Fuzzification of Some Cycle Related Graphs

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ABSTRACT

An evidence labeling of a graph is a process of labeling the vertices and edges and there by fuzzify the graph.The verex labeling is done by an injective function m and the edge labeling is done by an injective function ρ.The evidently labeled graph is a fuzzy graph. In this paper we shows the admissibility of evidence labeling in some cycle related graphs.

KEYWORDS– corona of a graph, cycle graph, evidence labeling, friendship graph, h-graph , middle graph.

INTRODUCTION

All graphs we consider in this paper are all finite, simple and undirected. The vertex set and edge set of graph are denoted by V(G) and E(G) respectively.
The first definition of fuzzy graph by Kaufmann was based on Zadeh's fuzzy relations. After that Rosenfield considered fuzzy relation on fuzzy sets and developed the structure of fuzzy graphs. After the work of Rosenfield, Yeh and Bang introduced various connectedness concepts of graphs and digraphs into fuzzy graphs. In [4], A.Nagoor Gani and D.Rajalaxmi(a)Subahashini introduced a new concept of fuzzy labeling. A graph is said to be a fuzzy labeling graph if it has a fuzzy labeling. In [2], we introduced fuzzy digraph labeling. In [5] we introduced a special type of labeling called evidence labeling.
A fuzzy graph G = (V, μ ,ρ) is a non empty set V together with a pair of functions $\mu : V \rightarrow [0,1]$ and
$\rho : V \times V \rightarrow [0,1]$ such that for all x, y in V , $\rho(x, y) \leq \mu(x) \wedge \mu(y)$. We call μ the fuzzy vertex set of G and ρ , the fuzzy edge set of G respectively. We denote the underlying graph of the fuzzy graph
G = (μ, ρ) by G* = (μ*, ρ*), where $\mu^* = \{x \in V : \mu(x) > 0\}$ and
$\rho^* = \{(x,y) \in V \times V : \rho(x,y) > 0\}$.
In this paper we identify some cycle related graphs that satisfies evidence labeling and proves the admissibility.

Definition

Let G =(V,E) be a crisp graph. Then G has an evidence labeling if there exist two injections m and ρ such that m: V*→[0,1] defined by $m(v_i) = \frac{i}{N}$, where $V^* = (v_1, v_{2,\dots,} v_N)$, $v_i \in V$ and ρ: E→[0,1] defined by

$\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N} \; m(v_k) \; if \quad (v_j, v_k) \in E \; and \; m(v_k) = \wedge_{(v_j,v_p) \in E} m(v_p) \; and \; i < j < k$

and

$\rho(v_i, v_j) = \frac{m(v_i) \wedge m(v_j)}{N^3} \quad if \quad (v_j, v_k) \notin E \; for \; any \; k,$ where $1 \le i \le N-1, 1 \le j \le N.$

The graph G which admits evidence labeling is called evidently labeled graph.

2 Main Results

Theorem 2.1

The cycle graph C_n admits evidence labeling.

Proof

Consider the path graph C_n, where $|V(G)|$ = n , $|E(G)|$= n.

Let V* =(v_1,v_2,....,v_n).

Define m : V*→[0,1] as m(v_i) =$\frac{i}{n}$, $1 \le i \le n$ and

ρ: E →[0,1] as ρ(v_i, v_{i+1}) =$\frac{i(i+2)}{n^3}$, $1 \le i \le n-2$.

ρ(v_i, v_n) =.$\frac{i}{n^4}$, $i = 1 , n-1$.

If m(v_i)=m(v_j) then i =j.

If possible ,suppose that ρ(v_i, v_{i+1}) = ρ(v_j, v_{j+1}) for some i≠j

$$\Longrightarrow \frac{i(i+2)}{n^3} = \frac{j(j+2)}{n^3}$$

$$\Longrightarrow i^2 - j^2 = 2(j-i)$$

$$\Longrightarrow i + j = -2, \text{ contradiction.}$$

Also, $\frac{1}{n^4} < \frac{(n-1)}{n^4} < \frac{i(i+2)}{n^3}$ for all i .

So ρ are injective.

Theorem 2.2

The corona of a cycle graph C_n admits evidence labeling.

Proof

Consider the corona G= $C_n \odot K_1$ = (V,E) of path graph P_n , where $|V(G)|$ = 2n ,
| $E(G)$|= 2n ,V(C_n)= { v_1,v_2,...,v_n }

Let V* =(v_1,v_2,...,v_n,v_{n+1},v_{n+2},...,v_{2n}).

