Nuclear Structure and Shape Phase Transitions in Actinides Thorium and Uranium Isotopes within Interacting Boson Model

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The potential energy surfaces (PES's) which describe the spherical to axially symmetric prolate deformed transition U (5) – SU(3) are obtained in the framework of the interacting boson model (sd – IBM), using the method of the interstice coherent states. The model has been shown to be applicable in the actinide region in particular it is able describe the shape transitions in$^{224\text{-}234}$Th and $^{230\text{-}238}$ U isotopes. The nuclei of the Thorium chain evolve from spherical to deformed shapes, while the nuclei of the Uranium chain are deformed and have rotational character. To get the model parameters, we performed a best fit of the IBM Hamiltonian to a selected set of energy levels, B(E2) transition rates and two- neutron separation energies by using a simulated search program. The energy and B(E2) ratios can serve as a guideline to appear the shape transitions. Our results support the interpretation of ^{226}Th as a critical point nucleus. The PES's also examined in terms of the essential parameters r_1, r_2 and the locus of the critical point in the r_1/r_2 space are given.

Keywords: Shape transitions, IBM, Intrinsic coherent states PES's, Essential parameters.

1. Introduction

In the simplest version of interacting boson model (IBM) [1] known as sd-IBM1, applicable to low-lying collective excitations in even-even nuclei, the basic building are s and d bosons, which hold angular momentum L= 0 and L= 2 respectively. These bosons interact via a Hamiltonian that is rotational invariant and number conserving and usually includes up to two-body interactions, although higher order terms have been sometimes included. The corresponding symmetry group is U(6) and posses only three possible dynamical symmetry limits, classified as U(5), SU(3) and O(6). The U(5) chain was initially proposed to describe vibrational nuclei. The other two chains describing axially symmetric deformed prolate rotor SU(3) and gamma soft O(6) nuclei respectively. For a nucleus, the Hamiltonian can be expressed as a linear combinations of the Casimir operators of all the three group chains and the nucleus can be represented as a point inside a Casten triangle [2] with the symmetry limits at the vertices.

Shape phase transition is one of the most significant topics in nuclear structure research. The critical point symmetries were introduced [3-6] in the framework of the geometric model, to predict the critical points of shape phase transitions. It was shown that the critical point of the phase transition from spherical transition U(5) to deformed gamma unstable O(6) transition and that between spherical vibrator and axially symmetric deformed prolate rotor SU(3) hold the symmetries E(5) [3] and X(5) [4] respectively. Both these symmetries focused on β degree of freedom, for which an infinite square well potential was imposed and were based on an approximate separation of β and γ dynamical modes. The most characteristic of X(5) is the $R_{4/2}$ ratio of 2.9, intermediate between vibrator 2 and rotator 3.33 values. The critical point in the phase transition from axially deformed to triaxial nuclei called Y(5) also has been analyzed [5]. Bonatsos et al [6] introduced the Z(5) critical point symmetry for the prolate to oblate nuclear phase transition. Since the introduction of these limits many theoretical and experimental studies on shape phase transition have been presented [7- 21].

According to catastrophe theory [22],the PES can be written in a special form in terms of two essential parameters r_1 and r_2 which gives the locus of the critical points in the plane formed by r_2, r_1. Also there are observable guides that are often used to study the shape through any isotopic chain like energy ratios, the electric quadruple transition rates and the two- neutron separation energies.

The purpose of this paper is to study the U(5)- SU(3) shape phase transition, and give a geometrical interpretation of the IBM analysis to the Th-U isotopes.The paper is organized as follows: The construction of the model Hamiltonian and the intrinsic coherent eigenstates are given in sections 2 and 3. Section 4 is devoted to study the PES's of the fitted IBM Hamiltonian and to analyze them in relation to the critical points. In section 5 we apply the formalism to Th-U isotopic chains and extract the model parameters. Finally in section 6, the conclusion of this work is presented.

2. Construction of IBM Hamiltonian for Shape Transitions

The sd-IBM1 include N valence bosons each of which has two states, on L= 0 (s- boson) state and an L=2 (d-boson) state with five orientations. The creation and annihilation operators for these bosons are denoted $(s^\dagger, d_\nu^\dagger;\ \nu = 0, \pm 1, \pm 2)$ and $(s, \tilde{d}_\nu, \nu = 0, \pm 1, \pm 2)$ respectively.

