

## ***Layered states are used to study the quantum dynamics of trapped ions under a dynamic field gradient.***

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### **Abstract**

Radiofrequency radiation can be used to couple the internal and motional states of ions using novel ion traps that offer either a static or a dynamic magnetic gradient field, which is necessary for conditional quantum logic. We demonstrate that the Hamiltonian characterizing this connection when a resonant dynamic gradient is present is the same Hamiltonian when a static gradient is present in a dressed state basis. The identical effective Lamb-Dicke parameter describes the coupling strength in both scenarios. When using a dynamic gradient field for cutting-edge studies with trapped ions, such as those in quantum information science, this understanding can be leveraged to get around challenging experimental criteria. Using a single resonant or detuned dynamic gradient field, this discovery simultaneously creates fresh experimental possibilities by for conditional multi-qubit dynamics, long-range coupling is induced.

**Keywords** - *Radiofrequency radiation, a dynamic magnetic gradient field.*

### **1. Introduction.**

The progress of experimental quantum information science has been significantly influenced by experiments with atomically trapped ions [1-3]. A prospective option for quantum simulations and scalable universal quantum computing that can outperform classical computers, trapped ions are well isolated from their surroundings and are well suited for examining fundamental quantum physics concerns [4]. An upper limit for the coherence time of ionic qubits is set by the coherence time of electromagnetic radiation in the optical or radio-frequency (RF) realm that is used to coherently produce internal electronic states acting as qubits. When two or more ions are contained within the same trap or trapping zone to simulate conditional quantum dynamics with two or more qubits, A quantum bus is created when the intrinsic dynamics of individual ions are connected to the collective vibration motion. Since a few decades ago, it has been common practice to couple ionic qubits via this quantum bus using laser light. Only when the Lamb-Dicke parameter, which measures the coupling strength, is used with light in and around the visible regime. in common traps takes on a sufficiently big value between internal and motional states [5]. driving only one vehicle optical control to select the appropriate ion from a group of trapped ions, typically separated by a few micrometres. Portion of the radiation



that is capable of being concentrated to a spot size smaller than the inter-ion spacing. a number of In experiments, deterministically setting up the quantum states of trapped ions has been successful. Even full quantum simulations and algorithms [6, 7] have been put into practice. When RF radiation is employed to directly control the ions' dynamics rather than taking the indirect route of imprinting RF signals onto optical beams and then guiding these optical beams towards trapped ions, the complexity of experimental setups can be significantly reduced. By using RF radiation directly, laser beam difficulties including frequency-, phase-, and amplitude noise, diffraction, and beam aiming instabilities in the optical domain can be avoided. When an additional, spatially variable field is given to an atom trap, it becomes possible to use RF radiation for coupling internal and motional dynamics. This magnetic gradient field may be static [10] or dynamic [11]. Even when excited by RF radiation, an effective Lamb-Dicke parameter develops in both situations due to magnetic gradient induced coupling. Spontaneous emission due to the limited lifespan of qubit states or spontaneous scattering brought on by non-resonant laser light generating Raman transitions are not an issue when using MAGIC for trapped ions as an adjunct to successful research based on laser-driven ion trap quantum logic. Conditional quantum dynamics based on magnetic gradient driven spin-spin coupling are resistant to thermal stimulation of the ions' vibrational motion, as is the case for some laser-based gates [22]. A critical benchmark for fault-tolerant quantum computing, single-qubit quantum gates driven by RF radiation have been accomplished with an error well below  $10^{-4}$  [23, 24]. A quantum byte (eight ions) could be addressed with a measured cross-talk between closely spaced, interacting ions in the  $10^5$  range using a static field gradient [20]. Three-qubit gates [26] and two-qubit gates [13, 14, and 17] were also demonstrated using MAGIC, which also offers new opportunities for quantum simulations and quantum computation [26, 27].

