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INVERSION FORMULA FOR TWO DIMENSIONAL FOURIER-LAPLACE TRANSFORM AND APPLICATION

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Abstract: The Fourier transform is a powerful tool to analyze signals and construct them to and from their frequency components. Fourier transformation (FT) has huge application in radio astronomy, sky observed by radio telescope is recorded as the FT of true sky termed as visibility in radio astronomy language and this visibility goes through Inverse Fourier transformation and de-convolution process to deduce the true sky image. Laplace transform have become an integral part of modern science, being used in a vast number of different disciplines.

The proposed paper provides the generalization of two dimensional Fourier-Laplace transform in the distributional sense. Proved Inversion Theorem as well as uniqueness theorem for two dimensional Fourier-Laplace transform. Apart from this application of Fourier-Laplace transform for solving partial differential equation is also studied.

Keyword: Fourier Transform, Fourier-Laplace Transform, Generalized function, Laplace Transform.

1. Introduction

Fourier transform continue to be a very important tool for the engineer, physicist and applied mathematician. It is also now useful to financial, economic and biological modelers as these disciplines become more quantitative. Fourier transform is also powerful in digital, wire and wireless communication. Human face recognition is, indeed, a challenging task, especially under illumination and pose variation. So the effectiveness of a simple face recognition algorithm is based on Fourier transform [1]. Fourier transform has huge application in radio astronomy, optical pattern recognition, signal processing, image processing and many more.

Laplace transform technique has been considered as an efficient way in solving different equations with integer order [2]. The name of this transform originates from a French mathematician Pierre Simon Laplace. Studying the theory and application of Laplace transform has become an essential part of any curriculum involving mathematics such as engineering, mathematics, physics and many other branches of science like nuclear physics. Even those going into fields such as chemistry sometimes are required to have an understanding of what a Laplace transform is. The most likely people to be using this transform would be engineers due to its applications in circuits in harmonic oscillators and systems such as HVAC systems and many other types of systems that deal with sinusoids and exponentials.

The primary use of this transform is to change an ordinary differential equation in a real domain into an algebraic equation in the complex domain, making the equation much easier to solve. The subsequent solution that is found by solving the algebraic equation is then taken and inverted by use of the inverse Laplace transform acquiring a solution for the original differential equation or ODE [3].

The purpose of this paper is to generalized two dimensional Fourier-Laplace transform in the distributional sense and to present the Inversion theorem for Distributional two dimensional Fourier-Laplace transform. The outline of the paper is as follows: Definitions are given in section 2 and section 3. In section 4, testing function spaces are described. Inversion theorem for two dimensional Fourier-Laplace transform is given in section 5. In section 6, Uniqueness theorem is given. Application of Fourier-Laplace transform are given in section 7. Lastly we conclude the paper.

The notations and terminology as per A. H. Zemanian [4], [5].

2. Definitions:

The Two Dimensional Laplace transform with the parameters $p, v,$ of function $f\left(x,y\right)$ denoted by $L\left[f\left(x,y\right)\right]=F\left(p,v\right)$ and is given by,

$$L\left[f\left(x,y\right)\right]=F\left(p,v\right)=\int_0^\infty\int_0^\infty e^{-px-vy}f\left(x,y\right)dxdy \tag{2.1}$$

The Two Dimensional Fourier transform with the parameters $s, u,$ of function $f\left(t,z\right)$ denoted by $F\left[f\left(t,z\right)\right]=F\left(s,u\right)$ and is given by,

$$F\left[f\left(t,z\right)\right]=F\left(s,u\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-i(st+uz)}f\left(t,z\right)dtdz \tag{2.2}$$

