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Some Complete- Graph Related Families Of Product Cordial (pc) Graphs.

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1.Abstract :

In this paper we discuss graph families related to complete graphs K_4 and K_5forProduct Cordial labeling. We show that One point union ofn copies of K_mi.e $(K_m)^n$, snake on K_5i.e.S(K_5,n) ,Fusion graph .$K_5FC_n$and KP(Cm,Cn,Pt) are product cordial..

Key words : edge, vertex, Product, cordial, completegraph, snake, union.

Subject Classification (AMS): 05C78.

2.Introduction

All graphs considered here are planar and simple graphs. for terminology and definition we refer G.F.Harare [6][7].Sundaram, Ponraj, and Somasundaram [8] introduced the notion of product cordial labelings. A product cordial labeling of a graph G with vertex set V is a function f :V→ {0,1} such that if each edge uv is assigned the label f(u)f(v), the number of vertices labeled with 0i.e$v_f(0)$and the number of vertices labeled with 1 i.e$v_f(1)$differ by at most 1, and thenumber of edges labeled with 0i.e. $e_f(0)$ and the number of edges labeled with 1 i.e. $e_f(1)$differ by at most1. A graph with a product cordial labeling is called as product cordial graph.(pc-graph)

A lot of work is done in this type of labelling so far.Sundaram, Ponraj, and Somasundaram[8] prove the following graphs most of it are union of different graphs,are product cordial graphs. trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; PmUPn; CmUPn; PmUK1;n; WmUFn (Fn is the fan Pn+K1); K1;mUK1;n; WmUK1;n; Wm U Pn; Wm U Cn; the total graph of Pn (the total graph of Pn has vertex setV (Pn) U E(Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C(t)^n$, the one-point union of t copies of Cn, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; CmUPn if and only if m+n is odd; Km;nUPs if s >mn; Cn+2 UK1;n; KnUKn;(n-1)/2 when n is odd; Kn UKn-1;n/2 when n is even.

Seoud and Helmi [] obtained the following results: Kn is not product cordial

for all n ≥4; Cm is product cordial if and only if m is odd; the gear graph Gm is

product cordial if and only if m is odd; all web graphs are product cordial; the C4-snake is product cordial if and only if the number of 4-cycles is odd; and

they determine all graphs of order less than 7 that are not product cordial.Vaidya and Barasara[9],[10],[11],[12]discuss product cordiality of closed helms, web graphs, ower graphs, double triangular snakes obtained from the path Pn if and only if n is odd, and gear graphs obtained from the wheel Wn if and only if n is odd.

In this paper We discuss snake on K_4i.e.S(K_4,n)and snake K_5 i.e. S(K_5,n) and show that for even n these snakes are product cordial.Further we show that fusion graph of K_5 and C_n

3.Definations

Defination3.1 K_4 snake S(K_4,n)

To obtain S(K_4,n) we start with a path P_{n+1} =(v_1,v_2,….v_n,v_{n+1}) .Take two new vertices $u_i$and u_{i+1} between every two vertices v_i and v_{i+1} and new edges (u_iu_{i+1}),(u_iv_i),(u_iv_{i+1}) and ($u_{i+1}v_i$),($u_{i+1}v_{i+1}$) for each i = 1,2..n.Note that here(v_i,u_i,u_{i+1},v_{i+1}) forms K_4.

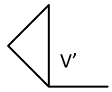
Defination3.2K_5 snake S(K_5,n) we start with a path P_{n+1} =(v_1,v_2,….v_n,v_{n+1}) .Take three new vertices u_i , u_{i+1} and u_{i+2} between every two vertices v_i and v_{i+1} and new edges (u_iu_{i+1})(u_iv_i)(u_iv_{i+1}),(u_iu_{i+2}), and ($u_{i+1}v_i$)($u_{i+1}v_{i+1}$),($u_{i+1}u_{i+2}$),($u_{i+2}v_{i+1}$),)($u_{i+2}v_i$) for each i = 1,2..n.Note that here(v_i,u_i,u_{i+1},u_{i+2},v_{i+1}) forms K_5 .i+1 is taken modulo n

Definition 3.3 : Fusion of vertices :Let v ϵV(G_1), v' ϵ V(G_2) where G_1 and G_2 are twographs. We fuse v and v' by replacing them with a single vertex say w and all edges incident with v in G_1 and that

withv' in G_2 are incident with u in the new graph G=G_1FG_2.

