# $(\alpha,\beta)$- Cut of N-Generated Intuitionistic Fuzzy Groups

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***Abstract*** — In this paper, we construct a new structure of Fuzzy set called $n-$generated Intuitionistic Fuzzy sets which is constructed from Intuitionistic Fuzzy Multi set and define $n-$ generated Intuitionistic Fuzzy subgroup and prove some basic properties and theorems based on $(\alpha,\beta)$-Cut of $n-$generated Intuitionistic Fuzzy sets and fuzzy subgroups

***Keywords***— Fuzzy set, multi-fuzzy set, fuzzy subgroup, multi-fuzzy subgroup, Intuitionistic fuzzy set, multi-Intuitionistic fuzzy set, $n-$generated fuzzy subset, $n-$generated fuzzy subgroups, n-generated intuitionistic fuzzy set, $n-$generated fuzzy level subsets, $(\alpha,\beta)$-Cut of Intuitionistic fuzzy subgroup, $(\alpha,\beta)$-Cut of $n-$generated Intuitionistic Fuzzy Group

## 1.INTRODUCTION

After the introduction of the concept of fuzzy sets b L.A.Zadeh [1], researchers were conducted the generalizations of the notion of fuzzy sets, A. Rosenfeld [2] introduced the concept of fuzzy group and the idea of "intuitionistic fuzzy set" was first published by K.T. Atanassov [3].Multi set theory was introduced by W.D.Blizard[4]. As a generalization of Multisets Yager [5] introduced the concept of Fuzzy Multi set (FMS). Shinoj. T.K and Sunil Jacob John [6] introduced the concept of Intuitionistic Fuzzy Multi sets and proved some basic operations such as union, intersection, addition, multiplication, etc. Cartesian product and $\alpha\beta$-cut of Intuitionistic Fuzzy Multi sets are defined and their various properties are discussed. A.solairaju, S.rethinakumar,M Maria Arockia Raj[7] introduce the concept of . $n-$ Generated fuzzy sets and its subgroups. P.K.Sharma develop the idea of $(\alpha,\beta)$-cut of intuitionistic fuzzy subgroup. In this chapter we introduce some basic properties of $(\alpha,\beta)$-cut of $n-$generated fuzzy subgroups of a group

## 2. PRELIMINARIES

**Definition.2.1. Let** $X$ **be a non-empty set. A** fuzzy set $A$ **drawn from** $X$ **is defined as** $A=\{x\in X/(x,\mu_A(x))\}$ **Where** $\mu_A(x):X\rightarrow[0,1]$ **is the membership function of the fuzzy set** $A$

**Example:2.2**

Let $X=\{1,2,3,4\}$ be a universal non empty set.A fuzzy set drawn from $X$ is as follows

$$A=\{(1,\ 0.1),(2,0.5),(3,\ 0.5),(4,0.9)\}$$

**Definition.2.3** Let $A$ be a fuzzy subset of a set $X$. For $t\in$[0, 1], $A_t=\{x\in X/A(x)\geq t\}$ is called a level fuzzy subset of $A$

**Example: 2.4**

Consider the fuzzy set

$A=\{(1,\ 0.1),(2,0.5),(3,\ 0.5),(4,0.9)\}$ .For $t=0.5\in[0,1]$ $A_t=\{x\in X\ /\ A(x)\geq 0.5\}=\{2,3,4\}$

**Definition2.5.** Let $X$ be a non empty set. An Intuitionistic Fuzzy set $A$ on $X$ is an object having the form $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle / x\in X\}$, where $\mu_A : X\to[0,1]$ & $\gamma_A : X\to[0,1]$ are the degree of membership and non- membership functions respectively with $0\leq\mu_A(x)+\gamma_A(x)\leq 1$

**Example:2.6**

Let $X=\{x,y,z,w\}$ be a universal non empty set. The Intuitionistic fuzzy set on X is as follows $A=\{\langle x,(0.3,0.2)\rangle,\langle y,(0.8,0.2)\rangle,\langle z,(1,0.5)\rangle\}$

