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N-Generated Fuzzy Groups and Its Level Subgroups

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Abstract- *In this paper, we define the algebraic structures of $n-$ generated fuzzy subgroups and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in $n-$ generated fuzzy subgroups. Characterizations of $n-$ generated level subsets of a $n-$ generated fuzzy subgroups of a group are given*

Keywords- *Fuzzy set, multi-fuzzy set, fuzzy subgroup, multi-fuzzy subgroup, anti-fuzzy subgroup, multi-anti fuzzy subgroup, $n-$ generated fuzzy subset, $n-$ generated fuzzy subgroups, $n-$ generated fuzzy level subsets,n-generated fuzzy level subgroups*

1.INTRODUCTION

After the introduction of the concept of fuzzy sets by L.A.Zadeh [1], researchers were conducted the generalizations of the notion of fuzzy sets A. Rosenfeld [2] Introduced the concept of fuzzy group and the idea of "Intuitionistic Fuzzy set" was first published by K.T. Atanassov [3]. W.D.Blizard [4] Introduced the concept of fuzzy multi-set theory. Also Shinoj .T.K and Sunil Jacob [6] produced some results in Intuitionistic Fuzzy Multi-sets. In this chapter we define $n-$ generated fuzzy sets and $n-$ generated fuzzy subgroups and some of their properties.

2. PRELIMINARIES

2.1 Definition

Let X be a non-empty set. A fuzzy set A on X is a mapping $A: X \rightarrow [0,1]$ and is defined as $A = \{x \in X / (x, \mu(x))\}$

2.2. Definition

Let X and Y be any two sets. Let $f: X \rightarrow Y$ be a function. If μ is a fuzzy set on X then the image μ under f is a fuzzy set on Y and is defined by

$f(\mu)(y) = v(y) = \sup_{x \in f^{-1}(y)} \mu(x), \ \forall \ y \in Y$ is called image of μ under f

2.3. **Definition**

Let X and Y be any two sets. Let $f : X \to Y$ be a function. If S is a fuzzy set on Y then the preimage of S under f is a fuzzy set on X and is defined by

$$\left(f^{-1}(S)\right)(x) = S\left(f(x)\right)$$

2.4. **Definition**

Let A be a fuzzy subset of a set X. For $t \in [0, 1]$, $A_t = \{x \in X \,/\, A(x) \geq t\}$ is called a level fuzzy subset of A

2.5. **Definition**

Let X be a non empty set. An Intuitionistic Fuzzy set A on X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$, where $\mu_A : X \to [0,1]$ & $\gamma_A : X \to [0,1]$ are the degree of membership and non- membership functions respectively with $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$

2.6. **Definition**

Let X be a non-empty set. A Fuzzy Multi set (FMS) A drawn from X is characterized by a function ' Count membership' of A denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp finite set drawn from the unit interval $[0,1]$.Then for any $x \in X$, the value $CM_A(x)$ is a crisp multi set drawn from $[0,1]$. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_{A_1}(x), \mu_{A_2}(x), \ldots\ldots\mu_{A_k}(x)\right)$ where $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq \ldots\ldots \geq \mu_{A_k}(x)$

$$A = \left\{\left\langle x : \left(\mu_{A_1}(x), \mu_{A_2}(x) \ldots\ldots \mu_{A_k}(x)\right)\right\rangle : x \in X\right\}$$

Example 2.7

Let $X = \{x, y, z, w\}$ be a universal non empty set.For each $x \in X$, we can write a Fuzzy Multi set as follows

$A = \{\langle x, (0.8, 0.7, 0.7, 0.6)\rangle, \langle y, (0.8, 0.5, 0.2)\rangle, \langle z, (1, 0.5, 0.5)\rangle\}$ Where

$CM_A(x) = (0.8, 0.7, 0.7, 0.6)$ with $0.8 \geq 0.7 \geq 0.7 \geq 0.6$

2.8. **Definition**

Let X be a non- empty universal set and let A be an Fuzzy Multi set on X. The n-generated Fuzzy set on X is constructed from the Fuzzy Multi set and is defined as

$$\lambda = \left\{ \left\langle x, \frac{1}{k}\left(\mu_{A_1}^{n}(x) + \mu_{A_2}^{n}(x) + ...\mu_{A_k}^{n}(x) \right) \right\rangle : x \in X \right\}$$

where $\mu_{A_1}^{n}(x) \geq \mu_{A_2}^{n}(x) \geq .... \geq \mu_{A_k}^{n}(x)$ and n is the dimension of the Fuzzy Multi set A

2.9.**Definition Multi-level subset**

Let A be a multi-fuzzy subset of X. For $t_i \in [0,1],\ i = 1,2,...,k,\ A_{t_i} = \{x \in X / A(x) \geq t_i\}$ is called multi-level subset of A