The Two Dimensional Fourier-Laplace transform with parameters $s, u, p, v,$ of $f\left(t,z,x,y\right)$ is defined as,

$$FL\left\{f\left(t,z,x,y\right)\right\}=F\left(s,u,p,v\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^\infty\int_0^\infty f\left(t,z,x,y\right)e^{-i\left\{(st+uz)-i(px+vy)\right\}}dtdzdxdy \tag{2.3}$$

where the kernel $K\left(s,u,p,v\right)=e^{-i\left\{(st+uz)-i(px+vy)\right\}}$

3. Distributional Generalized Two Dimensional Fourier-Laplace Transform (2DFLT)

For $f\left(t,z,x,y\right)\in FL^{\beta*}_{a,b,c,d,\alpha}$, where $FL^{\beta*}_{a,b,c,d,\alpha}$ is the dual space of $FL^{\beta}_{a,b,c,d,\alpha}$. It contains all distributions of compact support. The distributional two dimensional Fourier-Laplace transform is a function of $f\left(t,z,x,y\right)$ is defined as,

$$FL\{f(t,z,x,y)\} = F(s,u,p,v) = \langle f(t,z,x,y), \phi(t,z,x,y,s,u,p,v)\rangle, \tag{3.1}$$

Where $\phi(t,z,x,y,s,u,p,v) = e^{-i\{(st+uz)-i(px+vy)\}}$ and for each fixed $t(0<t<\infty)$, $z(0<z<\infty)$, $x(0<x<\infty)$ and $y(0<y<\infty)$. Also $s>0$, $u>0$, $p>0$ and $v>0$. The right hand side of (3.1) has a sense as an application of $f(t,z,x,y) \in FL^{\beta*}_{a,b,c,d,\alpha}$ to $\phi(t,z,x,y,s,u,p,v) \in FL^{\beta}_{a,b,c,d,\alpha}$.

4. Testing function space $FL^{\beta}_{a,b,c,d,\alpha}$ (S^{β}_{α}-type space)

Let I be the open set in $R_+ \times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I, the space $FL^{\beta}_{a,b,c,d,\alpha}$ is given by

$$FL^{\beta}_{a,b,c,d,\alpha} = \{\phi : \phi \in E_+ / \rho_{a,b,c,d,k,r,q,m,l,n}[\phi(t,z,x,y)]$$

$$= \sup_{I_1} \left| t^k z^r K_{a,b}(x) R_{c,d}(y) D_t^l D_x^q D_z^n D_y^m [\phi(t,z,x,y)] \right| \le C A^k k^{k\alpha} B^r r^{r\alpha} G^l l^{l\beta} H^n n^{n\beta} \}$$

where, $k,r,q,m,l,n = 0,1,2,3,........$ and where the constants A,B,G,H and C depend on the testing function ϕ.

The space $FL^{\beta}_{a,b,c,d,\alpha}$ is equipped with their natural Hausdorff locally convex topology $T^{\beta}_{a,b,c,d,\alpha}$.

This topology is generated by the total family of semi norm $\{\rho_{a,b,c,d,k,r,q,m,l,n}\}$.

5. Inversion Theorem for Two Dimensional Fourier-Laplace Transform

5.1. Lemma1: Let $FL\{f(t,z,x,y)\} = F(s,u,p,v)$ and $\text{supp}\, f \subset S_A \cap S_B$, where $S_A = \{t,z,: t,z \in R^n, |t|,|z| \le A, A>0\}$ and $S_B = \{x,y : x,y \in R^n, |x|,|y| \le B, B>0\}$, for $s>0$, $u>0$ and $\rho_1 < \operatorname{Re} p < \rho_2$ and $\sigma_1 < \operatorname{Re} v < \sigma_2$. Let $\phi \in D$ and

$$\psi(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \phi(t,z,x,y) e^{i\{(st+uz)-i(px+vy)\}} dtdzdxdy \tag{5.1.1}$$

Then for any fixed real numbers τ,τ' and r,r' with $-\infty<r<\infty$, $-\infty<r'<\infty$, $0<\tau<\infty$, $0<\tau'<\infty$.

$$\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} \left\langle f(t,z,x,y), e^{-i\{(st+uz)-i(px+vy)\}} \right\rangle \psi(s,u,p,v) ds du dw dl$$

$$=\left\langle f(t,z,x,y), \int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} e^{-i\{(st+uz)-i(px+vy)\}}\psi(s,u,p,v)dsdudwdl \right\rangle$$

Where $p=\rho+iw$, $v=\sigma+il$, also s,u,ρ,σ are fixed with $\sigma_1<v<\sigma_2$, $\rho_1<p<\rho_2$, $\sigma_1'<s<\sigma_2'$ and $\rho_1'<u<\rho_2'$.