**Definition.2.7**

Let $X$ be a non-empty set. A Fuzzy Multi set (FMS) $A$ drawn from $X$ is characterized by a function 'Count membership' of $A$ denoted by $CM_A$ such that $CM_A : X\to Q$ where $Q$ is the set of all crisp finite set drawn from the unit interval $[0,1]$.Then for any $x\in X$, the value $CM_A(x)$ is a crisp multiset drawn from $[0,1]$. For each $x\in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_{A_1}(x),\mu_{A_2}(x),........\mu_{A_k}(x)\right)$ where $\mu_{A_1}(x)\geq\mu_{A_2}(x)\geq........\geq\mu_{A_k}(x)$

$$A=\left\{\left\langle x:\left(\mu_{A_1}(x),\mu_{A_2}(x)......\mu_{A_k}(x)\right)\right\rangle : x\in X\right\}$$

**Example: 2.8**

Let $X=\{x,y,z,w\}$ be a universal non empty set.For each $x\in X$, we can write a Fuzzy Multi set as follows

$A=\{\langle x,(0.8,0.7,0.7,0.6)\rangle,\langle y,(0.8,0.5,0.2)\rangle,\langle z,(1,0.5,0.5)\rangle\}$ Where

$CM_A(x)=(0.8,\ 0.7,\ 0.7,\ 0.6)$ with $0.8\geq 0.7\geq 0.7\geq 0.6$

**Definition.2.9**

Let $X$ be a non- empty universal set and let $A$ be an Fuzzy Multi set on $X$. The n-generated Fuzzyset on $X$ is constructed from the Fuzzy Multi set and is defined as

$$\lambda=\left\{\left\langle x,\frac{1}{k}\left(\mu^n_{A_1}(x)+\mu^n_{A_2}(x)+.......\mu^n_{A_k}(x)\right)\right\rangle : x\in X\right\}$$

where $\mu_{A_1}^{n}(x) \geq \mu_{A_2}^{n}(x) \geq ....... \geq \mu_{A_k}^{n}(x)$ and $n$ is the dimension of the Fuzzy Multiset $A$

**Example: 2.10**

Let $X = \{x, y, z, w\}$ be a universal non empty set and let

$A = \{\langle x, (0.8, 0.7, 0.7, 0.6)\rangle, \langle y, (0.8, 0.5, 0.2)\rangle, \langle z, (1, 0.5, 0.5)\rangle\}$ Then

$\lambda = \{(x, \mu_A(x)), (y, \mu_A(y)), (z, \mu_A(z))\}$, where

$$\mu_A(x) = \frac{1}{4}\left[(0.8)^4 + (0.7)^4 + (0.7)^4 + (0.8)^4\right] = 0.32$$

$$\mu_A(y) = \frac{1}{3}\left[(0.8)^3 + (0.5)^3 + (0.2)^3\right] = 0.2$$

$$\mu_A(y) = \frac{1}{3}\left[(1)^3 + (0.5)^3 + (0.5)^3\right] = 0.41$$

i.e. $\lambda = \{(x, 0.32), (y, 0.2), (z, 0.4)\}$

**Definition.2.11**

Let A be a multi-fuzzy subset of X. For $t_i \in [0,1],\ i = 1, 2, ..., k,\ A_{t_i} = \{x \in X / A(x) \geq t_i\}$ is called multi-level subset of A

**Definition.2.12**

Let $\mu = (\mu_1, \mu_2, ..., \mu_k)$ and $\nu = (\nu_1, \nu_2, ..., \nu_k)$ be two multi-fuzzy sets in X of dimenstion k and n respectively. A multi-fuzzy mapping is a mapping $F : M^k FS(X) \to M^n FS(X)$ which maps each $\mu \in M^k FS(X)$ into a unique muti-fuzzy set $\nu \in M^n FS(X)$

**Definition.2.13**

A mapping $F : M^k FS(X) \to M^2 FS(X)$ is said to be an Atanassov Intuitionistic Fuzzy Sets Generating Maps(AIFSGM) if $F(\mu)$ is an Intuitionistic fuzzy set in $M^2 FS(X)$