In this study, we demonstrate that identical Hamiltonians can be used to describe the application of MAGIC to the addition of either a static or a dynamic gradient field to a Coulomb lattice of trapped ions. The Hamiltonian is featured in a dressed-state photo that was taken while, To achieve high-fidelity two-qubit gates, it is carefully avoided in current studies where a dynamic gradient field is used to maintain just a dynamic field gradient at the sites of the ions [13, 28, 29]. To prevent errors brought on by a non-zero offset field, another strategy is to utilise an additional dressing field [25]. Here, we demonstrate how the dynamic magnetic gradient field itself could be used to dress atomic states for conditional quantum gates, significantly reducing the amount of experimental work required to execute the dynamic MAGIC system.



we briefly review the unique scheme's properties before introducing it and going into more detail: I It does away with the laborious requirement to cancel out the dynamic field where the ions are. When using conditional quantum dynamics with more than two ions, this may be especially crucial. In order to produce qubits with extended coherence times, a reasonably strong and stable bias field does not need to be applied because the atomic states clothed by the dynamic gradient field are insensitive to background field noise (so-called clock states).

(iii) It becomes possible to apply a dynamic gradient field with the gradient pointing along the axis of the weakest ion confinement in a linear trap, which significantly strengthens the coupling between qubit states and motional states. (iv) With minimal cross-talk, it becomes simple to individually address ions exposed to such a dynamic field gradient. (v) It is demonstrated that the application of a single dynamic gradient field results in long-range spin-spin interaction between clothed qubits. This coupling's conditional dynamics can be applied to effective quantum computation and quantum simulations. The coupling between internal and motional states in the presence of a static (section 2) or dynamic (section 3) magnetic gradient field is discussed in the sections that follow. We recapitulate both approaches and integrate them into a single model, first by examining a single atom and then by showing their convergence for multi-qubit systems.

## 2. Static magnetic gradient

In the first part of this section, we describe how coupling between an atom's internal and motional states results from applying a static magnetic field gradient to a single trapped atom. The overview that follows is based on findings that were published in [10] and, for example, illustrated in [30]. The expression representing the coupling between internal states of various ions (spin-spin coupling) caused by the gradient field is then given. Next, a collection of ions subjected to a magnetic gradient field is taken into consideration. This expression for spin-spin coupling's derivation can be found in [31]. (see also [12, 32] for physical interpretations). We have included this information in order to establish the notation and make the article somewhat self-contained. The Hamiltonian characterizing a single atom  $j$  with energy level spacing  $\hbar \omega_0$  contained in a harmonic trap at position  $z_j$  extended up to first order in the field gradient is given by for a static magnetic gradient parallel to the  $z$ -axis.



$$H_{\text{static}} = \frac{1}{2} \hbar \omega_0 \sigma_z^{(j)} + \frac{\mu_z}{2} (B_0 + zB') \sigma_z^{(j)} + \hbar \nu_n a_n^\dagger a_n, \dots\dots\dots 1$$

With the ion's matrix element of the magnetic dipole moment  $z m$ , the magnetic field  $B(z) = B_0 + zB'$  and the Pauli-z matrix  $\sigma_z$ . Here,  $a_n^\dagger$  and  $a_n$  describe the creation and annihilation operators of the vibration mode in the harmonic trap with angular frequency  $\nu_n$ . The position  $z$  of ion  $j$  is given by  $z = z_j + \Delta z_j$  with its equilibrium position  $z_j$  and the displacement  $\Delta z_j$ . The displacement can be written in terms of the normal vibration mode  $n$  as

$$\Delta z_j = b_{j,n} q_n (a_n + a_n^\dagger) \dots\dots\dots 2$$

With the help of the coefficients  $1 < b_{j,n} < 1$  that reflect how strongly ion  $j$  participates in motional mode  $n$  (if only a single ion is considered, then  $b_{1,1} = 1$ ). Here,  $q_n = \sqrt{\hbar / 2m\nu_n}$  describes the atom's spatial extent in the ground state of the harmonic trapping potential. As a consequence, Hamiltonian equation (1) can be rewritten as

$$H_{\text{static}} = \frac{1}{2} \hbar \omega (z_j) \sigma_z^{(j)} + \hbar \nu_n a_n^\dagger a_n + \hbar \nu_n \varepsilon_{j,n} (a_n^\dagger + a_n) \sigma_z^{(j)} \dots\dots\dots 3$$

