International Journal in Physical and Applied Sciences
Volume 5 Issue 07, July 2018 ISSN: 2394-5710 Impact Factor: 4.657
Journal Homepage: http://ijmr.net.in, Email: irjmss@gmail.com
Blind Peer Reviewed Refereed Open Access International Journal

# SOME DOUBLE SEQUENCE SPACES: SPECIAL REFERENCE TO ORLICZ FUNCTIONS

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**Abstract**

The term sequence has a great role in real Analysis. The generalized notion of single sequences are the double sequences. Every double sequence is an infinite matrix. Various types of linear spaces of double sequences are constructed and their properties are studied. The difference operator is used on the double sequences to construct a new double sequence. The double sequences may be bounded, unbounded, convergent, non-convergent or some other. Depending on the behaviour of the sequences the sequence spaces are constructed and some properties of functional analysis are established.

***Keywords: Double, Sequence, Space, Orlicz etc.***

## 1. NOTATIONS

N := The set of all natural numbers., R = The set of all real numbers.

C := The set of all complex numbers., $\lim_k$ : means $\lim_{k\to\infty}$.

$\sup_k$ : means $\sup_{k\geq 1}$.

$\inf_k$ : means $\inf_{k\geq 1}$, unless otherwise stated.

$\sum_k$ : means summation over k = 1 to k = ∞, unless otherwise stated.

x := $(x_k)$, the sequence whose k th term is $x_k$., θ := (0, 0, 0, .....), the zero sequence.

$l_k$ := (0, 0, .., 1, 0, 0, ..), the sequence whose $k^{th}$ component is 1 and others are zeroes, for all
k ∈ N.

e := (1, 1, 1, 1, ......).

p := ($p_k$), the sequence of strictly positive reals.

w := {x = ($x_k$) : $x_k$ ∈ R (or C)}, the space of all sequences, real or complex.

$$l : \{x \in w : \sum_k |x_k| < \infty\}.$$

$l_\infty := \{x \in w : \sup_k |x_k| < \infty\}$, the space of bounded sequences.

$c_0 := \{x \in w : \lim_k |x_k| = 0\}$, the space of null sequences.

$c := \{x \in w : \lim_k x_k = l, \text{ for some } l \in \mathbb{C}\}$,

the space of convergent sequences. $l_\infty$, $c_0$, c are Banach spaces with the usual norm

$$\|x\| = \sup_k |x_k|.$$

## 2.DEFINITIONS

Definition 2.1. A paranorm is a function g: X → R which satisfies the following axioms: for any x, y, x0 ∈ X, λ, λ0 ∈ C [1],

(i) g(0) = 0;

(ii) g(x) = g(−x);

(iii) g(x + y) ≤ g(x) + g(y);

(iv) the scalar multiplication is continuous, that is λ → λ0, x → x0 imply λx → λ0x0.

In other words,

$|\lambda - \lambda 0| \to 0$, $g(x - x0) \to 0$ imply $g(\lambda x - \lambda 0x0) \to 0$.

A paranormed space is a linear space X with a paranorm g and it is written as (X, g) [2].

Any function g which satisfies all the conditions (i)-(iv) together with the condition

(v) $g(x) = 0$ if only if $x = 0$,

is called a total paranorm on X and the pair (X, g) is called total paranormed space.

Definition 2.2. A seminorm is a function $\nu : X \to R$, defined on a linear space X such that for all $x, y \in X$ [3],

(i) $\nu(x) = 0$ if $x = 0$;

(ii) $\nu(\alpha x) = |\alpha|\nu(x)$, for all scalars $\alpha$;

(iii) $\nu(x + y) \leq \nu(x) + \nu(y)$.

The property expressed by (ii) is called absolute homogeneity of ν and that expressed by (iii) is called subadditivity of ν. Thus, a seminorm is a real subadditive and absolutely homogeneous function on X.

Moreover, it follows from (ii) and (iii) that

$0 = \nu(0) = \nu(x + (-x)) \leq \nu(x) + \nu(-x) = 2\nu(x)$.

Whence a seminorm is necessarily non-negative. Also, a seminorm ν is convex on

X, since if $\lambda + \mu = 1$, $\lambda \geq 0$, $\mu \geq 0$ and $x + y \in X$, then

$\nu(\lambda x + \mu y) \leq |\lambda|\nu(x) + |\mu|\nu(y) = \lambda\nu(x) + \mu\nu(y)$.