**Definition2.14**

Let $f : X \to Y$ and $h : \prod M_i \to \prod L_j$ be functions. The Multi-fuzzy extension and the inverse of the extension are $f : \prod M_i^X \to \prod L_j^Y,\ f^{-1} : \prod L_j^Y \to \prod M_i^X$ defined by

$$f(A)(y) = \sup_{x \in f^{-1}(y)} h[A(x)],\ A \in \prod M_i^X, y \in Y$$

and

$f^{-1}(B)(x) = h^{-1}\left[B(f(x)\right],\ B \in \prod L_j^{\,Y}, x \in X$ where $h^{-1}$ is the upper adjoint of h . The function $h : \prod M_i \to \prod L_j$ is called the bridge function of the multi-fuzzy extension of $f$

**Definition.2.15**

Let $X$ be a non- empty universal set and let $A$ be an Fuzzy Multi set on $X$. The n-generated Fuzzyset on $X$ is constructed from the Fuzzy Multi set and is defined as

$$\lambda = \left\{ \left\langle x, \frac{1}{k}\left( \mu_{A_1}^{n}(x) + \mu_{A_2}^{n}(x) + .......\mu_{A_k}^{n}(x) \right) \right\rangle : x \in X \right\}$$

where $\mu_{A_1}^{n}(x) \geq \mu_{A_2}^{n}(x) \geq ....... \geq \mu_{A_k}^{n}(x)$ and $n$ is the dimension of the Fuzzy Multiset $A$

**Example: 2.16**

Let $X = \{x, y, z, w\}$ be a universal non empty set and let

$A = \left\{ \left\langle x, (0.8, 0.7, 0.7, 0.6) \right\rangle, \left\langle y, (0.8, 0.5, 0.2) \right\rangle, \left\langle z, (1, 0.5, 0.5) \right\rangle \right\}$ Then

$\lambda = \{(x, \mu_A(x)), (y, \mu_A(y)), (z, \mu_A(z))\}$, where

$$\mu_A(x) = \frac{1}{4}\left[ (0.8)^4 + (0.7)^4 + (0.7)^4 + (0.8)^4 \right] = 0.32$$

$$\mu_A(y) = \frac{1}{3}\left[ (0.8)^3 + (0.5)^3 + (0.2)^3 \right] = 0.2$$

$$\mu_A(y) = \frac{1}{3}\left[ (1)^3 + (0.5)^3 + (0.5)^3 \right] = 0.41$$

i.e. $\lambda = \{(x, 0.32), (y, 0.2), (z, 0.4)\}$

**Definition2.17**

Let $X$ be a non-empty set. A **Intuitionistic Fuzzy Multiset** $A$ denoted by IFMS drawn from $X$ is characterized by two functions' Count membership' of $A\left(CM_A\right)$ and 'Count non-membership of $A\left(CN_A\right)$ given respectively by $CM_A : X \to Q$ and $CN_A : X \to Q$ where $Q$ is the set of all crisp multi sets drawn from the unit interval [0,1], such that for each $x \in X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CM_A$ which is denoted by $\left( \mu_{A_1}(x), \mu_{A_2}(x), ........\mu_{A_k}(x) \right)$ where $\mu_{A_1}(x) \geq \mu_{A_2}(x) \geq ........ \geq \mu_{A_k}(x)$ and the corresponding nonmember ship sequence will be denoted by $\left( \nu_{A_1}(x), \nu_{A_2}(x), ........\nu_{A_k}(x) \right)$

such that $0 \le \mu_{A_i}(x) + \nu_{A_i}(x) \le 1, \quad i = 1,2,3......k$ for every $x \in X$ and is denoted by

$$A = \left\{ \left\langle x : \left( \mu_{A_1}(x), ... \mu_{A_k}(x) \right), \left( \nu_{A_1}(x), ... \nu_{A_k}(x) \right) \right\rangle : x \in X \right\}$$

**Remark:**.

Note that since we arrange the membership sequence in decreasing order, the corresponding non-membership sequence may not be in decreasing or increasing order.