Define m : V*→[0,1] as m(v_i) =$\frac{i}{2n}$, $1 \le i \le 2n$

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$.

$\rho(v_{n-1},v_n) = \frac{(n-1)}{(2n)^3}$.

$\rho(v_1,v_n) = \frac{2n}{(2n)^3}$.

$\rho(v_i, v_{i+n}) = \frac{i}{(2n)^4}$, $1 \le i \le n$.

Clearly m and ρ are injective.

Theorem 2.3

The middle graph of cycle graph C_n admits evidence labeling.

Proof

Consider the middle graph $M(C_n)$ of C_n, where $|V| = 2n$, $|E| = 3n$.

Let $V(C_n) = \{ v_1,v_2,...,v_n \}$ and add a vertex on each edge of C_n and label it as v_{n+i} on the edge (v_i, v_{i+n}) , $1 \le i \le n$.

Let $V^* = (v_1,v_2,....,v_n, v_{n+1}, v_{n+2},...,v_{2n})$.

Define m : V*→[0,1] as $m(v_i) = \frac{i}{2n}$, $1 \le i \le 2n$ and

ρ: E→[0,1] as $\rho(v_i, v_{i+n}) = \frac{i(i+1)}{(2n)^3}$, $1 \le i \le n-1$.

$\rho(v_i, v_{i+(n-1)}) = \frac{i(i+n)}{(2n)^3}$, $2 \le i \le n$.

$\rho(v_1, v_{2n}) = \frac{1}{(2n)^4}$.

$\rho(v_n, v_{2n}) = \frac{n}{(2n)^4}$.

Clearly m and ρ are injective.

Theorem 2.4

The H-graph of a cycle graph C_n admits evidence labeling.

Proof

Consider the H-graph G= (V,E) of cycle graph C_n, where $|V(G)| = 2n$, $|E(G)| = 2n - 1$ and $V(C_n) = \{v_1,v_2,...,v_n\}$. Let $V^* = (v_1,v_2,...,v_n,v_{n+1},v_{n+2},...,v_{2n})$.

Define m : V*→[0,1] as $m(v_i) = \frac{i}{2n}$, $1 \le i \le 2n$

Case I - when n is even

Without loss of generality assume that $(v_{\frac{n}{2}}, v_{\frac{3n+2}{2}})$ is an edge.

Define ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$, $n+1 \le i \le 2n-2$.

$\rho(v_{n-1},v_n) = \frac{(n-1)}{(2n)^4}$.

$\rho(v_{2n-1},v_{2n}) = \frac{(2n-1)}{(2n)^4}$.

$\rho(v_1,v_n) = \frac{1}{(2n)^4}$.

$\rho(v_{n+1}, v_{2n}) = \frac{(n+1)}{(2n)^4}.$

$\rho(v_{\frac{n}{2}}, v_{\frac{3n+2}{2}}) = \frac{n(3n+4)}{4(2n)^3}.$

Clearly m and ρ are injective.

Case II - when n is odd

Without loss of generality assume that $(v_{\frac{n+1}{2}}, v_{\frac{3n+1}{2}})$ is an edge.

Define ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(2n)^3}$, $1 \le i \le n-2$, $n+1 \le i \le 2n-2$

$\rho(v_{n-1}, v_n) = \frac{(n-1)}{(2n)^4}.$

$\rho(v_{2n-1}, v_{2n}) = \frac{(2n-1)}{(2n)^4}.$

$\rho(v_1, v_n) = \frac{1}{(2n)^4}.$

$\rho(v_{n+1}, v_{2n}) = \frac{(n+1)}{(2n)^4}.$

$\rho(v_{\frac{n+1}{2}}, v_{\frac{3n+1}{2}}) = \frac{n(3n+3)}{4(2n)^4}.$

Clearly m and ρ are injective.

Theorem 2.5

The friendship graph F_n admits evidence labeling.

Proof

Consider the friendship graph F_n , where $|V(F_n)| = 2n+1$,

$|E(G)| = 3n$, $V(C_3)=\{v_i, v_{i+1}, v_{2n+1}\}$, i=1,3,5,..., $2n-1$.

Let $V^* = (v_1, v_2, \dots v_n, v_{n+1}, v_{n+2}, \dots v_{2n+1})$, taking the common vertex as v_{2n+1}

Define m : V*→[0,1] as $m(v_i) = \frac{i}{2n+1}$, $1 \le i \le 2n+1$ and

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(2n+1)}{(2n+1)^3}$, $i = 1,3,5, \dots, 2n-1$.