Definition 2.3. Let X and Y be two nonempty subsets of the space w. Let $A = (a_{nk})$, (n, k = 1, 2, .....) be an infinite matrix with elements of real or complex numbers [4].

$$A_n(x) = \sum_k a_{nk} x_k,$$

provided the series converges. Then $Ax = (A_n(x))$ is called the A-transform of x.

Also

$$\lim_{n} Ax = \lim_{n\to\infty} A_n(x)$$

whenever it exists. If x ∈ X implies Ax ∈Y , we say that A defines a (matrix) transformation from X into Y and we denote it by A : X → Y [5].

By (X, Y ) we mean the class of matrices A that maps X into Y .

Definition 2.4. A continuous function M : R → R is called convex if

$$M\left(\frac{u+v}{2}\right) \leq \frac{M(u)+M(v)}{2}, \qquad \text{for all } u, v \in \mathbb{R}.$$

If in addition, the two sides of above are not equal for u= v, then we call M to be strictly convex.

Definition 2.5. A continuous function M : R → R is said to be uniformly convex if for any $\delta$ > 0 and any u > 0 there exists δ > 0 such that [6]

$$M\left(\frac{u+v}{2}\right) \leq (1-\delta)\frac{M(u)+M(v)}{2}, \qquad \text{for all } u, v \in \mathbb{R}$$

satisfying |u − v| ≥ $\delta$ max{|u|, |v|} ≥ $\delta$ u.

Remark 2.1. If M is convex function and M(0) = 0, then M(λx) ≤ λM(x) for all λ with 0 < λ < 1, [7].

Definition 2.6. An Orlicz function is a function M : [0, ∞) → [0, ∞) which is continuous, nondecreasing and convex with M(0) = 0, M(x) > 0 for x > 0 and M(x) → ∞, as x → ∞.

If convexity of M is replaced by M(x + y) ≤ M(x) + M(y), then it is called a Modulus funtion.

An Orlicz function M can always be represented in the following integral form M(x) = x 0 η(t)dt, where η is known as the kernel of M, is right differentiable for t ≥ 0, η(0) = 0, η(t) > 0, η is non-decreasing and η(t) → ∞ as t → ∞.

It has been used the idea of Orlicz sequence space;

$$l_M := \left\{ x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}$$

which is a Banach space with the norm

$$\|x\|_M = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1 \right\}.$$

Remark 2.2. An Orlicz function satisfies the inequality

$M(\lambda x) \leq \lambda M(x)$ for all $\lambda$ with $0 < \lambda < 1$.

Definition 2.7. If to every positive integer n, there is assigned a number an, then the collection ($a_1$, $a_2$, ..., $a_{n+1}$, ..) is said to be a sequence, denoted as (an). A sequence of Orlicz functions is a similar collection of Orlicz functions $M_k$ where k = 1, 2, 3, ... and is denoted by $M_k$. For more details of sequence of Orlicz functions.

**3. DOUBLE SEQUENCE**

In the case of one variable, we began with the study of sequences of numbers $x_j$, where the suffix j could be any integer. Here double sequences have a corresponding importance. These are sets of numbers $x_{jk}$ with two subscripts, which run through the sequence of all integers independently of each other, so that we have, for example, the numbers, [8].

$x_{11}$ $x_{12}$ $x_{13}$ ...

$x_{21}$ $x_{22}$ $x_{23}$ ...

$x_{31}$ $x_{32}$ $x_{33}$ ...

Definition 3.1. A double sequence $x = (x_{jk})$ has pringsheim limit L (denoted by P−lim x = L) provided that given ϙ > 0 there exists N ∈ N such that $|x_{jk}-L| <$ ϙ whenever j, k > N. We shall describe such an x more briefly as "P-convergent".

A double sequence $(x_{jk})$ is bounded if

$$\|x\| = \sup_{j,k \geq 0} |x_{jk}| < \infty.$$

Remark 3.1. In contrast to the case for single sequences, a P-convergent double sequences need not be bounded.

