A Note On Product Cordial Invariance Of Fusion Graphs

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1.Abstract:

In this paper we discuss product cordial invariance of fusion graph obtained by using $G_1= C_m$ and $G_2= FL(C_n)$ i.e.flag of Cn, denoted by $G= G_1FG_2$.We restrict our attention to $G_1 = C_3$ and $G_2= FL(Cn)$..In this case there are 3 different structrespopssible on G.We call it as type I, type II and type III.And in all the three cases the graph is product cordial (pc).

Key words :structure,Product Cordial, Invariance, Fusion , Graph, labeling, binary.

AMS Subject Classification 05C78.

2.Introduction :

Product cordial labeling was introduced by Somsundaran and others[9].Let Gbe a (p,q) graph.Define a function f: V(G) →{0,1}.This distribution of binary digits on vertices should be such that when for any edge (uv) on G label of (uv) is defined as f(u)f(v).the resultant binary label distribution on vertices and edges should follow $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$.This is also called as pairitycondition.The graph with product cordial labeling is called as product cordial (pc) graph.A lot of work has been done on pc labeling which could be observed in Dynamic Survey Of Graph Labeling [8].The graphs considered here are finite,simple and undirected.For terminology and definitions we refer [7],[8]

3.Results proved:

Theorem 3.1 Let G '= $FL(C_n)$The fusion graph of C_3 and G' denoted byG =C_3FG' is product cordial.(structure type I)

Proof: Let the C_3 be given by $(u_1e_1u_2e_2u_3e_3u_1)$ and the copy of $FL(C_n)$ fused at i^{th} (i = 1,2,3) vertex of C_3 be given by $C^i_n=(e^iv^i_1e^i_1v^i_2e^i_2,,..e^i_nv^i_1)$ and is joined by edge e^i tovertex v_i of C_3.

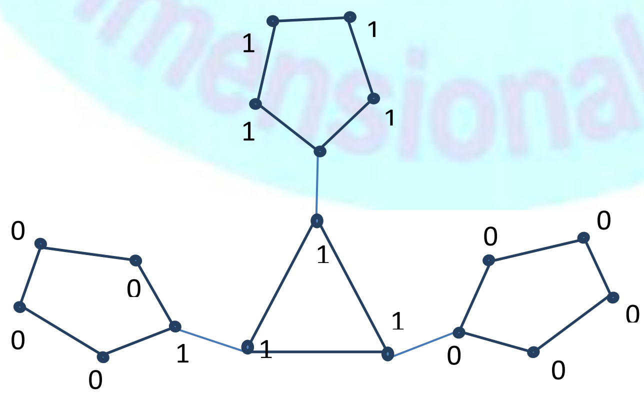

Fig 2. G' = $FL(C_5)$,Fusion of C_3 and G' denoted by C_3FG' with label distribution.

Define a function f:V(G)→{0,1} in the following way.

$f(u_i) = 1$, i = 1,2,3.

$f(v^i_j)$= 1 for all j = 1..n. andi =1
$f(v^i_j)$= 0 for all j = 1..n. andi = 2
$f(v^i_j)$= 1 for all j = 1..(x-1) } i = 3 and n = 2x or n = 2x+1

$f(v^i_j)$= 0 for all j = x,…n

The binary distribution of vertices and edges is given by :for n = 2x $v_f(0)$= 3x+1 and $v_f(1)$ =3x+2 and $e_f(0)$ = 3x+3 = $e_f(1)$ for n = 2x and

for n = 2x+1 we have $v_f(0)$= 3x+3 and $v_f(1)$ =3x+3 and $e_f(0)$ = 3x+4, $e_f(1)$=3x+5. Thus f is a product cordial function and G is product cordial graph.#

Using the cycle C3 and flag graph of cycle G' = FL(Cn) we obtain one other fusion graph called as type II.In which G' is fused with C_3 at a degree 3 vertex of G_2.The resultant graph is product cordial.

Theorem3.2 :C_3F G'(Type II) is product cordial.

