**An Evaluation of condensate size and release of energy of Quasi-two dimesnional condensate for varying trap Geometries and atom numbers**

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**Abstract**

 Using hybrid variational model, condensate sizeand release energy of quasi-two dimensinal condensate for varying trap geometries and atom numbers were Calculated. Our theoretically evaluated results are in good agreement with the experimental data.

**Introduction**

Bose-Einstein condensation of dilute atomic gases has been achieved in a variety of magnetic and optical dipole force traps with different geometries. There is considerable interest in studying the properties of these ultracold gases under conditions where the confinement gives a system with dimensionality less than 3.

 Recent experiments in optical lattices have observed the properties of a one-dimensional Tonks gas in which bosons show fermionic properties 1,2 Many other experiment: phase coherence between of lattice wells was observed;3,4 collective excitations of a one- dimensional gas were studies and tree body recombination rates is a correlated ID degenerate Bose gas measured. 5 All these experiments were carried out with many individual condensates in a lattice of tightly confining potential tubes formed at the inter- section of two optical standing waves. Tunnelling between individual wells was controlled through the beam intensities . A single optical potential well was used to confine a mixture of BEC and Ferm gas where the found to have a one-dimensional character. 6

 Other experiments used a one-dimensional lattice of BECs formed by a single standing wave. Each individual condensate was confined to an extreme pancake shaped potential well and had quasi-two dimensional properties; the tunnelling between the well could be controlled by adjusting the intersity of the standing wave an oscillation atomic current in an array of Josephson junctions was studied,7 number-squeezed staes were created8 and interference between independent condensates was observed.9

Two- dimensional Bose- condensates in a single potential were studied. 10-12 However, the new physics in this regime remains to be explored: a two-dimensional Bose gas in a homogenous potential does not undergo Bose- Einstein condensation (BEC); instead there is a Berenzinskii-Kosterlitz Thouless transition (a topological phase transition mediated by the spontaneous formation of vortex pairs) , a system that is superfluid even though it does not possess long-range order. This is counter to the usual picture of sperfluidity in three dimensions explaned in terms of a macroscopic wave function describing the whole system. A recent theoretical paper 13 discusses the dependence of the condensate coherence length on temperature. It shows that even for very low temperature, at a fraction of the critical temperature, the coherence length is much smaller than the condensate size due tostrong phase fluctuations. Early experiments on the KT transition were carried out with films of superfluid 4He14-15 and more ones include the onservation of quasi condensates in thin layers of spin-polarized hydrogen.16

 In recent experiments, BECs created in convertional three-dimensional magnetic traps have been put into the quasi-two-dimensional(Q2D) regime through the addition of an optical potential In this limit the interaction energy, proportional to the chemical potential  is on the order or smaller than the harmonic oscillator level spacing. Along the tightly confined axial direction the characteristics of the condensate are those of an ideal gas and the condensate width equals to harmonic oscillator length. Only when compressing the trap much further to a point where the condensate width becomes comparable to the scattering length one finds that the coupling constant g is chaning and becomes dependent on the density. 17,18 However, such a tight compression has not been achieved in recent experiments .The crossover to the Q2D regime was first observed in10 and in11by continuously removing atoms form a highly anisotropic trap to decrease the interaction energy. In the experiment described in 12 the Q2D crossover is observed by gradually increasing the trap anisotropy form moderate to very large values whilst keeping the number of atoms fixed.

**Mathematical Formula used in the Calculation**

 Condensates are usually trapped in harmonic potentials given by

 V ext = ------------- (1)

Where the (t) denot the trap anisotropies which can in general depend on time.A quasi-two dimensional trap has For large anisotropies the condensate shape along the z direction is very similar to the Gaussian Profile of an ideal gas. However, along the weakly confined x and y axes the condensate has parabolic shape characteristic of the hydrodynamic regime. To determine the dynamics of the quasi-two-dimensional condensate one uses a variational method,17 and define the trial wave function.

 = ------------- (2)

where the normalization constant Anis given by

  =  ------------- (3)

 The condensate width *li*(t) and phase (t) parameters are functions of time and their time evolution completely describes that of the condensate. The condensate density profile is at all time restricted to a parabolic shape in the radial plane and a Gaussian shape along the highly compressed axial direction. The Lagrangian density for the nonlinear Schrodinger equation is given by

---------- (4)

with the nonlinearity parameter g = where a is the scattering length, N is the number of atoms in the condensate and m is the atomic mass. After inserting the trial wave function(2) and into Eq.(4) the corresponding lagrangian is found through integration L= the four terms of Eq.(4) lead to

 L= L1+L2+L3+L4

---------- (5)

where we obtained the quantum pressure term18 for the x and y directions(where this term is divergent due to the sharp boundaries of the condensate wave function in the hydrodynamic regime) but retained it for the z direction where the condensate assumes the Gaussian shape of an ideal non-interacting gas (as the term proportional to . the quantum presuure term is crucial in describing the dynamics. The total energy per particle Etot and the chemical potential are given by

 E tot= Ekin+Epot+Eint,=Ekin+Epot+Eint ---------- (6)

where Ekin, Epot and Eint are the kinetic, potential and interaction energy, given by the last three terms of the Lagrangian(5) respectively, the Euler lagrange equations

 ---------- (7)

Yield the dynamic equagtion for the condensate whith 1i and phase  we find for the widths

  ---------- (8)

 After differentiating Eqs.(8) once more with respect to time one can express the resulting second equation in terms of the 1i alone

  ---------- (9)

where =1 for i=z and 0 otherwise. It is convenient to express the above equation in diemensionless quantities. so we introduces the dimensionless time  and widths di defined by

  ---------- (10)

where a0 = is the harmonic oscillator length . in terms of quantities Eq(9) can be rewritten as

  ---------- (11)

wherer the constant Cp = . To find ground state of Eqs.(11) one has to set the left side equal to zero and solve the remaining coupled nonlinear equations:

 ---------- (12)

This cannot be done analytically but it is straight toforward to find a numerical solution the (o), i= x,y,z are defined as the trap anisotropies at time t=0 when the condensate is in the ground state. The dio=di(o) are the ground state solution of Eq. (11) i.e. the solutions for the condensate widths di when the time derivative is set to zero.