with the position dependent level splitting  $w(z_j) = \omega_0 + m z_j (B_0 + z_j B') / \hbar$  and the coupling strength

$$|\varepsilon_{j,n}| = \frac{(\mu_z B' b_{j,n} q_n)}{(2 \hbar \nu_n)}$$

Every time a position dependent level split qubit is used, an interaction between the internal and external degrees of freedom is produced. To produce this contact, no extra RF or laser energy is required. Be aware that  $B$  denotes the gradient's magnitude in the static magnetic field. The application of a second driving field with an angular frequency  $\omega_D$  near the atomic resonance  $\omega$  is now being considered ( $z_j$ ). This driving field's and the imprisoned atom's interaction is described by



$$H_D = \frac{\hbar \Omega_D}{2} \sigma_x^{(j)} \exp[i(kz - \omega_D t)] + \text{h.c.} \dots\dots\dots 4$$

with  $k$  being the  $z$ -direction projection of the driving field's wave vector,  $k$ , onto the former. An increase in  $z$  indicates an interaction between internal and exterior degrees of freedom whose intensity is controlled by the Lamb-Dicke parameter

$\eta = b_{j,n} q_n k$  [5] around its equilibrium point. The wave number  $k = 2\pi/\lambda$  is too small (in typical ion traps) to produce a sizable  $\eta$ , however, when the applied radiation has a long wavelength, as in the RF regime, and as a result, the coupling between internal and motional states, which would make excitation of motional sidebands effective, is negligible. When  $\hbar k$  is low, the linear momentum that is imparted to the atom upon photon absorption or emission is insufficient to alter the trapped atom's motional state.

The transformation  $H = e^{S^{j,n}} H_0 e^{-S^{j,n}}$  [10] of H Static (equation (1)) leads to decoupling of internal and external degrees of freedom and we obtain

$$\tilde{H}_{\text{static}} = \frac{\hbar \omega_j}{2} \sigma_z^{(j)} + \hbar \nu_n \tilde{a}_n^\dagger \tilde{a}_n \dots\dots\dots 5$$

This reveals a direct interaction of the internal degrees of freedom: a long-range interaction between the ions' internal states (henceforth referred to as spins) induced by the static magnetic gradient described by

$$H_j = -\frac{\hbar}{2} \sum_{j < k} J_{j,k} \sigma_z^{(j)} \sigma_z^{(k)}, \dots\dots\dots 6$$

The operator  $S_j, n$  for a single ion  $j$  and mode  $n$  is given by

$$S_{j,n} = \varepsilon_{j,n} (a_n^\dagger - a_n) \sigma_z^{(j)}, \dots\dots\dots 7$$

signifying an internal-state dependent shift of the normal modes induced by the magnetic gradient.

The same transformation applied to the driving field Hamiltonian (equation (5)),



$$\tilde{H}_D = \frac{\hbar \Omega_D}{2} (\sigma_+ e^{\varepsilon_{j,n}(\tilde{a}_n^\dagger - \tilde{a}_n)} + \sigma_- e^{-\varepsilon_{j,n}(\tilde{a}_n^\dagger - \tilde{a}_n)}) (e^{i\omega_D t} + e^{-i\omega_D t}), \dots\dots\dots 8$$

which reveals the role of  $\varepsilon_{j,n}$  as a generalized Lamb-Dicke parameter. Expanding  $\tilde{H}_D$  in  $\varepsilon_{j,n}$  shows that motional sidebands of the internal resonance can be resonantly driven, for instance, if  $\omega_D = \omega_j + n\nu_n$ .