**Example:2.18**

Let $X = \{x, y, z, w\}$ be a universal non empty set The Intuitionistic Fuzzy Multiset on X is defined as

$$A = \begin{Bmatrix} \langle x, (0.3, 0.2), (0.4, 0.5) \rangle, \\ \langle y, (1, 0.5, 0.5), (0, 0.5, 0.2) \rangle, \\ \langle z, (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7) \rangle \end{Bmatrix}$$

**Definition.2.19**

Let $X$ be a non-empty set and let $A$ be an Intuitionistic fuzzy Multiset of $X$. An **n-generated intuitionistic fuzzy set** (NGIFS) on $X$ is an object of the form

$$\delta A = \left\{ \begin{pmatrix} x, \frac{1}{k}\left( \mu^n_{A_1}(x) + \mu^n_{A_2}(x) + ......\mu^n_{A_k}(x) \right), \\ \frac{1}{k}\left( \nu^n_{A_1}(x) + \nu^n_{A_2}(x) + ......\nu^n_{A_k}(x) \right) \end{pmatrix} : x \in X \right\} = \left\{ \langle x, \mu_A, \nu_A \rangle : x \in X \right\}$$

where $\mu_{\delta_A} : X \to [0,1]$ and $\nu_{\delta_A} : X \to [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \le \mu_{\delta_A}(x) + \nu_{\delta_A}(x) \le 1$, $n$ is the dimension of IFMS

**Example:2.20** Let $X = \{x, y, z, w\}$ be a universal non empty stand let

$$A = \begin{Bmatrix} \langle x, (0.3, 0.2), (0.4, 0.5) \rangle, \langle y, (1, 0.5, 0.5), (0, 0.5, 0.2) \rangle, \\ \langle z, (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7) \rangle \end{Bmatrix}$$

be an IFMS on $X$ then the N generated Intuitionistic Fuzzy set is constructed as follows

$$\delta A=\left\{\begin{array}{l}\left\langle x,\frac{1}{2}\left[(0.3)^2+(0.2)^2\right],\frac{1}{2}\left[(0.4)^2+(0.5)^2\right]\right\rangle,\left\langle \begin{array}{l} y,\frac{1}{3}\left[(1)^3+(0.5)^3+(0.5)^3\right], \\ \frac{1}{3}\left[(0)^3+(0.5)^3+(0.2)^3\right]\end{array}\right\rangle, \\ \left\langle \begin{array}{l} z,\frac{1}{4}\left[(0.5)^4+(0.4)^4+(0.3)^4+(0.2)^4\right] \\ ,\frac{1}{4}\left[(0.4)^4+(0.6)^4+(0.6)^4+(0.7)^4\right]\end{array}\right\rangle \end{array}\right\}$$

$$=\left\{\langle x,0.06,0.20\rangle,\langle y,0.41,\ 0.04\rangle,\langle z,\ 0.02,\ 0.13\rangle\right\}$$

**Definition. 2.21**

Let $A=\left\{\left\langle x,\lambda_A(x),\lambda_A(x)\right\rangle : x\in X\right\}$ and $B=\left\{\left\langle x,\lambda_B(x),\lambda_B(x)\right\rangle : x\in X\right\}$ be any two INGFS's of X, then

(1). $A\subseteq B \Leftrightarrow \mu_A(x)\le\mu_B(x)\ \ \&\ \ \nu_A(x)\ge\nu_B(x)\quad \forall\ x,y\in X$

(2). $A=B \Leftrightarrow \mu_A(x)=\mu_B(x)\ \ \&\ \ \nu_A(x)=\nu_B(x)\quad \forall\ x,y\in X$

(3). $A\cap B=\left\{\left\langle x,(\mu_A\cap\mu_B)(x),(\nu_A\cap\nu_B)(x)\right\rangle : x\in X\right\}$ where

$(\mu_A\cap\mu_B)(x)=\text{Min}\left\{\mu_A(x),\mu_B(x)\right\}=\mu_A(x)\wedge\mu_B(x)$

$(\nu_A\cap\nu_B)(x)=\text{Max}\left\{\nu_A(x),\nu_B(x)\right\}=\nu_A(x)\vee\nu_B(x)$

(4). $A\cup B=\left\{\left\langle x,(\mu_A\cup\mu_B)(x),(\nu_A\cup\nu_B)(x)\right\rangle : x\in X\right\}$ where

$(\mu_A\cup\mu_B)(x)=\text{Max}\left\{\mu_A(x),\mu_B(x)\right\}=\mu_A(x)\vee\mu_B(x)$

$(\nu_A\cup\nu_B)(x)=\text{Min}\left\{\nu_A(x),\nu_B(x)\right\}=\nu_A(x)\wedge\nu_B(x)$