After some algebra and using various symmetries the three coupled equations can be reduced to one polynomial equation introducing new dimensionless units Di defined as the ground state condensate widths 1i0 normalised by the axial harmonic oscillator length az, i.e., Di=1i0/az the polynomial equation can be written as

  ---------- (13)

where DZ = There is only one real and positive solution to this equation For the x and y widths we find

 ---------- (14)

One now examines the case where the anisotropy becomes very large. A solution to Eq.(13) is then given by neglection the first term on the right- hand side (RHS) and solving the remaining equation We find that  and thus the approximate solution is given by the axial harmonic oscillator length.

 Izo = ---------- (15)

It is the minimum width the condensate shape can attain and it is also the solution for the width of a noninteracting gas. for this reason the gas along the z direction is said to have the characteristics of and ideal noniteracting gas.

In order to calculate the release energy Erel of the condensate one takes equation 6(b) and one puts potential term equal to zero and the inertial to term is very small in that case

Ekin=Erel ---------- (16)

Then from lagragian equation (5) one obtains an expression for release energy

Erel=Ekin = ---------- (17)

After an initial calculation, when released form the trap, the condensate moves with constant velocity and after long enough time of flight one can approximate  where t is the time of flight. The release energy is then written as

 Erel ---------- (18)

**Discussion of Results**

Using the hybrid vibrational model developed by G. Hechenblaikner et. al19 we have evaluated the condensate size and release energy of quasi-two dimensional condensate for varying trap geometries and atom numbers. The data12 are taken or traps with initial oscillation frequency before release of Hz and 960 Hz. The atom numbers are 8x104 and 1.1x105. we have evaluated for two optical traps 1 and 2 respectively. Our theoretical value of the axial condensate size (in) shows that size increases with the radial frequency for both the traps. Our theoretically obtained values are also in good agreement with the experimental data12 the evaluated result for release energy(in the unit of) as a function of radial frequency indicates that the energy also increase with radial frequency for both the optical traps. These findings are in good agreement with the experimental data. Results an shown in table T1 and T2 respectively.

Table T1

Evaluated result for the condensate size as a function of radial frequency for two diiferent traps

|  |  |
| --- | --- |
| Redial frequency HZ | Axial size () |
| Theoretical result | Expt. results |
| Optical Part I | Optical Part II | Optical Part I | OpticalPart II |
| 2 | 50.5 | 32.6 | 48.2 | 34.8 |
| 4 | 52.8 | 38.5 | 50.5 | 36.2 |
| 5 | 55.2 | 40.2 | 52.4 | 38.5 |
| 6 | 57.8 | 42.6 | 54.7 | 40.6 |
| 7 | 58.2 | 44.3 | 55.6 | 42.5 |
| 8 | 59.6 | 46.8 | 56.5 | 45.5 |
| 10 | 60.5 | 50.5 | 58.8 | 48.6 |
| 15 | 65.6 | 52.4 | 60.4 | 50.2 |
| 20 | 69.7 | 55.3 | 62.7 | 53.7 |
| 25 | 72.6 | 57.8 | 67.7 | 54.8 |
| 30 | 75.3 | 59.2 | 70.4 | 55.7 |
| 40 | 79.6 | 63.5 | 72.5 | 58.2 |
| 45 | 82.5 | 65.2 | 75.5 | 60.5 |
| 50 | 85.6 | 67.8 | 79.8 | 63.5 |

Table T2

Evaluated result of the release energy in the unit of  as a function of radial frequency

|  |  |
| --- | --- |
| Radial frequency (Hz) | Release energy () |
| Theoretical result | Expt. results |
| Optical part I | Optical part II | OPtical part I | Optical part II |
| 2 | 0.352 | 0.278 | 0.365 | 0.299 |
| 4 | 0.376 | 0.293 | .0.387 | 0.338 |
| 5 | 0.395 | 0.318 | 0.418 | 0.375 |
| 6 | 0.417 | 0.336 | 0.455 | 0.417 |
| 7 | 0.428 | 0.352 | 0.517 | 0.446 |
| 8 | 0.435 | 0.387 | 0.566 | 0.502 |
| 10 | 0.458 | 0.415 | 0.598 | 0.566 |
| 15 | 0.556 | 0.448 | 0.653 | 0.612 |
| 20 | 0.605 | 0.463 | 0.698 | 0.667 |
| 25 | 0.702 | 0.486 | 0.738 | 0.708 |
| 30 | 0.817 | 0.538 | 0.854 | 0.807 |
| 40 | 0.935 | 0.607 | 0.966 | 0.917 |
| 50 | 1.106 | 0.779 | 1.138 | 1.108 |
| 60 | 1.158 | 0.958 | 1.200 | 1.123 |

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