In the summary above we already established the notation for N ions ( $1 \leq j, n \leq N$ ) even though, so far, a Single harmonically trapped atom ( $1 \leq j, n \leq N$ ) was considered. Now, we turn to the case of N ions confined in a linear trap: the generalization of equation (3) to N ions and its transformation by  $e^{S\hat{H}e^{-S}}$  static- with  $S = \sum_{j,n} S_{j,n} a_{j,n}$  leads to [31]

$$\tilde{H}_{\text{static}} = \frac{1}{2} \sum_j \hbar \omega(z_j) \sigma_z^{(j)} + \sum_n \hbar \nu_n \tilde{a}_n^\dagger \tilde{a}_n - \frac{\hbar}{2} \sum_{j < k} J_{j,k} \sigma_z^{(j)} \sigma_z^{(k)}. \dots\dots\dots 9$$

This demonstrates an interaction of the internal degrees of freedom that is direct: a long-range interaction between the internal states of the ions, hereafter referred to as spins, caused by the static magnetic gradient indicated by

$$H_j = -\frac{\hbar}{2} \sum_{j < k} J_{j,k} \sigma_z^{(j)} \sigma_z^{(k)}, \dots\dots\dots 10$$

Equation (4) is valid in the regime of a linear Zeeman Effect. In more general cases the coupling strength is given by  $\varepsilon_{j,n} = |\nabla_z w_j| b_{j,n} / \nu_n$  where  $w_j$  can be determined e.g. by the Breit-Rabi formula. With the coupling strength given by [31, 32, 43]

$$J_{j,k} = \sum_n \nu_n \varepsilon_{j,n} \varepsilon_{k,n}. \dots\dots\dots 11$$

Once more, we point out that no other radiation is involved in this interaction; it is purely caused by the magnetic gradient. As long as the total potential confining the ions consisting of the external trapping potential plus the Coulomb interaction between the ions remains a harmonic potential, this spin-spin coupling also allows for the creation of so-called hot quantum gates.



### 3. Dynamic magnetic gradient

Through a dynamic magnetic field  $B(z, t) = B(z) \cos(\omega_B t)$ , with a gradient of the amplitude perpendicular to the string of ions, the method described by Ospelkaus et al. [11] enables spin-spin coupling. The gradient of this dynamic magnetic field is assumed to point along the z-axis throughout the recapitulation of this scheme that follows. In order to provide the oscillating magnetic fields, the string of ions is assumed to be parallel to both the x-axis and the electrode, and the ion and the magnetic field are related via the matrix element  $\mu_x$  of the magnetic dipole moment. Similar to the static situation, we increase the magnetic field and express the position  $z$  as the equilibrium position  $z_j$  plus a tiny displacement  $\Delta z$ .

$$H_{\text{osci}} = \frac{\hbar\omega_0}{2} \sigma_z^{(j)} + \hbar\nu_n a_n^\dagger a_n + \sigma_x^{(j)} \cos(\omega_B t) \mu_x B_j \dots\dots\dots 12$$

As a consequence, the interaction Hamiltonian in the rotating wave approximation (RWA) is given by

$$H_{\text{osci,I}} = \hbar\sigma_+ (\Omega_j e^{-i\delta t} + \Omega_{j,n} a_n e^{-i(\delta+\nu_n)t}) + \text{h.c.} \dots\dots\dots 13$$

with the detuning  $d = \omega_B - \omega_0$ , the Rabi frequencies

$$\Omega_j = B_j \mu_x / (2\hbar), \dots\dots\dots 14$$

$$\Omega_{j,n} = B' \mu_x b_{j,n} q_n / (2\hbar). \dots\dots\dots 15$$

Equations (12) and (13) show that the magnetic gradient is responsible for the coupling between internal and motional degrees of freedom, just like in the static situation.

A spin-spin interaction can be produced by applying two dynamic magnetic fields with equal amplitude and opposite detuning close to the red- and blue-sidebands [11]. In general, it is preferable to have a spin-spin contact without any extra excitations from the carrier transition. The carrier is not excited because of resonant excitation (detuning of the driving fields by about  $\nu_n$ ). For high-fidelity two-qubit gates, a gradient and a weak absolute magnetic field strength are required. Therefore, An extensive amount of experimental work is put into determining the precise geometry of the electrodes that produce the magnetic fields. related to the ions' precise location [13, 28, 29].

#### 4. Dynamic MAGIC in a dressed-state basis.

In what follows, we demonstrate that when expressing the Hamiltonian for the dynamic case in a dressed-states basis, the approach using a resonant dynamic magnetic gradient for coupling internal and external degrees of freedom is equivalent to the static gradient approach. Furthermore, it is outlined how a non-resonant dynamic gradient allows for interesting variations of spin-motion coupling.