$\rho(v_i, v_{2n+1}) = \frac{i}{(2n+1)^4}.$

Clearly m and ρ are injective.

Theorem 2.6

The Helm graph H_n admits evidence labeling.

Proof

Consider the helm graph H_n , where $|V(H_n)| = 2n+1$,

$|E(H_n)| = 2n$, $V(Wn)=\{ v_{n+1}, v_{n+2}, \dots v_{2n}, v_{2n+1}\}$

Let $V^* = (v_1, v_2, \dots v_n, v_{n+1}, v_{n+2}, \dots v_{2n+1})$, taking the center vertex as v_{2n+1}

Define m : V*→[0,1] as $m(v_i) = \frac{i}{2n+1}$, $1 \le i \le 2n+1$ and ρ: E →[0,1] as $\rho(v_i, v_{n+i}) = \frac{i(2n+1)}{(2n+1)^3} = \frac{i}{(2n+1)^2}$, $1 \le i \le n$

$\rho(v_i, v_{2n+1}) = \frac{(n+i)}{(2n+1)^4}$. $n+1 \le i \le 2n-1$.

$\rho(v_{n+i}, v_{n+i+1}) = \frac{(n+i)(n+i+2)}{(2n+1)^3}$, $1 \le i \le n-1$.

$\rho(v_{2n}, v_{2n+1}) = \frac{2n}{(2n+1)^4}$

Clearly m and ρ are injective.

Theorem 2.7

The graph obtained by duplicating a vertex cycle graph C_n admits evidence labeling.

Proof

Let $V(C_n)=\{v_1,v_2,...,v_n\}$ .With out loss of generality, assume that v_1 is the duplicated vertex and it's copy be v_{n+1}.

Let $V^* =(v_1,v_2,...v_n,v_{n+1}\}$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{n+1}$, $1 \le i \le n+1$ and

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(n+1)^3}$, $1 \le i \le n-1$

$\rho(v_n, v_{n+1}) = \frac{(n)}{(n+1)^4}$.

$\rho(v_2, v_{n+1}) = \frac{2}{(2n+1)^4}$

Clearly m and ρ are injective.

Theorem 2.8

The graph obtained by duplicating an arbitrary edge by a new vertex in cycle graph C_n admits evidence labeling.

Proof

Let $V(C_n)=\{v_1,v_2,...,v_n\}$ .With out loss of generality, assume that $e=(v_1,v_n)$ is the duplicated edge by the vertex v_{n+1}.

Let $V^* =(v_1,v_2,...,v_n,v_{n+1}\}$.

Define $m : V^* \to [0,1]$ as $m(v_i) = \frac{i}{n+1}$, $1 \le i \le n+1$ and

ρ: E →[0,1] as $\rho(v_i, v_{i+1}) = \frac{i(i+2)}{(n+1)^3}$, $1 \le i \le n-1$

$\rho(v_n, v_{n+1}) = \frac{(n)}{(n+1)^4}$.

$\rho(v_1, v_{n+1}) = \frac{1}{(2n+1)^4}$

Clearly m and ρ are injective.

CONCLUSIONS

In this paper we prove some theorems which shows the admissibility of evidence labeling in some cycle related graphs like middle graph, friendship graph etc. Evidence labeling is a best tool to fuzzify the crisp graphs especially cycle related graphs.

REFERENCES

[1]Mathew Varkey T K and Sunoj B S, "*ADCSS-Labeling of 2-Tuple Graphs of Some Graphs",*IOSR Journal of Mathematics,Vol.12,Issue 5 Ver.V(Sep-Oct2016), pp 12-15.
[2] Mathew Varkey T K and Sreena T D , "*Fuzzy Digraph Labeling".* Aryabhatta Journal of Mathematics and Informatics,Vol.9, Issue-01,(January-June,2017), pp 627-631
[3]Frank Harary,"*Graph Theory"* ,Avalon Publishing,1969.
[4] A.Nagoor Gani and D .Rajalaxmi (a)Subahashini ,"*Properties of Fuzzy Labeling*", Graph,Applied Mathematical Sciences,Vol.6,2012,no.70, pp 3461-3466.
[5]MathewVarkey T K and Sreena.T.D, "*Fuzzification of Some Path Related Graphs* "[Communicated].

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