They satisfy the usual boson commutation relations

$$[s\,, s^\dagger] = 1\,, \qquad [\tilde{d}_\mu, d_\nu^\dagger] = \delta_{\mu\nu} \tag{1}$$

$$[s, d_\nu^\dagger] = [\tilde{d}_\nu, s^\dagger] = 0$$

Thus, the Hilbert space of the sd-IBM carries an irreducible representation of the group U(6), which has several subgroups and different phases. To describe the actinide region, the Hamiltonian used in this paper is restricted to the U(5) – SU(3) transition in the form of combination of three Casimir invariant operators (the definition for the Casimir operators have been taken from Ref [23]).

$$\mathrm{H} = \epsilon\, \mathrm{C}_1[\mathrm{U}(5)] + \frac{1}{2}\mathrm{K}_1\mathrm{C}_2[\mathrm{O}(3)] - \frac{1}{2}\mathrm{K}_2\mathrm{C}_2[\mathrm{SU}(3)] \tag{2}$$

whereC_n[G] is the n th-order Casimir operators of the group G, with

$$\mathrm{C}_1[\mathrm{U}(5)] = \hat{\mathrm{n}}_\mathrm{d} \tag{3}$$

$$\mathrm{C}_2[\mathrm{O}(3)] = 2\hat{\mathrm{L}}.\hat{\mathrm{L}} \tag{4}$$

$$C_2[SU(3)] = 2\,\hat{Q}_\chi.\hat{Q}_\chi + \frac{3}{4}\hat{L}.\hat{L}\,, \chi = -\sqrt{7}/2 \tag{5}$$

where $\hat{n}_d$, $\hat{L}$ and $\hat{Q}_\chi$ are the d-boson number operator, the angular momentum operator and the quadrupole operator respectively.

The multipole operators are defined as

$$\hat{n}_d = \sum_\mu d_\mu^\dagger\,.\,d_\mu \tag{6}$$

$$\hat{L} = \sqrt{10}\,[d^\dagger \times \tilde{d}]^{(1)} \tag{7}$$

$$\tilde{Q} = [s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} - \frac{\sqrt{7}}{2}[d^\dagger \times \tilde{d}]^{(2)} \tag{8}$$

where $[d^\dagger \times \tilde{d}]^{(L)}$stand for the L tensor coupling of the d-boson creation and annihilation operators($\tilde{d}_\mu = (-1)^\mu d_\mu$) with μ= L, L-1, L-2, …, -L is the angular momentum projection and the dot (.) denoting the scalar product defined as$\hat{T}_L.\hat{T}_L = \sum_\mu(-1)^\mu\,\hat{T}_{L,\mu}\hat{T}_{L,-\mu}$ where $\hat{T}_{L,\mu}$is the μ-component of the operator $\hat{T}_L$.

3. Construction of the Intrinsic Coherent State

To analyze the PES's and the critical behavior, the intrinsic coherent state method is used, which introduces the shape variables β and γ in the IBM. In such formalism the ground state of the nucleus with N valance bosons can be expressed by an orthogonal combination of the$s^\dagger$and$d^\dagger$ operators. We define the boson creation operator $\Gamma_c{}^\dagger$ as:

$$\Gamma_c{}^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + \beta\,\cos\gamma\,d_0^\dagger + \frac{1}{\sqrt{2}}\beta\sin\gamma\left(d_2^\dagger + d_{-2}^\dagger\right)\right] \tag{9}$$

Therefore, we get for the ground state the eigenfunction

$$|c\rangle = \frac{1}{\sqrt{N!}}\left(\Gamma_c^\dagger\right)^N|0\rangle \tag{10}$$

where$|0\rangle$ is the boson vacuum state

For γ=o (axial symmetry)

$$\Gamma_c^\dagger = \frac{\left(s^\dagger + \beta\,d_0^\dagger\right)}{\sqrt{1+\beta^2}} \tag{11}$$

The parameters β and γ are the variational parameters related to the shape variables in the geometric collective model (GCM) [12, 14, 24].

The expectation values of the above Casimir operators in equations (3-5) in the ground state coherent state equation (10) are:

$$\langle c|C_1[U(5)]|c\rangle = N\frac{\beta^2}{1+\beta^2} \tag{12}$$

$$\langle c|C_2[O(3)]|c\rangle = 12N\frac{\beta^2}{1+\beta^2} \tag{13}$$

$$\langle c|C_2[SU(3)]|c\rangle = 10N\frac{1}{1+\beta^2} + 10N\frac{\beta^2}{1+\beta^2} + N(N-1)\frac{\left(8\beta^2 + 4\sqrt{2}\beta^3\cos 3\gamma + \beta^4\right)}{(1+\beta^2)^2} \tag{14}$$

4.Geometric Analysis, PES's and Critical Points

The collective properties of the nucleus are illustrated by the PES's describing all deformation effects. We can get the PES by calculating the expectation value of the transitional Hamiltonian equation (2) on the intrinsic coherent state equation (10) using the above expectation values of the Casimir operators, yield:

$$E(N,\beta,\gamma) = N\frac{A_2\beta^2 + A_3\beta^3\cos 3\gamma + A_4\beta^4}{(1+\beta^2)^2} + A_0 \tag{15}$$

where

$$A_2 = \epsilon + 6k_1 - 4(N-1)k_2 \tag{16}$$

$$A_3 = -2\sqrt{2}(N-1)k_2 \tag{17}$$

$$A_4 = \epsilon + 6k_1 - \frac{1}{2}(N-1)k_2 \tag{18}$$

$$A_0 = -5k_2 N \tag{19}$$

The shape of the nucleus defined through the equilibrium value of the deformation parameters β and γ, which are obtained by minimizing the ground state PES's. The parameter γ represents the departure from axial symmetry. That is$\gamma = 0^\circ$ *and* $\gamma = 60^\circ$ stand for an axially symmetric deformed nucleus, prolate and oblate respectively, where any other value correspond to a triaxial shape.