Proof: Let the cycle C_3 be given by v1e1v2e2v3e3v1 and the pendent vertex at vi be v''_i and cycle Cn at vertex vi be given by ($v^i e^i_1 v^i_2 e^i_2 \ldots e^i_n v^i$) ,i= 1,2,3.Define a function f:V(G)→{1,0} as follows,

$f(v_i)$=1 i =1,2,3.

$f(v''_1)$ = 0,$f(v^i_j)$ = 0, i= 1,j = 2,3,4..n

$f(v''_2)$ = 1,$f(v^i_j)$ = 1, i= 2,j = 2,3....nn = 2x+1 or n = 2x depends on n is odd or even.

$f(v''_3)$ = 0,$f(v^i_j)$ = 1, i= 3,j = 2,3,.,x

$f(v^i_j)$ = 0, i= 3 j = x+1,..,n

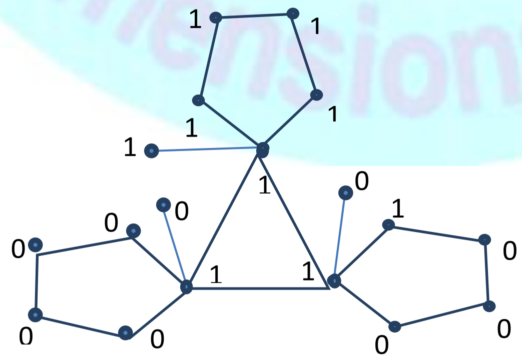

Fig 2. G' = FL(C_5),Fusion of C3 and G' denoted by G= C_3FG' with label distribution.

Structure type II

for n = 2x we have vertex distribution as vf(0) = n+x+1 and vf(1)= n+x+2

and the edge distribution is ef(0) = n+x+3= ef(1)

for n = 2x+1 we have vertex distribution as vf(0) = n+x+2= vf(1)

and the edge distribution is ef(0) = n+x+4 and ef(1)=n+x+3#

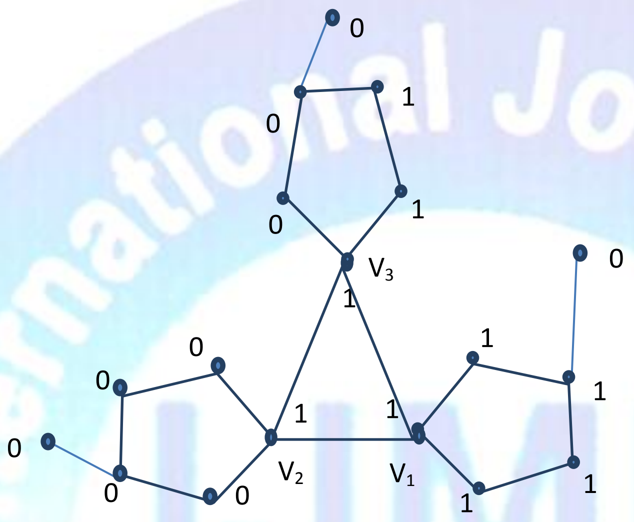

Fig 3. G' = FL(C_5),Fusion of C3 and G' denoted by G= C_3FG' with label distribution.

Structure type III

Theorem 3.3: C_3F G'(Type III) is product cordial.

Proof: Let the cycle C_3 be given by v1e1v2e2v3e3v1 and cycle C^in at vertex vi be given by ($v^i e^i_1 v^i_2 e^i_2 \ldots e^i_n v^i$) , i= 1,2,3.and the pendent vertex at v^i_3 be v''_{i3} (i = 1,2,3). Define a function f:V(G)→{1,0} as follows,

f(v_i)=1 i =1,2,3.

f(v''_{13}) = 0,f(v^i_j) = 1, i= 1,j = 2,3,4..n

f(v''_{23}) = 0,f(v^i_j) = 0, i= 2,j = 2,3....n

And when n = 2x+1 or n = 2x

f(v''_{33}) = 0,f(v^i_j) = 1, i= 3,j = 2,3,.,x

f(v^i_j) = 0, i= 3, j = x+1,..,n

On vertices,labels arev_f(0)=n+x+1, v_f(1) = n+x +2 and that on edges we have label numbers e_f(0)= n+x +3=e_f(1) for n =2x

and for n = 2x+1 we have labels on vertices, are v_f(0)=n+x+2,v_f(1) = n+x +2 and that on edges we have label numbers e_f(0)= n+x +4, e_f(1) = n+ x+3.