5.2. Lemma2: Let a,b,c,d,ρ,r and τ and a',b',c',d',σ,r' and τ' be real numbers with $c<s<d$, $c'<u<d'$, $a<p<b$, and $a'<v<b'$. Also, let $\phi\in D$ then

$$\frac{1}{\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(t,z,x,y)\frac{\sin(t-g)r}{(t-g)}\frac{\sin(z-j)r'}{(z-j)}\frac{\sinh(x-e)\tau}{(x-e)}\frac{\sinh(y-q)\tau'}{(y-q)}dtdxdzdy = A(g,j,e,q)$$

converges in $FL_{a,b,c,d,\alpha}$ to $\phi(g,j,e,q)$ as $r\to\infty$, $\tau\to\infty$, $r'\to\infty$, $\tau'\to\infty$.

5.3. Inversion Theorem

Statement: Let $FL\{f(t,z,x,y)\}=F(s,u,p,v)$, for $s>0$, $u>0$ and $\rho_1<\operatorname{Re}p<\rho_2$ and $\sigma_1<\operatorname{Re}v<\sigma_2$. Also, let r,r',τ,τ' be a real variables such that $-\infty<r<\infty$, $-\infty<r'<\infty$, $-\infty<\tau<\infty$, and $-\infty<\tau'<\infty$. Then, in the sense of convergence in D^*,

$$f(t,z,x,y)=\lim_{\substack{r\to\infty\\ \tau\to\infty\\ r'\to\infty\\ \tau'\to\infty}}\frac{1}{16\pi^4}\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} F(s,u,p,v)e^{i\{(st+uz)-i(px+vy)\}}dsdudpdv,$$

where, $p=\rho+iw$, $v=\sigma+il$. Also s,u,ρ,σ are fixed real numbers with $-r<s<r$, $-r'<u<r'$, $\rho_1<p<\rho_2$, $\sigma_1<v<\sigma_2$.

Proof: Let, $\phi\in D$. Choose the real numbers c,d,c',d' such that $c<s<d$, $c'<u<d'$ and the real numbers a,b,a',b' are such that $\rho_1<a<p<b<\rho_2$, $\sigma_1<a'<v<b'<\sigma_2$, we have to show that

$$\langle f(t,z,x,y),\phi(t,z,x,y)\rangle=\lim_{\substack{r\to\infty\\ \tau\to\infty\\ r'\to\infty\\ \tau'\to\infty}}\left\langle\frac{1}{16\pi^4}\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} F(s,u,p,v)e^{i\{(st+uz)-i(px+vy)\}}dsdudpdv,\phi(t,z,x,y)\right\rangle \tag{5.3.1}$$

Now the integral on s,u,p and v is a continuous function of t,z,x and y and therefore the right hand side of (5.3.1) without the limit notation can be written as

$$\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}F(s,u,p,v)e^{i\{(st+uz)-i(px+vy)\}}dsdudwdldtdzdxdy, \qquad (5.3.2)$$

where, $p=\rho+iw,\ v=\sigma+il$ and $r,\tau,r',\tau'>0$.

Since $\phi(t,z,x,y)$ is of bounded support and the integrand is a continuous function of t,z,x,y,s,u,w,l, the order of integration may be changed and we write,

$$\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}F(s,u,p,v)e^{i\{(st+uz)-i(px+vy)\}}dsdudwdldtdzdxdy,$$

$$=\frac{1}{16\pi^4}\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}F(s,u,p,v)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)e^{i\{(st+uz)-i(px+vy)\}}dtdzdxdydsdudwdl$$

$$=\frac{1}{16\pi^4}\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}\left\langle f(g,j,e,q),e^{-i\{(sg+uj)-i(pe+vq)\}}\right\rangle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)e^{i\{(st+uz)-i(px+vy)\}}dtdzdxdydsdudwdl$$

$$=\left\langle f(g,j,e,q),\frac{1}{16\pi^4}\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}e^{-i\{(sg+uj)-i(pe+vq)\}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)e^{i\{(st+uz)-i(px+vy)\}}dtdzdxdydsdudwdl\right\rangle$$

