(\frac{\gamma}{\delta+1} \right)^4 \left(\frac{t_2^{4(\delta+1)+\beta+2} + t_3^{4(\delta+1)+\beta+2}}{(2(\delta+1)+1)(4(\delta+1)+\beta+2)} \right) \right] \\
 &\quad + \alpha \left[\frac{-1}{(\beta+1)} \left(t_2^{\beta+1} t_1 + t_3^{\beta+1} t_2 - \frac{t_2^{\beta+2}}{\beta+2} \right) - \frac{\gamma}{(\delta+1)(\delta+\beta+2)} \left(t_2^{\delta+\beta+2} t_1 + t_3^{\delta+\beta+2} t_2 - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\
 &\quad \frac{1}{2(\delta+1)+\beta+1} \left(t_2^{2(\delta+1)+\beta+1} t_1 + t_3^{2(\delta+1)+\beta+1} t_2 - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) + \frac{\gamma}{(\delta+1)(\beta+1)} \left(t_2^{\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{\delta+\beta+3}}{\delta+\beta+3} \right) \\
 &\quad + \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\delta+\beta+2} \left(t_2^{\delta+\beta+2} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{\delta+\beta+2} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{2\delta+\beta+4}}{2\delta+\beta+4} \right) + \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{2(\delta+1)+\beta+1} \\
 &\quad \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{\delta+2}}{\delta+2} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{\delta+2}}{\delta+2} - \frac{t_2^{3(\delta+1)+\beta+2}}{3(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^2 \frac{1}{\beta+1} \\
 &\quad \left(t_2^{\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\beta+2}}{2(\delta+1)+\beta+2} \right) - \left(\frac{\gamma}{\delta+1} \right)^3 \frac{1}{\delta+\beta+2} \\
 &\quad \left(t_2^{\delta+\beta+2} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{\delta+\beta+2} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{2(\delta+1)+\delta+\beta+3}}{2(\delta+1)+\delta+\beta+3} \right) - \left(\frac{\gamma}{\delta+1} \right)^4 \frac{1}{2(\delta+1)+\beta+1} \times \\
 &\quad \left(t_2^{2(\delta+1)+\beta+1} \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} + t_3^{2(\delta+1)+\beta+1} \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - \frac{t_2^{4(\delta+1)+\beta+2}}{4(\delta+1)+\beta+2} \right) + I_p \left[t_2 + \frac{\gamma}{\delta+1} \left(t_1^{\delta+1} t_2 - \frac{t_2^{\delta+2}}{\delta+2} \right) + \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\
 &\quad \left. \left(t_1^{2(\delta+1)} t_2 + \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 t_1^{\delta+1} \frac{t_2^{\delta+2}}{\delta+2} \right) - t_1 - \frac{\gamma}{\delta+2} t_1^{\delta+2} - \left(\frac{\gamma}{\delta+1} \right)^2 \left(t_1^{2(\delta+1)+1} + \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 \frac{t_1^{2\delta+3}}{\delta+2} \right) \right] \\
 &\quad \left. \left(t_1^{2(\delta+1)} t_2 + \frac{t_2^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 t_1^{\delta+1} \frac{t_2^{\delta+2}}{\delta+2} \right) - t_1 - \frac{\gamma}{\delta+2} t_1^{\delta+2} - \left(\frac{\gamma}{\delta+1} \right)^2 \right. \\
 &\quad \left. \left(t_1^{2(\delta+1)+1} + \frac{t_1^{2(\delta+1)+1}}{2(\delta+1)+1} - 2 \frac{t_1^{2\delta+3}}{\delta+2} \right) \right] + r_w(d_c) \left\{ \alpha c \frac{t_1^{\beta+1}}{\beta+1} - \alpha \frac{t_3^{\beta+1}}{\beta+1} \right\} + r_w(T_{wc}) r_w(T_{ws}) \Bigg\}
 \end{aligned}$$

----- (13)

subject to $sI_T \leq r_w(T_{ws})$

Using Lagrange multipliers method, by using Kuhn-Tucker necessary condition. Solve the equations, the values of t_1^* , t_2^* and t_3^* are obtained. Substituting t_1^* , t_2^* and t_3^* in (6), (7), (9), (10) and (13) the optimum values I_T^* , I_{p1}^* , θ_L^* , I_m^* and TC^* are obtained.

7. Numerical example:

The following values of the parameter in proper unit were considered as input for the numerical result

$$IP = 1000, c = 2, \alpha = 700, \beta = 3, \gamma = 0.03, \delta = 0.02, s = 50 \text{ sqft}$$

$$\tilde{S}_c = (100000, 200000, 300000), \tilde{h}_c = (5000, 6000, 7000),$$

$$\tilde{d}_c = (500, 600, 700), \tilde{T}_{wc} = (500, 1000, 1500), \tilde{T}_{ws} = (4000, 5000, 6000).$$

Using MATLAB software, the optimum values I_T^* , I_{p1}^* , θ_L^* , I_m^* and TC^* are tabulated.

Comparison table 7.1

Mode 1	S_c Rs	h_c Rs	d_c Rs	T_{wc} Rs	T_{ws} sqft	t_3^* /month	I_T^* /Ton	I_{p1}^* /Ton	θ_L^* /Ton	I_m^* /Ton	TC^* Rs
Crisp1	100000	5000	500	500	4000	2.5	1,149	970	74.9	2,149	1,18,84,000
Crisp2	200000	6000	600	1000	5000	2.5373	1,296	976	76.0	2,296	1,07,85,000
Crisp3	300000	7000	700	1500	6000	2.5414	1,100	979	76.2	2,100	1,21,80,000
Fuzzy	180000	5400	540	900	4500	2.826	1,325	980	68.5	2,325	1,03,11,000

Observation:

From the above table, it should be noted that compared to crisp model, the fuzzy model is very effective method, because of the time consuming in fuzzy analysis and the optimal results are obtained easily.

- i. The average total cost is obtained in fuzzy model is less than the crisp model.
- ii. The optimal values I_T^* , I_{p1}^* , I_m^* in fuzzy model are higher than the crisp model.
- iii. The deterioration level in fuzzy model is less than the crisp model.

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