$Deg_G u = deg_{G1}(v)_+ deg_{G2}(v')$ and $|V(G)| = |V(G_1)| + |V(G_2)| - 1$, $|E(G)| = |E(G_1)| + |E(G_2)|$

Let G_1= 

G_2= 

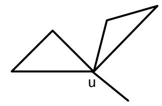
G= 

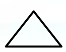
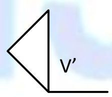
Figure3.1 fusion of vertex v and v' at vertex u

The fusion of two vertices in the same graph is described in [5].

Definition3.4: Fusion graph :let G_1 and G_2 be two graphs with $|V(G_1)|=P$. Take P copies of G_2. Choose a same fixed vertexv' in each copy of G_2.Toeach vertex in G1, fuse v' from one copy each of G_2. The resultant graph is fusion graph of G_1 and G_2 denoted by G_1 F G_2.

Note that $|V(G_1 F G_2)| = |V(G_1)| + |V(G_2)| - 1$.

$|E(G_1 F G_2)| = P|E(G_2)| + E(G_1)$

G_1= 
G_2 = 

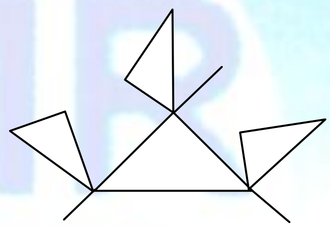
$G=G_1FG_2$ = 

Figure3.2 Fusion of graph G1 and G2

Definition3.5: Path union of graph G is Pm(G) .It is basically a path on points n and taking n copies of graph G. We fuse a copy of G at the same fixed vertex of G at each point of (verterx) of P_n.

Take G = C3 and m = 4 We get $P_4(C_3)$

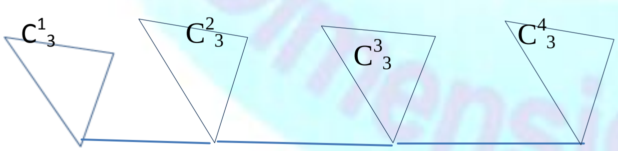


Figure 3.3: $P_4(C_3)$

4.Theorems with Proofs

Theorem 4.1:K_5FC_n is pc

Proof:Let the K_5 be given by (v_1,v_2,v_3,v_4,v_5) . The different copies of Cn fused with K5 are

C^i_n,i = 1,2,3,4,5.The consecutive vertices on i^{th} copy of Cn are v^i_j, with $v^i_1=v_i$, j=1,2..n and i= 1,..5

We define a pc function on V(G),G =K_5FCn as follows:

f:V(G) →{0,1} given by

f(v^i_j),=1 for i =1 and j=1,2..n,

f(v^i_j),=0 for i =2 and j=2..n

case n = 2x+1

Let t =3x-4.Write t =q(2x+1) +p,0≤p≤2x and q=0,1

Subcase q = 1

f(v^i_j),=1 for i =4 and j=1,2..p

f(v^i_j),=0 for i =4 and j=p+1,..n

f(v^i_j),=1 for i =5 and j=3

f(v^i_j),=0 for i =5 and j=2,4,5,6..n

f(v^i_j),=1 for i =3 and j=1,2..n

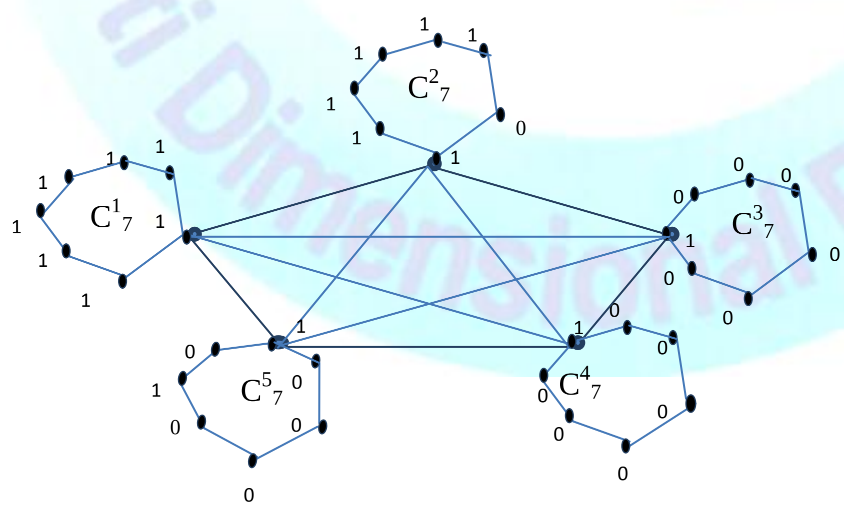


Fig 4.1 vertex labels of K_5FC_7

Subcase q=0,

$f(v^i_j)$,=1 for i =3 and j=2..t

$f(v^i_j)$,=0 for i =3 and j>t

$f(v^i_j)$,=0 for i =4 and j=2,3,..n

$f(v^i_j)$,=1 for i =5 and j=3

$f(v^i_j)$,=1 for i =5 and j=2,4,5…n

case n = 2x

Let t =3x-5.Write t =q(2x) +p,0≤p<2x ; q= 0 or 1

Subcase q = 1

$f(v^i_j)$,=1 for i =3 and j=1,2..n

$f(v^i_j)$,=1 for i =4 and j=1,2..p

$f(v^i_j)$,=0 for i =4 and j=p+1,..n

$f(v^i_j)$,=1 for i =5 and j=3

$f(v^i_j)$,=0 for i =5 and j=2,4,5,6..n

subcase q=0,

$f(v^i_j)$,=1 for i =3 and j=2..t

$f(v^i_j)$,=0 for i =3 and j>t

$f(v^i_j)$,=0 for i =4 and j=2,3,..n

$f(v^i_j)$,=1 for i =5 and j=3

$f(v^i_j)$,=1 for i =5 and j=2,4,5…n

Theorem 4.2 $S(K_5,n)$ is pc iff n is even.

Proof: The copy of K_5 at ith vertex of path P_n be K^i_5, i= 1,2,3,4…n

We define a pc function as following.

fF:V(G)→{0,1}

f(v) = 1 for all v in K^i_5,i= 1,2…,n/2

f(v) = 0 for all v in K^i_5,i =n/2, n/2+1,…..n

f(v)= 1 for all v∈(v_1,v_2,v_3..v_t,t =n/2)

f(v)= 0 for all v∈(v_{t+1},,v_{t+2},…n)

The label numbers are $v_f(0)$= t.5=$v_f(1)$and $e_f(0)$= 10t+n/2; $e_f(1)$= 10t+n/2-1#

Theorem4.3:One point union of n copies of K_mi.e $(K_m)^n$is pciff n is even.

Proof: In $(K_m)^n$let the different copies of K_m be $K^i_{m,}$i = 1to n and the vertices on K^i_m be $v^i_{j,}$ j= 1,2..m.where $v^i_1$is vertex common to all copies of K_m for all I =1..n

Define f : V($(K_m)^n$) →{0.1} as follows.

f(v^i_j)=1 for i= 1,3,5,7… and all j

f(v^i_j)= 0 for i= 2,4,6,8,.. and j =2,3,4,..m

We have

If n= 2x,$v_f(1)$= x(m-1)+1; $v_f(0)$=x(m-1) and on edges $e_f(0)$=xm(m-1)=$e_f(1)$ It follows that the graph is pc#

Theorem4.4A Kayak paddle G = KP(Cm,Cn,Pt) is pc for all m,n and t:m,n≥3,t≥2.

Proo: Let m≤n.Let the vertices on Cm be $v_1,v_2,..v_m$, with v_1 being 3 degree verterx and vertices on path being $v_1=p_1,p_2=v_{m+1},p_3=v_{m+2}.,p_t=v_{m+t-1}(p_t=u_1)$,the vertices on C_n be $u_1=p_t,u_2=v_{m+t},..u_n=v_{m+t+n-2}$,with u_1 being 3 degree verterx.

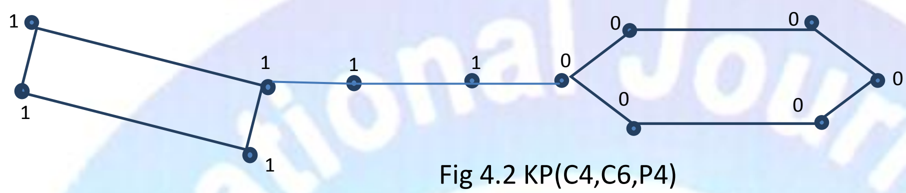


Fig 4.2 KP(C4,C6,P4)

The total number of vertices on G =Tv=m+n+t-2. Let k = Tv/2 for even Tv and k =(Tv+1)/2 otherwise.

Define a pc function f:V→{0,1} as follows:

$f(v_i)=1$ for all i = 1,2,..m,..k ;$f(v_i)= 0$ for all i>k.#

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