(5). $A+B=\left[\left\{x,\left(\mu_A+\nu_A-\mu_A\nu_A\right),\left(\mu_B+\nu_B-\mu_B\nu_B\right)\right\};x\in X\right]$

(6). If $A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle : x\in X\right\}$, then

$7.(A)^C=A=\left\{\left\langle x,1-\mu_A(x),1-\nu_A(x)\right\rangle : x\in X\right\}$

**Definition.2.22**

Let $G$ be a group. A fuzzy subset $A$ of $G$ is said to be a **fuzzy subgroup** of $G$ if

$(i).A(xy)\ge\min\left\{A(x),A(y)\right\}$

$(ii).A(x^{-1})\ge A(x)\quad \forall\ x,y\in G$

**Definition2.23**

An IFS $A=\left\{\left(x,\mu(x),\nu(x)\right): x\in X\right\}$ of a group $G$ is said to be **intuitionistic fuzzy subgroup** of $G$ (In short IFSG) if (i). $\mu(xy)\ge\mu(x)\wedge\mu(y)$, (ii). $\mu(x^{-1})=\mu(x)$, (iii). $\nu(xy)\le\nu(x)\wedge\nu(y)$, (iv). $\nu(x^{-1}) = \nu(x),$ for all $x, y \in G$

**Definition2.24**

An MIFS $A = \left\{ \left( x, \mu_{A_i}(x), \nu_{A_i}(x) \right) : x \in X \right\}$ of a group $G$ is saidto be **multi- intuitionistic fuzzy subgroup** of $G$ (In short MIFSG) if

(i). $\mu_{A_i}(xy) \geq \mu_{A_i}(x) \wedge \mu_{A_i}(y)$ (ii). $\mu_{A_i}(x^{-1}) = \mu_{A_i}(x)$

(iii). $\nu_{A_i}(xy) \leq \nu_{A_i}(x) \wedge \nu_{A_i}(y)$

(iv). $\nu_{A_i}(x^{-1}) = \nu_{A_i}(x),$ for all $x, y \in G$

**Definition.2.25**

An NGIFS $\lambda = \left\{ \left( x, \mu_A(x), \nu_A(x) \right) : x \in X \right\}$ of a group $G$ is said to be **n-generated intuitionistic fuzzy subgroup** of $G$ (In short NGIFSG) if

(i). $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$

(ii). $\mu_A(x^{-1}) = \mu_A(x)$

(iii). $\nu_A(xy) \leq \nu_A(x) \wedge \nu_A(y)$

(iv). $\nu_A(x^{-1}) = \nu_A(x),$ *for all* $x, y \in G$

**Definition.2.26**

An IFSG $A = \left\{ \left( x, \mu(x), \nu(x) \right) : x \in X \right\}$ of a group $G$ is said to be **intuitionistic fuzzy normal subgroup** of $G$ (IFNSG) if

(i). $\mu(xy) = \mu(yx)$

(ii). $\nu(xy) = \nu(yx)$ for all $x, y \in G$

**Remark.2.27**

An IFSG $A$ of a group $G$ is normal iff (i). $\mu(g^{-1}xg) = \mu(x)$

(ii). $\nu(g^{-1}xg) = \nu(x)$ for all $x, \in A,\ g \in G$

**3.** $(\alpha, \beta)$ **- Cut of N-Generated Intuitionistic Fuzzy Set (In short NGIFS) and their properties.**

**Definition3.1.**

Let $A$ be an n-generated intuitionistic fuzzy set(NGIFS) on $X$. Then $(\alpha, \beta)$ -Cut of $A$ is a crisp subset of the NGIFS of $A$ is given by

$C_{\alpha,\beta}(A) = \left\{ x \in X \,/\, \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta \right\}$ where

$\alpha,\beta\in[0,1]$, with $\alpha+\beta\le 1$ $\mu_A(x)=\frac{1}{k}\sum_{i=1}^{k}\mu^n_{A_i}(x),\ \nu_A(x)=\frac{1}{k}\sum_{i=1}^{k}\nu^n_{A_i}(x),\ n>1,$

**Proposition:3.2**

If $A$ and $B$ be two NGIFS's of a universe set $X$, then the following holds

(i). $C_{\alpha,\beta}(A)\subseteq C_{\delta,\theta}(A)$ if $\alpha\ge\delta$ and $\beta\le\theta$