That shows that the three structures of fusion graph as in type I,type II and type III are not isomorphic but All three of them have product cordial labeling.That explains pc invariance under fusion graph.

Theorem 3.4: Fusion of Shel S_5 with $K_{1,n}$ given by G = $S_5FK_{1,n}$is product cordial.

Proof: Let the C5 cycle on shel be given by ($v_1e_1v_2e_2..e_5v_1$) ,the two chords are (v_1v_3) and (v_1v_4).The n pendent vertices at vi be given by $v^i_1,v^i_2,..v^i_n$.Thus there are 5(n + 1)+2 edges and 5n + 5 vertices on G.

Define a function f:V(G)→{0,1} by

$f(v_i)$ = 1 for I = 1,2,3,4 and $f(v_5)$ = 0

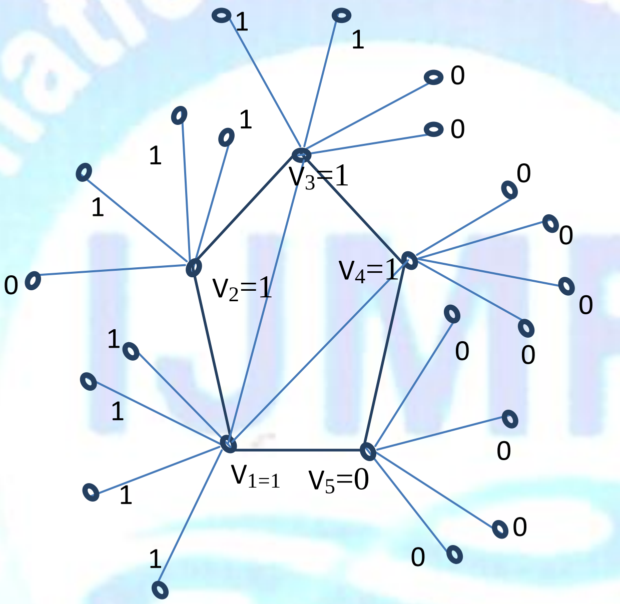

Fig 2 : $S_5FK_{1,4}$ shows binary label distribution

$f(v^i_1)$=1 for i = 1 and $f(v^i_1)$=0 for i=2,3,4,5.

$f(v^i_2)$=1 for i = 1,2 and $f(v^i_2)$=0 for i= 3,4,5

$f(v^i_j)$=1 for j = 2,3,..nand i = 1,2.

$f(v^i_j)$=0 for j is odd number >2 and $f(v^i_j)$=1 for j is even number >2 for i = 3

$f(v^i_j)$=0 for all j = 3,4,..n and i = 4,5

for n 1 we have label distribution given by $v_f(0)$ = 5 = $v_f(1)$ and $e_f(0)$ = $e_f(1)$ =6

$v_f(0)$ = 5n = $v_f(1)$ and $e_f(0)$ = 2+ (n/2)*5;$e_f(1)$ =$e_f(0)$ +1;

Thus f is a product cordial function and G is product cordial graph.#

We prove a more general result on fusion of graph obtained by cycle Cn and Flag graph of any graph G_1i.e.G = CnFL(G_1) for product cordial property.

The point of fusion being 3-degree vertex on Flag.FL(G_1)

Theorem 3.5: Let G1 be a (p,q) graph.G = C_{2n}FL(G_1) is product cordial for all G_1 and all cycles C_{2n}.

Proof: Let the vertices on cycle Cn be v_1,v_2..v_{2n}.The pendent vertex at v_i be v^iThe copy of G_1 fused at vertex v_i be G^i..Then v^i_j stands for j^{th} vertex of G^i .

Define f : V(G)→ {0,1} as follows.

$f(v^i)$= 0 for all i= 1,2..n.

f(vi) = 1 for all i = 1,2,…n.