For this purpose, we transform the Hamiltonian H<sub>0</sub> given in equation (12) into the rotating frame of the S ion with d = ωB - ω<sub>0</sub>, resulting in

$$H_I = \frac{\mu_x}{2} (\sigma_+ e^{-i\delta t} + \sigma_- e^{i\delta t}) \times [B_j + B' b_{j,n} q_n (a_n^\dagger + a_n)] + \hbar \nu_n a_n^\dagger a_n$$

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for a single ion and a single mode. For details on the matrix elements  $z m$  and  $x m$

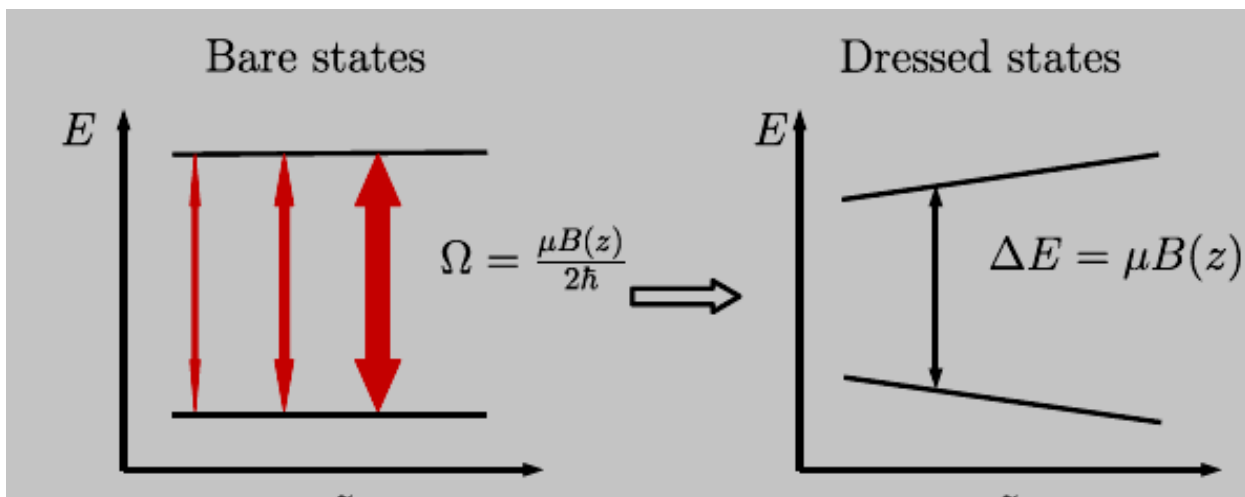


Figure 1. Left: the interaction strength  $\Omega$  between a spatially varying driving field  $B(z, t) = B(z)\cos(\omega B t)$  and a two-level atom.

Right: in the dressed-state picture this position dependent interaction leads to a position dependent level splitting  $\Delta E$  of states  $|\pm\rangle$

The eigenstates of the operator  $\sigma_+ \exp(-i\delta t) + \sigma_- \exp(i\delta t)$  are given by

$$|\pm\rangle = \frac{1}{\sqrt{2}} (e^{i\delta t/2} |g\rangle \pm e^{-i\delta t/2} |e\rangle)$$

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using just the atomic states  $g$  and  $e$ . For  $d = 0$ , the states are equivalent to the often used time-independent clothed states. To compare the states for  $d \neq 0$  in detail to the typically clothed states, see section 4.1. For  $d \neq 0$ , the states vary from the dressed states and become time dependent. Therefore, the Pauli-z matrix in the base is provided by

$$\Sigma_z = T(\sigma_+ e^{-i\delta t} + \sigma_- e^{i\delta t}) T^\dagger \dots\dots\dots 18$$

with the unitary transformation

$$T = \frac{e^{-i\delta t/2}}{\sqrt{2}} \left( |+\rangle + |-\rangle \right) \langle g| + \frac{e^{i\delta t/2}}{\sqrt{2}} \left( |+\rangle - |-\rangle \right) \langle e|. \dots\dots\dots 19$$