(By lemma 1)

The order of integration for the repeated integral herein may be changed because again $\phi(t,z,x,y)$ is of bounded support and the integrand is a continuous function of t,z,x,y,s,u,w,l upon doing this we obtain

$$=\left\langle f(g,j,e,q),\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}e^{-i(sg+uj)}e^{-(pe+vq)}e^{i(st+uz)}e^{(px+vy)}dsdudwdldtdzdxdy\right\rangle$$

(By Fubini's theorem)

$$=\left\langle f(g,j,e,q),\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)\int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'}e^{(t-g)is}e^{p(x-e)}e^{(z-j)iu}e^{v(y-q)}dsdudwdldtdzdxdy\right\rangle$$

$$=\left\langle f(g,j,e,q),\frac{1}{16\pi^4}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\phi(t,z,x,y)\left[\int_{-r}^{r}e^{(t-g)is}ds\right]\left[\int_{-r'}^{r'}e^{(z-j)iu}du\right]\left[\int_{-\tau}^{\tau}e^{p(x-e)}dp\right]\left[\int_{-\tau'}^{\tau'}e^{v(y-q)}dv\right]dtdzdxdy\right\rangle$$

$$= \left\langle f(g,j,e,q), \frac{1}{16\pi^4} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \phi(t,z,x,y) \frac{2\sin(t-g)r}{(t-g)} . \frac{2\sin(z-j)r'}{(z-j)} \right.$$

$$\left. \frac{2\sinh(x-e)\tau}{(x-e)} . \frac{2\sinh(y-q)\tau'}{(y-q)} dtdzdxdy \right\rangle$$

$$= \left\langle f(g,j,e,q), \frac{1}{\pi^4} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \phi(t,z,x,y) \frac{\sin(t-g)r}{(t-g)} . \frac{\sin(z-j)r'}{(z-j)} . \frac{\sinh(x-e)\tau}{(x-e)} . \frac{\sinh(y-q)\tau'}{(y-q)} dtdzdxdy \right\rangle$$

Taking $r \to \infty$, $r' \to \infty$, $\tau \to \infty$, $\tau' \to \infty$ and using lemma 2, we get

$$= \langle f(g,j,e,q), \phi(g,j,e,q) \rangle$$

$$\therefore \lim_{\substack{r\to\infty \\ \tau\to\infty \\ r'\to\infty \\ \tau'\to\infty}} \left\langle \frac{1}{16\pi^4} \int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} F(s,u,p,v) e^{i\{(st+uz)-i(px+vy)\}} dsdudpdv, \phi(t,z,x,y) \right\rangle = \langle f, \phi \rangle,$$

This completes the proof.

6. Uniqueness Theorem

If $FL\{f(t,z,x,y)\} = F(s,u,p,v)$, for $s,u,p,v \in \Omega_f$, and $FL\{g(t,z,x,y)\} = G(s,u,p,v)$, for $s,u,p,v \in \Omega_g$, $s > 0$ and $u > 0$ and $\rho_1 < \operatorname{Re} p < \rho_2$ and $\sigma_1 < \operatorname{Re} v < \sigma_2$. If $\Omega_f \cap \Omega_g$ is not empty and if $F(s,u,p,v) = G(s,u,p,v)$, for $s,u \in \Omega_f \cap \Omega_g$ and $p,v \in \Omega_f \cap \Omega_g$ then $f = g$ in the sense if equality $D^*(I)$.

Proof: f and g must assign the same value to each $\phi \in D$. By inversion theorem and equating $F(s,p)$ and $G(s,p)$ in

$$\langle f - g, \phi(t,z,x,y) \rangle =$$

$$\lim_{\substack{r\to\infty \\ \tau\to\infty \\ r'\to\infty \\ \tau'\to\infty}} \left\langle \frac{1}{16\pi^4} \int_{-r}^{r}\int_{-r'}^{r'}\int_{-\tau}^{\tau}\int_{-\tau'}^{\tau'} (F-G)(s,u,p,v) e^{i\{(st+uz)-i(px+vy)\}} dsdudpdv, \phi(t,z,x,y) \right\rangle = 0$$

Thus $f = g$ in $D^*(I)$.