$f(v^i_j)$ =1 ,for all i= 1..n and for all j = 1,2..p.

$f(v^i_j)$ =0 ,for all i= n+1,n+2,..2n , and for all j = 1,2..p.

The binary distribution of vertices and edges will be given by,

$v_f(0)$ =n + np; $v_f(1)$= n+np; and on edges $e_f(0)$= nq+2n=$e_f(1)$.This clearly shows that f is product cordial function.#

Theorem 5: Let C_3 be the given cycle.. Take 3 copies of C_n.At each of these fuse a path P_m with one of it's pendent vertex.Let this graph be called as G1.Fuse the other pendent vertex of each copy of G1 to different three vertices of main C_3.The resultant graph is the fusion graph G =C_3FG1.Then G is product cordial.

Proof: Let the cycle be (v_1,v_2,v_3).The vertices of G1 fused at vi be given by v_i=v^i_1,v^i_2,..v^i_m ,v^i_{m+1},..,v^i_{m+n-1}.

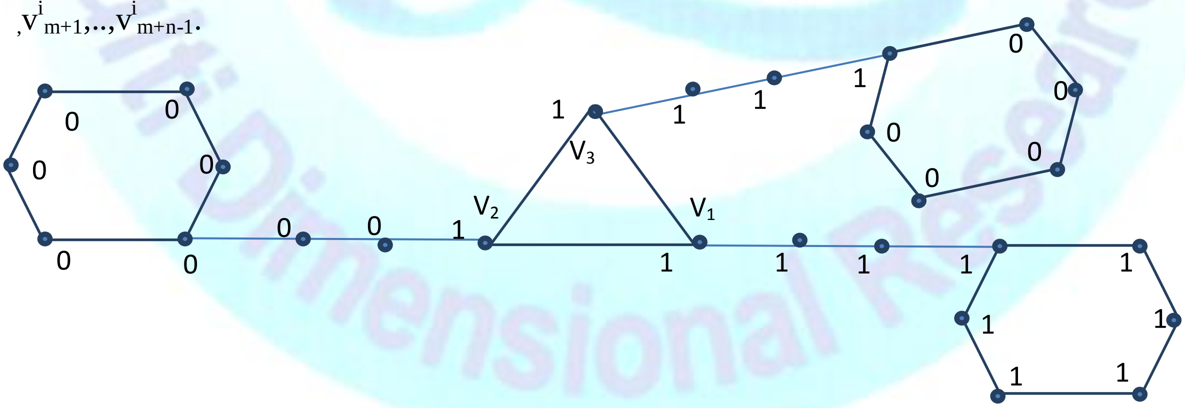

Fig 3 G= C_3 FG1 and G1 is obtained by fusing a pendent vertex of P_4 to C_6

Define a function f : V(G) → {0,1} as follows

$f(v_i)$ = 1 for all i=1,2,3

$f(v^i_j)$ = 1 for all j= 2,3..m+n-1 and i =1

$f(v^i_j) = 0$ for all j= 2,3,,..m+n-1 and i =2

when i =3 take k= m +n-5 if k= 2x then

$f(v^i_j) = 1$ for all j= 2..x,x+1 and $f(v^i_j) = 0$, for x+2 to m+n-1

If k = 2x+1 then $f(v^i_j) = 1$ for all j= 2,3..,x+2. and $f(v^i_j) = 0$, for x+3 to m+n-1

The binary distribution on vertices and edges is :

For m+n-5 = 2x we have, $v_f(0)$ =n+m+x+1 =vf(1) and ef(0)=m+n+x+3 ,ef(1) = m+n+x+2

For m+n-5 =2x+1 we have,$v_f(0)$ =n+m+x+1 ,vf(1) = m+n+x+2and ef(0)=m+n+x+3 ,ef(1) = m+n+x+3.

Thus the given graph is pc.

4.Future scope:

In above discussion of fusion graphs under pc invariance we type III structure can be further modified by attaching the pendent edge to different points of Cn.From isomorphic point of view the graphs may be different but for binary distribution of labels we expect to get same distribution as in type III.

5.Acknoledgement:

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