This relation shows that the strength of the interaction between the bare states (proportional to  $s_+ + s_-$ ) transforms into the level splitting of the state (proportional to  $S_z$ ) as displayed in figure 1. The Hamiltonian  $\hbar \omega_{na_n}$  describing the energy of the vibration modes is invariant under all these transformations. As a consequence, the resulting Hamiltonian  $H| \pm \rangle = THT + i(dtT(t))T(t)^\dagger$  leads to

$$H_{|\pm\rangle} = \frac{\mu_x B_j}{2} \Sigma_z + \hbar \omega_n a_n^\dagger a_n + \frac{\mu_x B' b_{j,n} q_n}{2} (a_n^\dagger + a_n) \Sigma_z + \frac{\hbar \delta}{2} \Sigma_x. \dots\dots\dots 20$$

The first three terms have the exact same shape as the static gradient field Hamiltonian in equation (3). The transformation matrix  $T$  becomes time dependent for  $d \neq 0$  and produces an additional term, which leads to the final term. The identifications are obtained by comparing the Hamiltonians'  $H$ , equation (20), and  $H_{static}$ , equation (3)

$$\omega(z_j) = \frac{\mu_x B_j}{\hbar} \dots\dots\dots 21$$



$$\epsilon_{j,n} = \frac{\mu_x B' b_{j,n} q_n}{2\hbar\nu_n} \dots\dots\dots 22$$

The coupling between motional and internal degrees of freedom in static and dynamic MAGIC is described by the exact same effective Lamb- Dicke parameter using the appropriate matrix element of the atomic magnetic moment, as shown by a comparison with equation (4). We can express equations (14) and (15) as  $w(z_j) = 2W_j$  and

$$\epsilon_{j,n} = \Omega_{j,n}/\nu_n \dots\dots\dots 23$$

Demonstrating the relationship between the sideband Rabi frequency and the coupling constant  $\epsilon_{j,n}$ . Be aware that the position dependence of the level splitting  $w(z_j)$  and not the detuning determine the effective Lamb-Dicke parameter  $\epsilon_{j,n}$ . When using single-qubit rotations, the detuning specifies the pertinent reference frame to use.

Selecting a dynamic gradient field with zero detuning prevents single-qubit rotations brought on by the additional term  $(d/2)S_x$ . This gradient field causes the spin-motion coupling. This extra singlequbit rotation will have an impact on the evolution as a whole if  $d \neq 0$ . Depending on the specifics of a two-qubit gate, this could result in a decreased effective coupling because of quick oscillations (for more details see appendix A).

**4.1. Spin-motion coupling using time-independent dressed states:**

Below, we go over a different strategy that can help you comprehend the impact of detuning the dynamic gradient field. We take time-independent dressed states into consideration for this reason. The RWA describes how a two-level atom with energy splitting  $w_0$  interacts with a classical driving field with frequency  $w_B$  in the driving field's rotating frame.

$$H = -\frac{\hbar\delta}{2}\sigma_z + \frac{\hbar\Omega}{2}(\sigma_+ + \sigma_-) \dots\dots\dots 24$$

with  $\tan(2q) = W/d$ , are called dressed states. We transform equation (12) first into the rotating frame of the driving field and consecutively into the dressed-state picture resulting in



$$H_{\text{dress}} = \hbar \sqrt{\delta^2 + \Omega^2} \tilde{\Sigma}_z + \hbar \nu_n a_n^\dagger a_n + \frac{\mu_x B' b_{j,n} q_n}{2} (a_n^\dagger + a_n) [\sin(2\theta) \tilde{\Sigma}_z + \cos(2\theta) \tilde{\Sigma}_x] \dots\dots\dots 25$$