(ii). $C_{1-\beta,\beta}(A)\subseteq C_{\alpha,\beta}(A)\subseteq C_{\alpha,1-\alpha}(A)$

(iii). $A\subseteq B\Rightarrow C_{\alpha,\beta}(A)\subseteq C_{\alpha,\beta}(B)$

(iv). $C_{\alpha,\beta}(A\cap B)=C_{\alpha,\beta}(A)\cap C_{\alpha,\beta}(B)$

(v). $C_{\alpha,\beta}(A\cup B)\supseteq C_{\alpha,\beta}(A)\cup C_{\alpha,\beta}(B)$ equal if $\alpha+\beta=1$

**Proof. (i).**

Let $x\in C_{\alpha,\beta}(A)\Rightarrow$ $\mu_A(x)\ge\alpha,\ \nu_A(x)\le\beta$ . ....(1)

Given $\delta\le\alpha\le\mu_A(x)$ and $\theta\ge\beta\ge\mu_B(x)$.

Hence $\mu_A(x)\ge\delta$ and $\nu_A(x)\le\theta$

This implies $x\in C_{\delta,\theta}(A)$ ...... (2).

From (1) and (2) $C_{\alpha,\beta}(A)\subseteq C_{\delta,\theta}(A)$

(ii).Since $\alpha+\beta\le 1$, implies that $1-\beta\ge\alpha$ and $\beta\le\beta$

By part.(i). $C_{1-\beta,\beta}(A)\subseteq C_{\alpha,\beta}(A)$ .......... (3)

Again $\alpha+\beta\le 1$, implies that $\alpha\ge\alpha$ and $\beta\le 1-\alpha$ By part.(i). $C_{\alpha,\beta}(A)\subseteq C_{\alpha,1-\alpha}(A)$ .......... (4)

From (3) and (4) $C_{1-\beta,\beta}(A)\subseteq C_{\alpha,\beta}(A)\subseteq C_{\alpha,1-\alpha}(A)$

(iii). $x\in C_{\alpha,\beta}(A)\Rightarrow$ $\mu_A(x)\ge\alpha,\ \mu_B(x)\le\beta$

$A\subseteq B\Rightarrow\mu_B(x)\ge\mu_A(x)\ge\alpha,$ and $\nu_B(x)\le\nu_A(x)\le\beta$

$\Rightarrow\mu_B(x)\ge\alpha,$ and $\nu_B(x)\le\beta$ and so $x\in C_{\alpha,\beta}(B)$ be a Hence $C_{\alpha,\beta}(A)\subseteq C_{\alpha,\beta}(B)$

(iv).Since $A\cap B\subseteq A$ and $A\cap B\subseteq A$

Therefore by part (i)

$C_{\alpha,\beta}(A\cap B)\subseteq C_{\alpha,\beta}(A)$ and $C_{\alpha,\beta}(A\cap B)\subseteq C_{\alpha,\beta}(B)$

$\Rightarrow C_{\alpha,\beta}(A\cap B)\subseteq C_{\alpha,\beta}(A)\cap C_{\alpha,\beta}(B)$ ......... (5)

Let $x\in C_{\alpha,\beta}(A)\cap C_{\alpha,\beta}(B)\Rightarrow x\in C_{\alpha,\beta}(A)$ and $x\in C_{\alpha,\beta}(B)$

$\Rightarrow \mu_A(x) \geq \alpha$ & $\nu_A(x) \leq \beta$ and $\mu_B(x) \geq \alpha$ & $\nu_B(x) \leq \beta$

$\Rightarrow \mu_A(x) \geq \alpha$ & $\mu_B(x) \geq \alpha$ and $\nu_A(x) \leq \beta$ & $\nu_B(x) \leq \beta$

$\Rightarrow \mu_A(x) \geq \alpha \wedge \mu_B(x) \geq \alpha \geq \alpha$ and $\nu_A(x) \geq \alpha \vee \nu_B(x) \leq \beta$

$\Rightarrow \left(\mu_A \cap \mu_B\right)(x) \geq \alpha$ and $\left(\nu_A \cap \nu_B\right)(x) \leq \beta$