A spherical nucleus has a global minimum in the PES at β = 0 while the deformed one has absolute minimum at β≠ 0. For any stable equilibrium state the first derivative of E (N, β, γ) with respect to β must be zero and the second derivative must be positive. Thus one obtain

$$A_3\,\beta^3-(4A_4-2A_2)\beta^2-3A_3\beta-2A_2=0 \tag{20}$$

$$A_3\,\beta^5-(6A_4-3A_2)\beta^4-8A_3\beta^3+(6A_4-8A_2)+3A_3\beta+A_2>0 \tag{21}$$

Thus, to find the extrema in β,one needs to solve the cubic equation

$$2\sqrt{2}(N-1)k_2\beta^3+[2\in+12k_1+6(N-1)k_2]\beta^2-6\sqrt{2}(N-1)k_2\beta+[2\in+12k_1+8(N-1)k_2]=0 \tag{22}$$

For $\in=0$, the cubic equation becomes

$$\beta^3+\frac{3}{\sqrt{2}}\alpha\beta^2-3\beta+\frac{1}{\sqrt{2}}(3\alpha-7)=0 \tag{23}$$

whereα is positive and is given by

$$\alpha=1+\frac{2}{N-1}\left(\frac{k_1}{k_2}\right) \tag{24}$$

To analyze the critical behavior for the energy functional equation (15), the antispinodal point occur when E(N, β, γ) becomes flat at β = 0 or when$\frac{\partial^2 E}{\partial\beta^2}|_{\beta=0}=0\ (A_2=0)$, which yield

$$\left(\frac{k_1}{k_2}\right)_a=\frac{3}{2}(N-1),\qquad N_a=1+\frac{3}{2}\left(\frac{k_1}{k_2}\right),\qquad \alpha_a=\frac{7}{3} \tag{25}$$

The critical point occur when $A_3^2=4A_2A_4$ which yield

$$\left(\frac{k_1}{k_2}\right)_c=\frac{3}{4}(N-1),\qquad N_c=1+\frac{3}{4}\left(\frac{k_1}{k_2}\right),\qquad \alpha_c=\frac{5}{2} \tag{26}$$

The equilibrium value of β occur when the first order derivative of E (N, β, γ) with respect to β vanishes $\frac{\partial E}{\partial\beta}=0$.

For a given $\left(\frac{k_1}{k_2}\right)_a$ shape transition occurs by changing N. For large N, $\alpha_N=1$ and the minimum occur at$\beta_e=\sqrt{2}$.

If k_1 is also equal to zero (pure SU(3)), then $N_c=1$ which implies deformed shapes for all N and the equilibrium value of β is$\beta_e=-\frac{A_3}{4A_4}=-\sqrt{2}$.

Figure (1) illustrate the PES's corresponding to the two terms $C_2[O(3)]$and $C_2[SU(3)]$ of the Hamiltonian (2) at $k_1 = 42$ KeV and $k_2 = 8$ KeV for five cases of boson numbers N = 2, 4, 8, 10, 14.

For a special case, if the original Hamiltonian contains only the d-bosn number operator $\hat{n}_d$ and the quadrupole operator $\hat{Q}(H = \epsilon\,\hat{n}_d - kQ.Q)$, then the PES can be written in the form

$$E(N,\beta,\gamma) = \left(\epsilon - \frac{9}{4}k\right)\frac{N\beta^2}{1+\beta^2} - \frac{N(N-1)k}{(1+\beta^2)^2}\left[4\beta^2 + 2\sqrt{2}\beta^3\cos 3\gamma + \frac{1}{2}\beta^4\right] \qquad (27)$$