with  $\Sigma_j$  denoting the Pauli-matrices in the dressed state picture. The factor between square brackets in the third term on the right-hand side of equation (26) is of the form  $n \Sigma$  with  $|n|^2 = 1$  and can thus be interpreted as a generator of a spin-rotation around the axis  $n = (\cos 2\theta, 0, \sin 2\theta)^T$ . The motional degree of freedom is now coupled to  $n \Sigma$ . As a consequence, a detuning causes a change in the direction  $n \Sigma$  of the coupling, but does not change the coupling strength itself given by the effective Lamb-Dicke parameter  $e_{j,n}$ . The states specified in equation (17) and the dresses defined in equation (25) vary in that (i) the states are defined in the reference frame of the ion and the dresses are defined in the reference frame of the driving field, respectively. (ii) For  $d \neq 0$ , are time-dependent, while the states specified in equation (17) and the dresses defined in equation (25) vary in that (i) the states are defined in the reference frame of the ion and the dresses are defined in the reference frame of the driving field, respectively. (ii) For  $d \sigma = 0$ , whereas dress are time independent for  $d \sigma \neq 0$ , are time dependent. (iii) In the basis, a detuning  $d \neq 0$  results in an extra driving term, but in the dressed-state image, it results in a modification in the driving term. while it causes the rotation axis  $n$  to shift in the dressed-state image. In all scenarios, the detuning has no influence on the coupling strength as indicated by the effective Lamb-Dicke parameter  $e_{j,n}$ . Between the states as specified in equation (17) and the clothing as defined in equation (25) there are the following differences: In the reference frame of the ion and the driving field, respectively,  $I$  are defined and dressed. (ii) For  $d \sigma = 0$ , whereas dress are time independent for  $d \sigma \neq 0$ , are time dependent. (iii) In the dressed-state image, a detuning  $d \sigma = 0$  results in a shift in the rotation axis  $n$ .

**4.2. Long-range spin–spin coupling using a dynamic gradient field.**

Equations (10) and (11), which describe spin-spin coupling between all pairs of ions in the static condition, extending equation (3)'s expression for the situation of a single ion to that of several ions. Using a dynamic gradient as an example Equation (20), where the magnitude of the static gradient has been substituted by the magnitude of the dynamic gradient, leads us to the same Hamiltonian as in the static situation. After that, the extension to a line of  $N$  ions oriented along



Equations (10) and (11) explain a spin-spin interaction that is caused by the dynamic on the z-axis once more Gradient. The potency of the dynamic gradient field's spin-spin coupling between states is just by entering equation (22) for  $e_{j,n}$  into (11). Given that the coupling equations We won't repeat equation (11) here because it describes the coupling constants  $J_j$  and  $k$  in both the static and dynamic cases. As was experimentally shown in the static gradient case [14, 26], this interaction might be employed for conditional quantum dynamics without the necessity for driving sideband resonances.

The contributions from multiple radial vibration modes to the spin-spin coupling in equation (11) largely cancel each other if a dynamic field with a gradient in the radial direction is applied to an ion string. If there are just two ions present and the center-of-mass (COM) mode and the stretch mode are almost degenerate due to the trap parameters, this cancellation is very important.

Because static and dynamic MAGIC are equivalent, it is possible to apply results from static MAGIC, such those from [26, 33, 34], to dynamic MAGIC.

## 5. Conclusion

As a result, it is not necessary to cancel the dynamic magnetic field at the ion point for the approach suggested dynamic MAGIC paired with clothed states. On the other hand, the off-set field  $B_j$  at location  $z_j$  helps to produce the dressed state's energy splitting. We demonstrate that the Hamiltonian characterizing this connection when a resonant dynamic gradient is present is the same Hamiltonian when a static gradient is present in a dressed state basis. The identical effective Lamb-Dicke parameter describes the coupling strength in both scenarios. When using a dynamic gradient field for cutting-edge studies with trapped ions, such as those in quantum information science, this understanding can be leveraged to get around challenging experimental criteria. Using a single resonant or detuned dynamic gradient field, this discovery simultaneously creates fresh experimental possibilities by for conditional multi-qubit dynamics, long-range coupling is induced. The novel concept introduced here—with the features summarized in the introduction and discussed in more detail above—should allow for a decisive reduction of experimental complexity and, at the same time, opens new perspectives for an RF-based approach to quantum computation and quantum simulations with trapped ions.



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