$\Rightarrow x \in C_{\alpha,\beta}(A \cap B)$ ……………….(6)

From (5) and (6), $C_{\alpha,\beta}(A \cap B) = C_{\alpha,\beta}(A) \cap C_{\alpha,\beta}(B)$

**(v).** since $A \subseteq A \cup B$ and $B \subseteq A \cup B$

Therefore by part (i)

$C_{\alpha,\beta}(A) \subseteq C_{\alpha,\beta}(A \cup B)$ and $C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A \cup B)$

$\Rightarrow C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A \cup B)$ ……… (7)

Let $x \in C_{\alpha,\beta}(A \cup B) \Rightarrow \left(\mu_A \cup \mu_B\right)(x) \geq \alpha$ and $\left(\nu_A \cup \nu_B\right)(x) \leq \beta$

$\Rightarrow \mu_A(x) \geq \alpha$ & $\nu_A(x) \leq \beta$ and $\mu_B(x) \geq \alpha$ & $\nu_B(x) \leq \beta$

$\Rightarrow \mu_A(x) \vee \mu_B(x) \geq \alpha$ and $\nu_A(x) \wedge \nu_B(x) \leq \beta$

If $\mu_A(x) \geq \alpha$, then $\nu_A(x) \leq 1 - \mu_A(x) \leq 1 - \alpha = \beta$

$\Rightarrow x \in C_{\alpha,\beta}(A) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$

Similarly if $\mu_B(x) \geq \alpha$, then $\nu_B(x) \leq 1 - \mu_B(x) \leq 1 - \alpha = \beta$

$\Rightarrow x \in C_{\alpha,\beta}(B) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$

We see that $x \in C_{\alpha,\beta}(A \cup B) \Rightarrow x \in C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$

$\Rightarrow C_{\alpha,\beta}(A \cup B) \subseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$ …………..(8)

From (7) and (8), $C_{\alpha,\beta}(A \cup B) \supseteq C_{\alpha,\beta}(A) \cup C_{\alpha,\beta}(B)$

**Theorem: 3.3** If $A$ is n-generated intuitionistic fuzzy subgroup of a group $G$. Then $C_{\alpha,\beta}(A)$ is a subgroup of a group $G$, where $\mu_A(e) \geq \alpha,\ \nu_A(e) \leq \beta$ and $e$ is the identity element of a group $G$

**Proof:**

Clearly $C_{\alpha,\beta}(A) \neq \phi$ as $e \in C_{\alpha,\beta}(A)$

Let $x, y \in C_{\alpha,\beta}(A)$ be any two elements. Then $\mu_A(x) \geq \alpha,\ \nu_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha,\ \nu_A(y) \leq \beta$

$\Rightarrow \mu_A(x) \wedge \mu_A(y) \geq \alpha$ and $\nu_A(x) \vee \nu_A(y) \leq \beta$

As $A$ is n-generated intuitionistic fuzzy subgroup of $G$.

Therefore $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha$ and

$\mu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y) \leq \beta$ , $\mu_A(xy^{-1}) \geq \alpha$ and $\nu_A(xy^{-1}) \leq \beta$

$\Rightarrow xy^{-1} \in C_{\alpha,\beta}(A)$. Hence $C_{\alpha,\beta}(A)$ is a subgroup of $G$

**Theorem: 3.4**

If $A$ is n-generated intuitionistic fuzzy normal subgroup of a group $G$.Then $C_{\alpha,\beta}(A)$ is a normal subgroup of a group $G$, where $\mu_A(e) \geq \alpha,\ \nu_A(e) \leq \beta$ and $e$ is the identity element of a group $G$

**Proof:**

Let $x \in C_{\alpha,\beta}(A)$ and $g \in G$ be any element. Then $\mu_A(x) \geq \alpha,\ \nu_A(x) \leq \beta$. Also $A$ be n-generated intuitionistic fuzzy normal subgroup of a group $G$.Therefore $\mu(g^{-1}xg) = \mu(x)$ and $\nu(g^{-1}xg) = \nu(x)$ " $x$,Î $A$, $g$ Î $G$

Therefore $\mu_A(g^{-1}xg) = \mu_A(x) \geq \alpha$ and $\nu_A(g^{-1}xg) = \nu_A(x) \leq \beta$

This implies $\mu_A(g^{-1}xg) \geq \alpha$ and $\nu_A(g^{-1}xg) \leq \beta$

That is $g^{-1}xg \in C_{\alpha,\beta}(A)$

Hence $C_{\alpha,\beta}(A)$ is a normal subgroup of $G$

**Definition (3.9):**

Let $G$ be a group and $A$ be a NGIFSG of a group $G$.