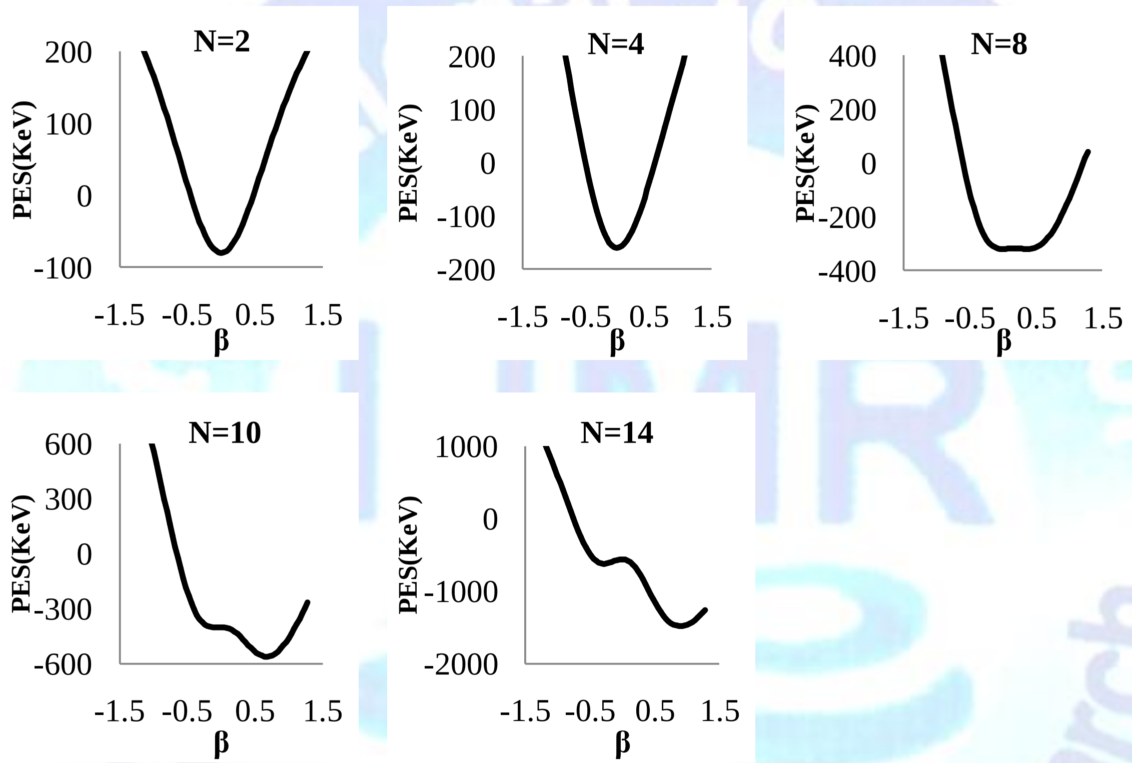

Figure (1) PES's as a function of deformation parameter β for the model parameters $\epsilon = 0, k_1 = 42$ KeV and $k_2 = 42$ KeV at five values of boson number N_B = 2, 4, 8, 10, 14.

The essential parameters r_1 and r_2[22] for the above equation take the forms

$$r_1 = \frac{-4k + \frac{\epsilon+\frac{9}{4}k}{N-1}}{3k + \frac{\epsilon+\frac{9}{4}k}{N-1}} = \frac{1-(4-e)\lambda}{1+(3-e)\lambda} \qquad (28)$$

$$r_2 = \frac{4\sqrt{2}k}{3k + \frac{\epsilon+\frac{9}{4}k}{N-1}} = \frac{4\sqrt{2}\lambda}{1+(3-e)\lambda} \qquad (29)$$

$$\text{with}\, e = \frac{\frac{9}{4}}{N-1} \quad , \lambda = \frac{k}{\epsilon}(N-1) \qquad (30)$$

Equation (27) can be rewritten in terms of the control parameter λ as:

$$\varepsilon(\mathrm{N},\beta,\gamma)=\frac{\mathrm{E}(\mathrm{N},\beta,\gamma)}{\epsilon\mathrm{N}}=\frac{\grave{A}_2\beta^2+\grave{A}_3\beta^3\cos3\gamma+\grave{A}_4\beta^4}{(1+\beta^2)^2}+\grave{A}_0 \quad (31)$$

where

$$\grave{A}_2=1-\frac{16\mathrm{N}-25}{4\mathrm{N}-4}\lambda \quad (32)$$

$$\grave{A}_3=-2\sqrt{2}\lambda \quad (33)$$

$$\grave{A}_4=1-\frac{2\mathrm{N}-11}{4\mathrm{N}-4}\lambda \quad (34)$$

$$\grave{A}_0=-\frac{5}{\mathrm{N}-1}\lambda \quad (35)$$

For antispinodal point, $\lambda_a=\frac{4\mathrm{N}-4}{16\mathrm{N}-25}$ (36)

For critical point, $81\lambda_c^2+36(\mathrm{N}-2)\lambda_c-8(\mathrm{N}-1)=0$ (37)

For equilibrium, yield the cubic equation

$$4\sqrt{2}(\mathrm{N}-1)\beta^3+[4(\mathrm{N}-1)+3(4\mathrm{N}-1)\lambda]\beta^2-12\sqrt{2}(\mathrm{N}-1)\lambda\beta+[4(\mathrm{N}-1)-(16\mathrm{N}-25)\lambda]=0 \quad (38)$$

Figure(2) illustrate the PES's for the U(5) – SU(3) transition for boson number N = 10. We find that the PES's exhibit a strong dependence on the $\frac{k_2}{\epsilon}(or\ \lambda)$. For $\lambda>\lambda_a$ the PES are very shallow, while for $\lambda=\lambda_a$ the PES display the typical flat bottomedcurves expected at the critical point ($\lambda_a=0.2666$). With increasing λ, the PES becomes deep. i.e a phase transition happen.