2.10. Definition Multi-Fuzzy Mapping

Let $\mu = (\mu_1, \mu_2, ..., \mu_k)$ and $\nu = (\nu_1, \nu_2, ..., \nu_k)$ be two multi-fuzzy sets in X of dimenstion k and n respectively. A multi-fuzzy mapping is a mapping $F : M^k FS(X) \to M^n FS(X)$ which maps each $\mu \in M^k FS(X)$ into a unique multi-fuzzy set $\nu \in M^n FS(X)$

2.11. Definition Atanassov Intuitionistic Fuzzy Sets Generating Maps (AIFSGM)

A mapping $F : M^k FS(X) \to M^2 FS(X)$ is said to be an Atanassov Intuitionistic Fuzzy Sets Generating Maps(AIFSGM) if $F(\mu)$ is an Intuitionistic fuzzy set in $M^2 FS(X)$

2.12. Definition Multi-Fuzzy extensions of functions

Let $f : X \to Y$ and $h : \prod M_i \to \prod L_j$ be functions . The Multi-fuzzy extension and the inverse of the extension are $f : \prod M_i^{\,X} \to \prod L_j^{\,Y}$, $f^{-1} : \prod L_j^{\,Y} \to \prod M_i^{\,X}$ defined by $f(A)(y) = \sup_{x \in f^{-1}(y)} h[A(x)],\ A \in \prod M_i^{\,X}, y \in Y$ and $f^{-1}(B)(x) = h^{-1}[B(f(x)],\ B \in \prod L_j^{\,Y}, x \in X$ where h^{-1} is the upper adjoint of h .

The function $h : \prod M_i \to \prod L_j$ is called the bridge function of the multi-fuzzy extension of f.

2.13. Definition

Let X and Y be any two sets. Let $f : X \to Y$ be a function. If λ is a $n-$generated fuzzy set on X then the image of λ under f is a $n-$generated fuzzy set on Y and is defined by $f(\mu)(y) = v(y) = \sup_{x \in f^{-1}(y)} \lambda(x), \ \forall\ y \in Y$ is called image of λ under f

2.14. Definition

Let X and Y be any two sets. Let $f : X \to Y$ b a function. If λ is an $n-$generated fuzzy set on Y then the pre image of λ under f is a $n-$generated fuzzy set on X and is defined $\left(f^{-1}(\lambda)\right)(x) = \lambda(f(x))$

2.15. Definition:

Let λ be an $n-$generated fuzzy set on X. For $t \in [0, 1]$, a level $n-$generated fuzzy subset of λ_t is defined by $\lambda_t = \{x \in X / \lambda(x) \ge t\}$

2.16. Properties of n-generated fuzzy set

Let k be a positive integer and A and B be two fuzzy multi- sets of dimension k and if $A^G = \{(x, \lambda(x)); x \in X\}$ & $B^G = \{(x, \gamma(x)); x \in X\}$

$n \in N$, where $\lambda(x) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(x)$, $\gamma(x) = \frac{1}{k}\sum_{i=1}^{k} v_i^n(x)$ Then

(1). $A^G \subseteq B^G \Leftrightarrow \lambda(x) \le \gamma(x)$

(2). $A^G = B^G \Leftrightarrow \lambda(x) = \gamma(x)$

(3). $A^G \cup B^G = \lambda(x) \cup \gamma(x)$

$$= \left[\left(x, \max\left[\lambda(x), \gamma(x)\right]\right); x \in X\right]$$

(4). $A^G \cap B^G = \lambda(x) \cap \gamma(x)$

$$= \left[\left(x, \min\left[\lambda(x), \gamma(x)\right]\right); x \in X\right]$$

(5). $A + B = \left[\left\{x, \left(\lambda(x) + \gamma(x) - \lambda(x)\gamma(x)\right)\right\}; x \in X\right]$

(6). If $A^G = \{(x, \lambda(x)); x \in X\}$, then $\left(A^G\right)^C = \{(x, 1 - \lambda(x)); x \in X\}$

2.17. Definition

Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if

$(i).A(xy) \geq \min\{A(x), A(y)\}$

$(ii).A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

2.18 . **Definition**

Let G be a group. A fuzzy subset A of G is said to be an anti-fuzzy subgroup of G if
(i). $A(xy) \leq max\{A(x), A(y)\}$, (ii). $A(x^{-1}) = A(x) \quad \forall\, x, y \in G$

2.19. **Definition**

Let G be a group. A multi-fuzzy subset A of G is said to be an multi-fuzzy subgroup of G if $(i).A(xy) \geq \min\{A(x), A(y)\}$ $(ii).A(x^{-1}) \geq A(x) \quad \forall\, x, y \in G$

2.20. **Definition**

Let G be a group. A multi-fuzzy subset A of G is said to bean multi-anti-fuzzy subgroup of G if $(i).A(xy) \leq \max\{A(x), A(y)\}$

$(ii).A(x^{-1}) = A(x) \quad \forall\, x, y \in G$

2.21. **Definition**

Let G be a group. A $n-$generated fuzzy subset λ of a group G is called a $n-$generated fuzzy subgroup of G if