Let $x \in G$ be a fixed element. Then the set

$xA = \left\{\left(g, \mu_{xA}(g), \nu_{xA}(g)\right) : g \in G\right\}$ where $\mu_{xA}(g) = \mu_A(x^{-1}g)$ and $\nu_{xA}(g) = \nu_A(x^{-1}g)$ for all $g \in G$ is called n generated intuitionistic fuzzy left coset of $G$ determined by $A$

Similarly,the set $Ax = \left\{\left(g, \mu_{Ax}(g), \nu_{Ax}(g)\right) : g \in G\right\}$ where $\mu_{Ax}(g) = \mu_A(g\,x^{-1})$ and $\nu_{Ax}(g) = \nu_A(g\,x^{-1})$ for all $g \in G$ is called n generated intuitionistic fuzzy right coset of $G$ determined by $A$ and $x$

**Theorem: 3.5**

Let $A$ be NGIFSG of a group $G$ and $x$ be any fixed element of $G$. Then (i). $x \cdot C_{\alpha,\beta}(A) = C_{\alpha,\beta}(xA)$

(ii). $C_{\alpha,\beta}(A)\cdot x = C_{\alpha,\beta}(Ax) \quad \forall\ \alpha,\beta \in [0,1], \text{ with } \alpha+\beta \le 1$

**Proof:**

(i). $C_{\alpha,\beta}(xA) = \{g \in G : \mu_A(g) \ge \alpha \ \&\ \nu_A(\mathrm{g}) \le \beta\}$ and

$x\cdot C_{\alpha,\beta}(xA) = x\cdot\{y \in G : \mu_A(y) \ge \alpha \ \&\ \nu_A(y) \le \beta\}$

$= \{xy \in G : \mu_A(y) \ge \alpha \ \&\ \nu_A(y) \le \beta\}$

Put $xy = g$ so that $y = x^{-1}g$

Therefore $x\cdot C_{\alpha,\beta}(A) = \{g \in G : \mu_A(x^{-1}\mathrm{g}) \ge \alpha \ \&\ \nu_A(x^{-1}\mathrm{g}) \le \beta\}$

$= \{g \in G : \mu_{xA}(\mathrm{g}) \ge \alpha \ \&\ \nu_{xA}(\mathrm{g}) \le \beta\} = C_{\alpha,\beta}(xA)$

Thus $x \cdot C_{\alpha,\beta}(A) = C_{\alpha,\beta}(xA) \quad \forall\ \alpha,\beta \in [0,1], \text{ with } \alpha+\beta \le 1$

(ii). $C_{\alpha,\beta}(Ax) = \{g \in G : \mu_{Ax}(g) \ge \alpha \ \&\ \nu_{Ax}(\mathrm{g}) \le \beta\}$ and

$C_{\alpha,\beta}(xA)\cdot x = \{y \in G : \mu_A(y) \ge \alpha \ \&\ \nu_A(y) \le \beta\}\cdot x$

$= \{yx \in G : \mu_A(y) \ge \alpha \ \&\ \nu_A(y) \le \beta\}$

Put $yx = g$ so that $y = gx^{-1}$

Therefore $C_{\alpha,\beta}(A)\cdot x = \{g \in G : \mu_A(\mathrm{g}\,x^{-1}) \ge \alpha \ \&\ \nu_A(\mathrm{g}\,x^{-1}) \le \beta\}$

$= \{g \in G : \mu_{Ax}(\mathrm{g}) \ge \alpha \ \&\ \nu_{Ax}(\mathrm{g}) \le \beta\} = C_{\alpha,\beta}(Ax)$

Thus $C_{\alpha,\beta}(A)\cdot x = C_{\alpha,\beta}(Ax) \quad \forall\ \alpha,\beta \in [0,1], \text{ with } \alpha+\beta \le 1$

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