Definition 3.2. A double sequence $x = x_{jk}$ of real numbers is called almost convergent to a limit L if

$$\lim_{p,q \to \infty} \sup_{m,n \geq 0} \left| \frac{1}{pq} \sum_{j=m}^{m+p-1} \sum_{k=n}^{n+q-1} -L \right| = 0.$$

That is, the average value of $(x_{jk})$ taken over any rectangle $\{(j, k) : m \leq j \leq m + p - 1, n \leq k \leq n + q - 1\}$ tends to L as both p and q tend to $\infty$, and this convergence is uniform in m and n. The notion of almost convergence for single sequence was introduced.

Remark 3.2. As in case of single sequences, every almost convergent double sequence is bounded. But in contrast to the case for single sequences, a convergent double sequence need not be almost convergent. However every bounded convergent double sequence is almost convergent.

Definition 3.3. A double sequence space E is said to be solid if $(\alpha_{i,j}x_{i,j}) \in E$, whenever $(x_{ij}) \in E$, for all double sequences $(\alpha_{ij})$ of scalars with $|\alpha_{ij}| \leq 1$, for all $i, j \in N$.

Let $K = \{(n_i, k_j) : i, j \in N; n_1 < n_2 < n_3 < .... \text{ and } k_1 < k_2 < k_3 < ...\} \subseteq N \otimes N$ and E be a double sequence space. A K-step space of E is a sequence space

$$\lambda_K^E = \{(\alpha_{i,j}x_{i,j}) : (x_{i,j}) \in E\}$$

A canonical pre-image of a sequence $(x_{n_i}, k_j) \in E$ is a sequence $(b_n, k) \in E$ defined as follows:

$$b_{nk} = \begin{cases} a_{nk} & \text{if } (n,k) \in K, \\ 0 & \text{otherwise }. \end{cases}$$

A canonical pre-image of step space λ E K is a set of canonical pre-images of all elements in λ E K.

A double sequence space E is said to be monotone if it contains the canonical pre-images of all its step spaces.

A double sequence space E is said to be symmetric if $(x_{ij}) \in E$ implies $(x\pi(i),\pi(j)) \in E$, where π is a permutation of N.

Lemma 3.1. A sequence space E is solid implies E is monotone.

The folowing inequality will be used throughout

$$|x_{jk} + y_{jk}|^{p_{jk}} \leq D(|x_{jk}|^{p_{jk}} + |y_{jk}|^{p_{jk}})$$

where$x_{jk}$ and $y_{jk}$ are complex numbers, $D = \max(1, 2^{H-1})$ and $H = \sup_{j,k} p_{jk} < \infty$ .

**4. CONCLUSION**

So it is concluded that this article is devoted to the background materials which begins with the notations and conventions and some basic definitions such as Paranorm, Seminorm, Regular matrix, Convex and Uniformly convex functions, Orlicz function, Difference sequences, Statistical convergence, 2-norm and n-normed spaces, Ideal, σ-mean, Double sequences etc., which are needed throughout the work. This article concludes with an introduction to the double sequences which also includes some elementary properties.

**REFERENCES**

[1]. C. Bektas, On some difference sequence spaces defined by a sequence of Orlicz functions, J. Zhejiang Univ. SCIENCE A 7(12), 2093-2096 (2006).

[2]. Esi, Ayhan&Acikgoz, Mehmet. (2009). On Some New Sequence Spaces via Orlicz Function in a Seminormed Space. AIP Conference Proceedings. 1168. 10.1063/1.3241420.

[3]. Khan, Vakeel. (2008). On a new sequence space defined by Orlicz functions. StudiaUniversitatisBabeş-Bolyai. Mathematica. 55. 10.1501/Commua1_0000000181.

[4]. N. Faried, M.S.S. Ali and H.H. Sakr, On generalized fractional order difference sequence spaces defined by a sequence of modulus functions, Math. Sci. Lett. 6(2), 163- 168 (2017).

[5]. O. Duyar and S. Demiriz, On some new generalized difference sequence spaces and their topological properties, J. New Result Sci. 6, 1-14 (2014).

[6]. P. Baliarsingh and S. Dutta, A unifying approach to the difference operators and their applications, Bol. Soc. Paran. Mat. 33(1), 49-57 (2015).

[7]. P. Baliarsingh, Some new difference sequence spaces of fractional order and their dual spaces, Appl. Math. Comput. 219, 9737-9742 (2013).

[8]. Raj, Kuldip& Sharma, Ajay & Sharma, Sunil. (2013). Sequence spaces defined by a Musielak-Orlicz function in 2-normed spaces. Journal of Computational Analysis and Applications. 15.