$(i).\lambda(xy) \geq \min\{\lambda(x), \lambda(y)\}$

$(ii).\lambda\left(x^{-1}\right) = \lambda(x) \quad \forall\, x, y \in G$ where $\lambda(x) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(x),\ \ \lambda(y) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(y)$

& $\lambda(xy) = \frac{1}{k}\sum_{i=1}^{k} \mu_i^n(xy)$

2.22. **Definition**

Let G be a group. An $n-$generated fuzzy subset λ of a group G is called an $n-$generated anti- fuzzy subgroup of G if

$(i).\lambda(xy) \leq \max\{\lambda(x), \lambda(y)\}$

$(ii).\lambda\left(x^{-1}\right) = \lambda(x) \quad \forall\, x, y \in G$

3. Properties of n-generated- Level Subsets of an n- generated Fuzzy subgroups

In this chapter we introduce the concept of $n-$generated level fuzzy subset of a $n-$generated fuzzy subgroup

3.1. Definition:

Let λ be a $n-$generated fuzzy subgroup of a group G. For any $t=(t_1,t_2,....t_k,..)$ where $t_i \in [0,1]$ for all i, we define the $n-$generated level subset of λ as $L(\lambda;t)=\{x \in G/\lambda(x) \geq t\}$

Theorem.3.2:

Let λ be an $n-$generated fuzzy subgroup of a group G. For any $t=(t_1,t_2,....t_k,..)$ where $t_i \in [0,1]$ for all i suchthat $t \leq \lambda(e)$ where $'e'$ is the identity element of G, $L(\lambda;t)$ is a subgroup of G.

Proof:

Let $x, y \in L(\lambda;t) \Rightarrow \lambda(x) \geq t$ and $\lambda(y) \geq t$

Now, $\lambda(xy^{-1}) \geq \text{Min}\{\lambda(x), \lambda(y)\}$

$\geq \text{Min}\{t,t\}$

$\Rightarrow \lambda(xy^{-1}) \geq t$

$\Rightarrow xy^{-1} \in L(\lambda;t)$

$\Rightarrow L(\lambda;t)$ is a subgroup of G.

Theorem3.3:

Let G be a group and let λ be an $n-$generated fuzzy subset of a group G such that $L(\lambda;t)$ is a subgroup of G. Then for any $t=(t_1,t_2,....t_k,..)$ where $t_i \in [0,1]$ for all i such that $t \leq \lambda(e)$ where $'e'$ is the identity element of G, λ is an $n-$generated fuzzy subgroup of G

Proof:

Let $x, y \in G$ and $\lambda(x)=r$ & $\lambda(y)=s$

where $r=(r_1,r_2,....r_k,...)$, $s=(s_1,s_2,...s_k,....)$, for $r_i, s_i \in [0,1]$ for all i

Suppose $r < s$

Now $\lambda(x)=r \Rightarrow x \in L(\lambda;r)$

And now $\lambda(y)=s>r \Rightarrow y \in L(\lambda;r)$

Therefore $x, y \in L(\lambda;r)$.

As $L(\lambda;r)$ is a subgroup of G, $xy^{-1} \in L(\lambda;r)$

Hence $\lambda(xy^{-1}) \geq r = \min \{r,s\}$

$\geq \min \{\lambda(x), \lambda(y)\}$

That is, $\lambda\left(xy^{-1}\right) \geq \min \{\lambda(x), \lambda(y)\}$

Hence λ is a $n-$generated fuzzy subgroup of G.

Theorem3.4:

Let λ be an $n-$generated fuzzy subgroup of a group G and $'e'$ is the identity element of G If two $n-$generated level fuzzy subgroups $L(\lambda;r)$, $L(\lambda;s)$ for $r=\left(r_1,r_2,....r_3,..\right)$, $s=\left(s_1,s_2,...s_k,...\right)$ where $r_i,s_i \in [0,1]$ for all i and $r,s \leq \lambda(e)$ with $r<s$ of λ are equal, then There is no x in G such that $r \leq \lambda(x) < s$

Proof:

Let $L(\lambda;r)=L(\lambda;s)$

Suppose there exists $x \in G$ such that $r \leq \lambda(x) < s$

Then $L(\lambda;s) \subseteq L(\lambda;r)$

$\Rightarrow x \in L(\lambda;r)$, but $x \notin L(\lambda;s)$

This contradicts our assumption that $L(\lambda;r)=L(\lambda;s)$

Hence there is no $x \in G$ such that $r \leq \lambda(x) < s$

Conversely, suppose that there is no $x \in G$ such that $r \leq \lambda(x) < s$, then by definition $L(\lambda;s) \subseteq L(\lambda;r)$

Let $x \in L(\lambda;r)$ and there is no $x \in G$ suchthat $r \leq \lambda(x) < s$

Hence $x \in L(\lambda;s)$ and therefore $L(\lambda;r) \subseteq L(\lambda;s)$

Hence $L(\lambda;r)=L(\lambda;s)$

CONCLUSION

In this chapter we have propounded the concept of n-generated fuzzy sets. It is directly proportional to Multi-fuzzy set theory

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