7. Application of Fourier-Laplace Transform for Solving Partial Differential Equation

In this section the partial differential equation of heat conduction for the interior of wedge shape solid is solved.

7.1: Find the conventional function $u = u(t,x,z)$ on the domain $\{(t,x,z): -\infty < t < \infty, 0 < x < \infty, z > 0\}$ which satisfies the differential equation:

$$a\frac{\partial u}{\partial t} = bx^2\frac{\partial^2 u}{\partial x^2} + cx\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial z^2} \qquad (7.1)$$

with the following boundary conditions.

a) As $z \to 0^+$, $u(t,x,z)$ converges in the sense of D^* to a generalized function $f(t,x) \in FL^*_{a,b,\alpha}$.
b) As $z \to 1^-$, $u(t,x,z)$ converges in the sense of D^* to a generalized function $g(t,x) \in FL^*_{a,b,\alpha}$.

Now we apply the distributional Fourier-Laplace transform to (7.1) and using an operational transform formula, the equation (7.1) converts to

$$a.S\overline{U} = bp(p+1)\overline{U} + cp\overline{U} + \frac{\partial^2 \overline{U}}{\partial z^2},$$

where, $\overline{U}(s,p,z) = FL\{u(t,x,z)\}$,

$$\therefore \frac{\partial^2 \overline{U}}{\partial z^2} = \left(a.S - bp^2 - (b+c)p\right)\overline{U}$$

$$\therefore \frac{\partial^2 \overline{U}}{\partial z^2} = P(s,p)\overline{U} \text{ where } P(s,p) = \left(a.S - bp^2 - (b+c)p\right)$$

$$\therefore \overline{U} = A(s,p)e^{\sqrt{P(s,p)}z} + B(s,p)e^{-\sqrt{P(s,p)}z} \qquad (7.2)$$

where, $A(s,p)$ and $B(s,p)$ are the unknown generalized function which do not depend on z. In view of the boundary condition (a) and (b) we get,

$$F(s,p) = A(s,p) + G(s,p)$$

$$G(s,p) = A(s,p)e^{-\sqrt{P(s,p)}} + B(s,p)e^{\sqrt{P(s,p)}} \qquad (7.3)$$

Solving above equations,

$$A(s,p)=\frac{F(s,p)e^{\sqrt{P(s,p)}}-G(s,p)}{e^{\sqrt{P(s,p)}}-e^{-\sqrt{P(s,p)}}} \tag{7.4}$$

$$B(s,p)=\frac{G(s,p)-F(s,p)e^{-\sqrt{P(s,p)}}-G(s,p)}{e^{\sqrt{P(s,p)}}-e^{-\sqrt{P(s,p)}}} \tag{7.5}$$

Where, $F(s,p)=\left\langle f(t,x), e^{-i(st-ipx)}\right\rangle$ and

$G(s,p)=\left\langle \mathrm{g}(t,x), e^{-i(st-ipx)}\right\rangle$

Equation (7.2) is

$$\bar{U}=A(s,p)e^{-\sqrt{P(s,p)}z}+B(s,p)e^{\sqrt{P(s,p)}z} \tag{7.6}$$

Where, A and B as in (7.4) and (7.5)

Furthermore, $A(s,p)e^{-\sqrt{P(s,p)}z}$ and $B(s,p)e^{-\sqrt{P(s,p)}z}$ are smooth functions of s and p for each $z>0$.

Therefore, we can apply the inverse Fourier- Laplace Transform to get,

$$u(t,x,z_i)=\frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\bar{U}e^{i(st-ipx)}dsdp \ ,$$

where, $\bar{U}$ is as in (7.6)

8. Conclusions

This paper is mainly focused on the generalization of two dimensional Fourier-Laplace transform in the distributional sense and proved inversion theorem for two dimensional Fourier-Laplace transform with the help of two lemmas. Also described the application of Fourier-Laplace transform for solving partial differential equation.

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