If we eliminate the degrees of freedom connected with the s boson in the quadrupole - quadrupole interaction, the PES takes the form

$$\mathrm{E}(\mathrm{N},\beta,\gamma)=\mathrm{N}\frac{(1-4\lambda)\beta^2-2\sqrt{2}\lambda\beta^3\cos3\gamma+(1-\frac{1}{2}\lambda)\beta^4}{(1+\beta^2)^2} \quad (39)$$

The corresponding antispinodal and critical points are located at

$$\lambda_a=\frac{1}{4},\lambda_c=\frac{2}{9}$$

and the two essential parameters r_1,r_2 of the shape diagram becomes

$$r_1 = \frac{1-4\lambda}{1+3\lambda}, \qquad r_2 = \frac{4\sqrt{2}\lambda}{1+3\lambda} \tag{40}$$

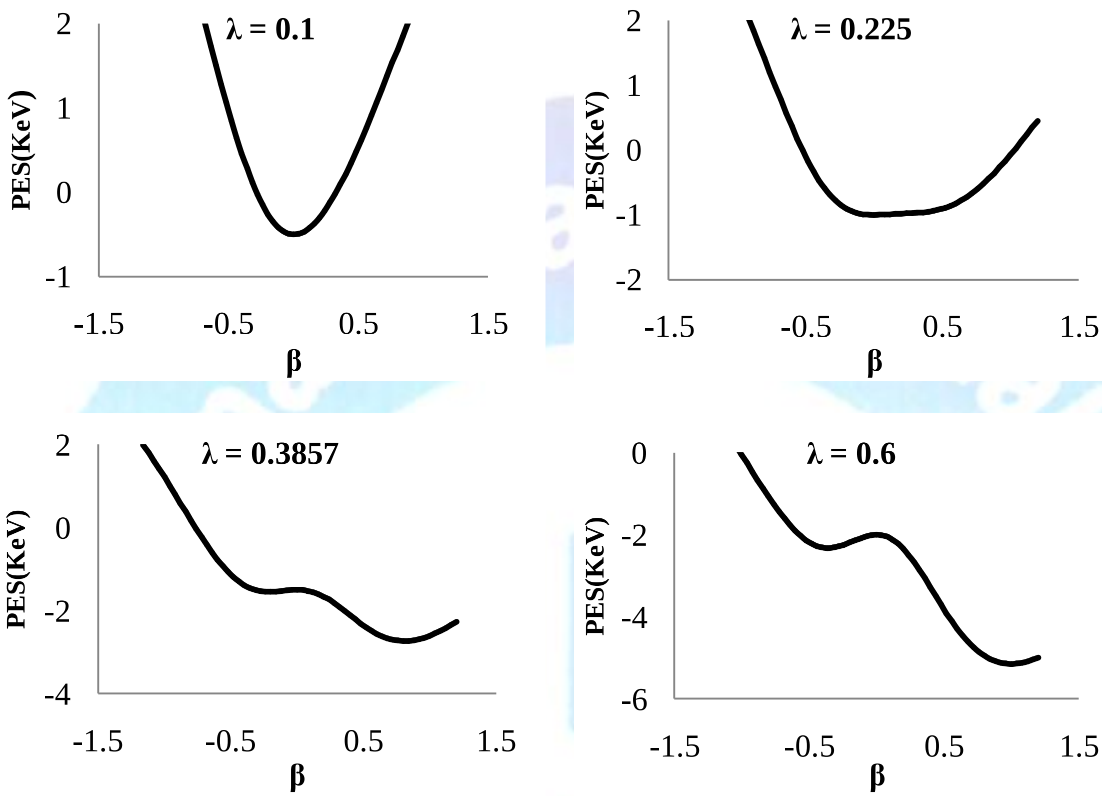

Figure (2) PES's as a function of deformation parameter β for different values of the control parameter λ at N =10. The middle curve is at critical value λ = 0.2666.

Therefore

$$(r_1)_a = 0, \qquad (r_2)_a = \frac{4\sqrt{2}}{7} \tag{41}$$

$$(r_1)_c = \frac{1}{15}, \qquad (r_2)_c = \frac{8\sqrt{2}}{15} \tag{42}$$

The equilibrium value of β is given by solving the equation

$$\sqrt{2}\lambda\beta^3 + (1+3\lambda)\beta^2 - 3\sqrt{2}\lambda\beta + (1-4\lambda) = 0 \tag{43}$$

The deformation parameter β at the critical point $(\lambda_c = \frac{2}{9})$ is given by$\beta_c = \frac{1}{2\sqrt{2}}$ (44)

4. Numerical Calculations for Thorium and Uranium Isotopic Chains

We applied our formalism to the even-even Th and U isotopes. Firstly we use the spectroscopic properties: the energy levels $2_1^+, 4_1^+, 6_1^+, 8_1^+, 0_2^+, 2_2^+, 3_1^+$and 4_2^+, B(E2) transition probabilities$2_1^+ \longrightarrow 0_1^+$, $4_1^+ \longrightarrow 2_1^+$, $2_2^+ \longrightarrow 0_1^+$ and $0_2^+ \longrightarrow 2_1^+$ and the two neutron separation energies defined as $S_{2n} = BE(N_8) - BE(N_8 - 1)$ obtained in the framework of our proposed IBM Hamiltonian equation (2) for even-even $^{224-234}$Th and $^{230-238}$U isotopic chains to fit the experimental data. In order to estimate the best set of model parameters we used aComputer simulation search program with the add of PHINT code [25]. The x^2 test is used to perform the fitting. The x^2 function is defined in the standard way as:

$$x^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{x_i(data) - x_i(IBM)}{x_i(data)}\right]^2$$

where N is the number of experimental data points, The x_i (data) andx_i(IBM) are respectively the experimental and calculated spectroscopic property. The best model parameters A_0, A_2, A_3 and $A_4$are listed in Table (1). With these parameters the PES's are calculated and presented in Figures (3, 4) as a function of deformation parameter β. From the figures we observe that the Th isotopes provide an example of transitions from spherical to axially symmetric deformed shape. At neutron number N = 134 (N_B = 8), the spectrum look like those U(5)-limit, around N = 136 (N_B=9) there seems to be the X(5) critical point symmetry, finally at N = 144 (N_B = 13)the behavior look like the rotational shape or SU(3)-limit. For U isotopic chain, all nuclei are mainly prolate deformed and have rotational SU(3) character.

One of the best signatures of shape transition is the behavior of energy ratios. So the most important characteristic feature of X(5) critical point symmetry is the$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$ energy ratio of 2.91 intermediate between vibrator $R_{4/2} = 2$ and rotor $R_{4/2} = 3.33$. In Figure (5), we give the calculated IBM energy ratios $R_{I/2} = \frac{E(I_1^+)}{E(2_1^+}$ for the ground state band for the ^{224,234}Th (N_B = 8, 13)and$^{230, 238}$U(N_B = 11, 15)nuclei compared with the dynamical symmetries U(5) and SU(3)predictions.

Table (1) Values of the model parameters A_0, A_2, A_3 and A_4 (in MeV) as derived in fitting procedure used in the calculations for Th and U isotopic chains. N_B is the total number of bosons.

isotope	N_B	A_1	A_2	A_3	A_4
^{224}Th	8	-0.2800	14.752400	-1.108743	16.124400
^{226}Th	9	-0.3375	16.148475	-1.527350	18.034750
^{228}Th	10	-0.3750	17.624750	-1.909188	19.987250
^{230}Th	11	-0.4125	19.067125	-2.333452	21.954625
^{232}Th	12	-0.4500	20.440500	-2.800142	23.905500
^{234}Th	13	-0.4875	21.785075	-3.309259	25.880075

^{230}U	11	-0.4124	19.050625	-2.333452	21.938125
^{232}U	12	-0.4500	20.422500	-2.800142	23.887500
^{234}U	13	-0.4875	21.710975	-3.309259	25.805975
^{236}U	14	-0.5250	23.007250	-3.860803	27.784750
^{238}U	15	-0.5625	24.209625	-4.454772	29.722125

For Th nuclei, this ratios varies from the values which correspond to vibration around a spherical shape to the characteristic value for excitation of a well deformed rotor. For U nuclei, the energy ratios are almost deformed shape. We have seen that the IBM can simultaneously reproduce the X(5) energy ratios. Similar conclusions are given from the behavior of reduced quadrupole transition probabilities B(E2) values. Figure (6) provide us by the B(E2) ratios $\frac{B(E2,I+2\to I)}{B(E2,2_1^+\to 0_1^+)}$ at the critical boson number N= 9compared to the dynamical symmetries U(5) and SU(3) predictions.

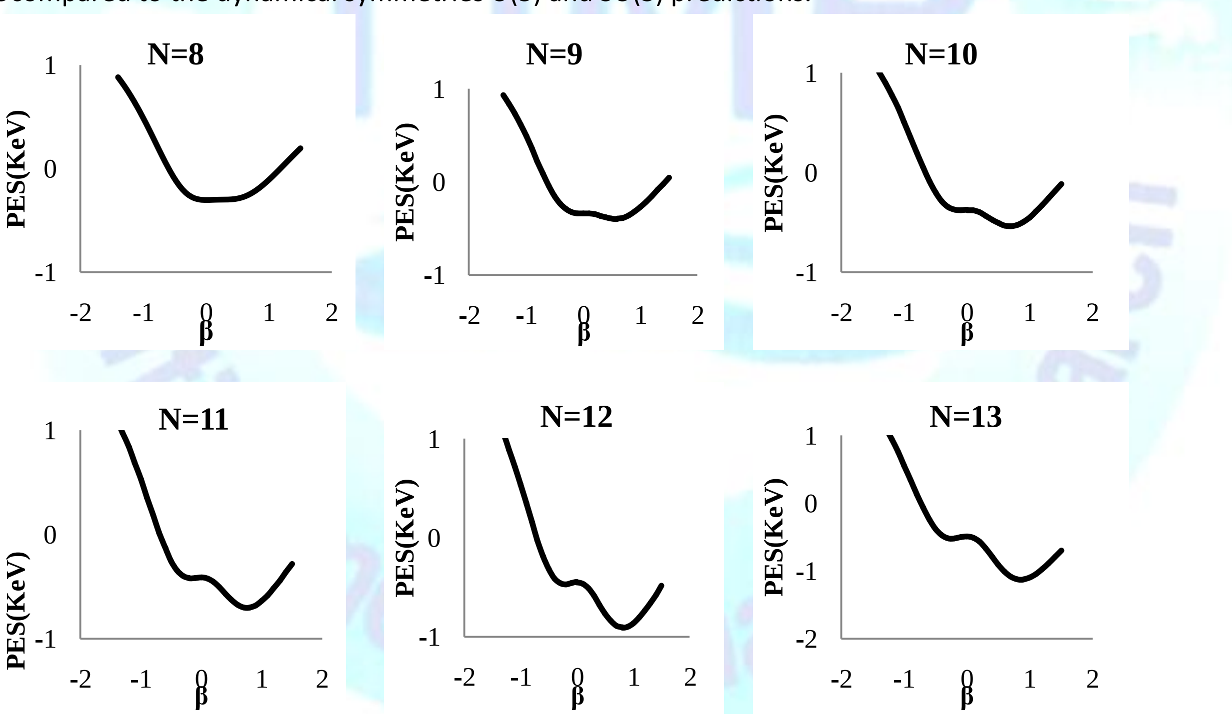

Figure (3) PES's calculated from IBM with intrinsic coherent state versus deformed parameter β corresponding to U(5) – SU(3) shape phase transition for even-even$^{224-234}$Thisotopic chain. The total number of bosons is N_B = 8-13 and $= -\sqrt{7}/2$.

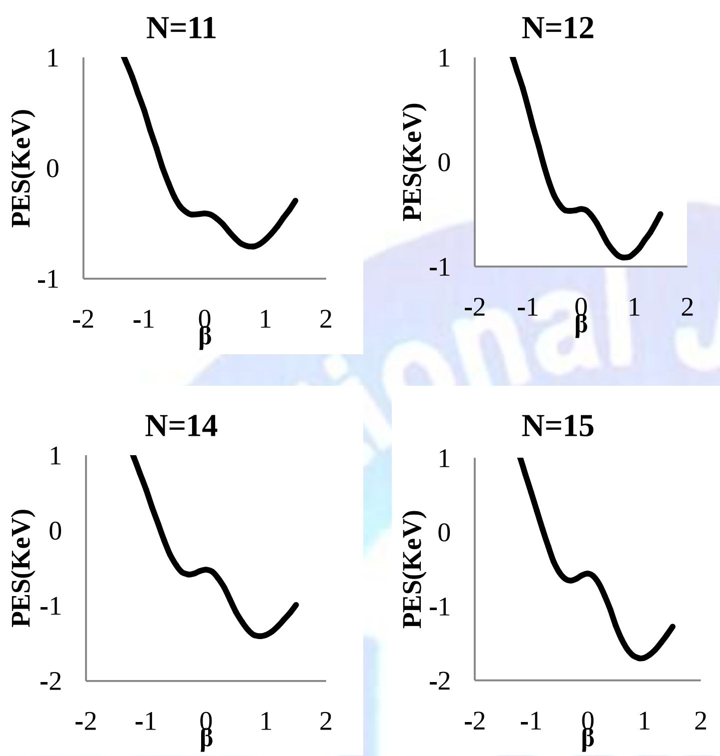

Figure (4) The same as in figure (3) but for 230-238Uisotopic chain. The total number of bosons is N_B = 11-15

An analysis with catastrophe theory shows that the values of the essential parameters r_1, r_2 for Th and U isotopic chains which exihibt a transitional region between the U(5) – SU(3)is characterized by a straight line.

$$r_1 = 1 - \frac{7}{4\sqrt{2}} r_2 , \quad -\frac{4}{3} \leq r_1 \leq 1$$

The pure two dynamical symmetry limits U(5) and SU(3) are (0, 0) and (1.88, -1.33) respectively. The numerical values of r_1, r_2 are listed in Table(2).

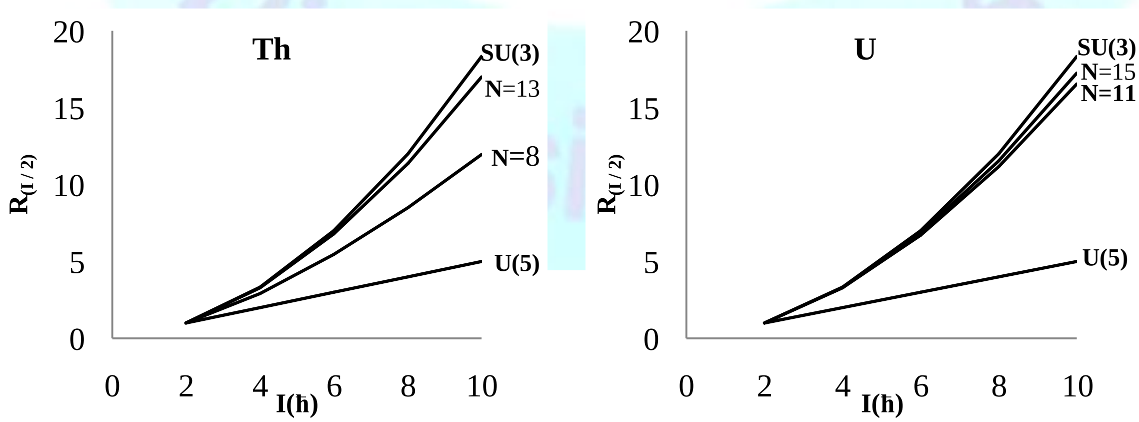

Figure (5) The calculated IBM energy ratios $R_{(I/2)} = E(I_1^+)/E(2_1^+)$ for the ground state band for $^{224,\,234}$Th (N = 8, 13) and $^{230,\,238}$U (N = 11, 15) compared with the dynamical symmetries U(5) and SU(3) prediction.

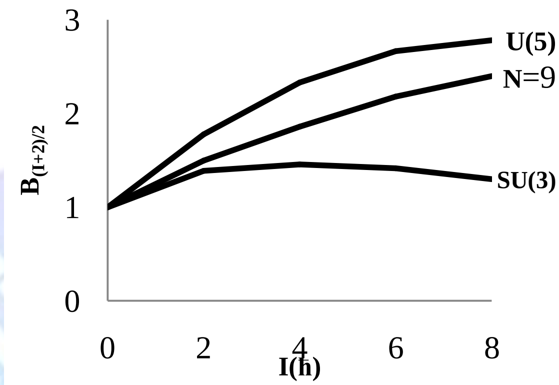

Figure (6) The calculated values of the ratios $B((I+2)/2) = \frac{B(E2, I+2 \to I)}{B(E2, 2_1^+ \to 0_1^+)}$ as a function of the spin I for the ground state band of ^{226}Th at the critical boson number N = 9 compared to the dynamical symmetries U(5) and SU(3) predictions.

Table (2) The essential parameters r_1, r_2, for Th and U isotopic chains.

isotope	N_B	r_1	r_2	isotope	N_B	r_1	r_2
^{224}Th	8	0.8431	0.1266	^{230}U	11	0.7673	0.1879
^{226}Th	9	0.8103	0.1532	^{232}U	12	0.7466	0.2047
^{228}Th	10	0.7885	0.1708	^{234}U	13	0.7260	0.2213
^{230}Th	11	0.7675	0.1878	^{236}U	14	0.7065	0.2371
^{232}Th	12	0.7468	0.2046	^{238}U	15	0.6870	0.2528
^{234}Th	13	0.7267	0.2207				

5. Conclusion

In this paper, we have studied the evolve from spherical to prolate deformed shape transition in $^{224-234}$Th and $^{230-238}$U isotopes in framework of Casimir and multipole forms of the general one – and two – body IBM Hamiltonian on the basis of intrinsic coherent state formalism. We have performed a best fit of U(5) – SU(3) sd-IBM Hamiltonian to a selected set of energy levels, B(E2) transition rates and sometwo – neutrons separation energies. The control parameter of the quadrupole operator was fixed at $-\frac{\sqrt{7}}{2}$. All calculations have been done as a function of the number of bosons (isotropic chains). We have found a good agreement between theory and experimental data particularly in the vibrational and rotational limit. It is shown that the thorium nuclei exhibit a shape transition from spherical vibrator to axially deformed rotator when moving from the lighter to heavier isotopes, while all nuclei of uranium isotopic chain are deformed and have rotational character of SU(3). The energy and B(E2) ratios are examined and also the calculations are analyzed in terms of the essential parameters r_1 and r_2 of catastrophe theory.

The outcome of this work can be utilized in separation and enrichment of thorium and uranium isotopes based on the nuclear shape deformation, which led to the nuclear field shift ion chromatography process [26].

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