



## AN ANALYSIS OF THE IMPACT OF TRADE SANCTIONS ON THE INCIDENCE OF CHILD LABOUR

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**ABSTRACT:** *Although the incidence of child labour has declined in recent years in the aggregate, the experience of last fifteen years is not very satisfactory to achieve United Nations goal of ending child labour in all its forms by 2025. The purpose of this paper is to develop a general equilibrium model, with co-existence of both formal and informal sectors, to analyze the efficacy of trade sanctions and liberalized trade policy in reducing the incidence of child labour. Informal sectors use child labour to produce a non-traded intermediary for the tariff protected import competing formal sector. The analysis observes that instruments like liberalized trade policy improves child labour problem, though, under reasonable conditions trade sanctions may worsen the situation.*

**Keywords:** Child labour; trade sanctions; trade liberalization; informal sector; general equilibrium.

**JEL classification:** F10, J13, J22, O17.

### 1. Introduction:

Child labour is a multi-dimensional problem which adversely affects proper mental and physical well-being of working children. It lowers human capital accumulation which in turn reinforces the problem itself generating more number of low skill and less educated workers with reduced future earning ability. Sheer economic necessity can be viewed as one of the most important cause of this disconcerting problem. According to ILO (2017) children engaged in hazardous work are considered to be the worst form of child labour. Reports of international bodies like UNICEF and ILO reflect that absolute figure of child labour on the global scale are substantially high. According to the Bureau of Statistics of the ILO, worldwide 152 million are victims of child labour between age group 5-17 years in 2016. The percentage of world's children aged between 5-14 years engaged as child labour is 9.9 in 2012 (Source: ILO 2017) while that in 2000 was 16. In India the percentage of working children between age group 5-14 years in 2011 was 3.9 while that in 2001 was 5.0 (Source: ILO 2017). Census data of different years also confirms that in India number of child labour in absolute terms has increased over the decades though a continuous declining trend has been observed with respect to the child labour



participation rate. If the estimate includes child labour engaged in unpaid household jobs then one might get a relatively grim picture.

Globalization has a complex relationship with child labour problem. According to some economists, to maintain competitive edge developing countries will be induced to use child labour more intensively, as a result of trade openness. Economists like Cigno et al. (2002) support this view. According to them trade openness as a result of globalization does have a negative effect on child labour. Some others with completely different view are of the opinion that, in developing economies, increased employment and earning opportunities, as a result of openness, will induce development for all. Globalization will lead to enhanced demand for skilled labour leading to more investment in education lessening the attractiveness for child labour. Ray (2000) concluded that limited access to education and poor quality of education imparted contribute to the apathy of poor parents to send their children to school. Uncertainty in the future returns from skill formation and school education compels them to send their children to work, instead, which brings immediate return. Thus costs and benefits of education influence the parents' decision whether a child would go to work or not. Study of Ravallion and Wodon (1999) investigated how enrollment subsidy and other educational incentive policies reduced child labour incidence in Bangladesh. The study revealed that Food For Education programme in Bangladesh reduced child participation rate in work by twenty percent for girls and thirty percent for boys<sup>1</sup>. Chaudhuri and Dwivedi (2007) concluded that the policy of subsidy on education might be helpful in controlling the child labour problem. However Chandrasekher and Mukhopadhyay (2006) reached a related but different conclusion that even completely free primary education will not necessarily ensure hundred percent attendances.

However, one of the root causes of supply of child labour is the abject poverty of the parents, which compels them to have large families and children to go out in the job market and earn their own means of livelihood. Basu and Van (1998) and Baland and Robinson (2000) supported this view. Baland and Robinson (2000) have shown that child labour may persist *'either when parents leave their children no bequests or when capital markets are imperfect'*. In Basu and Van (1998), on the other hand, poverty is the driving force behind the incidence of child labour. Parents do not send their children to work when household income exceeds some threshold family income from adult wage, as in their analysis child labour and adult labour are substitutes. In contrast to the existing view that poverty reduction will eradicate this social evil, Dwivedi and Chaudhuri (2010) have shown that reduction in poverty is not a necessary condition for the problem of child labour to improve in the developing economies.

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<sup>1</sup> Chaudhuri (2010) provided a completely different picture in terms of a general equilibrium structure.



There are some theoretical papers, which have investigated the efficacy of various trade and non-trade policies to assess the impact on child labour. These analyses are very important particularly when it was believed that liberalized policies would take the developing countries into higher growth orbits and benefits of which would definitely trickle down to the bottom of the society. In terms of a two goods and two period model Jafarey and Lahiri (2002) studied the impact of trade sanctions on the child labour incidence in the presence of credit market imperfection<sup>2</sup>. However, they have laid special emphasis on the role of the credit market in financing the cost of children's education. In the presence of satisfactory schooling quality and relatively high return to education, credit market imperfection may boost the positive impact on the incidence of child labour. In terms of a competitive general equilibrium model, in the presence of adult unemployment, Gupta (2002) examined the effectiveness of trade sanctions as a means of curbing the prevalence of child labour. His study shows that trade sanctions, though reduce the unemployment problem of adult labour, may have perverse effect on the supply of child labour. Thus, in contrast to the existing view, the paper depicted that the supply of child labour varies inversely with the adult unemployment rate. But trade and fiscal policies, which affect the effective producers' price of the product produced by adult workers, may help in reducing the prevalence of the evil in the system. However, in Gupta (2002) child labour is used in a separate sector for producing an internationally traded commodity. This assumption seems to be unrealistic because in reality it is hard to find any sector producing internationally traded commodity using only child labour. Also this paper did not consider the impact of liberalized trade policies. Efficacy of trade and non-trade policies on the incidence of child labour use has also been investigated by Chatterjee and Ray (2016) in terms of a three factor general equilibrium model. Their study finds that, although trade policy is ineffective in eradicating child labour, the use of non-trade policy is quite effective in reducing the problem. Chaudhuri and Dwibedi (2006) analyzed, in terms of a theoretical model with informal sectors, the desirability of trade liberalization in agriculture in developed nations in controlling the child labour problem. In another paper Dwibedi and Chaudhuri (2014) have shown that though agricultural subsidy policy fails to eradicate this social evil, economic growth through FDI can play a positive role in this respect.

It can be concluded from the above study that governments should resort to different trade and non-trade policies to accelerate the rate of fall of child labour use. The objective of the present paper is to enquire the effectiveness of trade sanctions as a policy for reducing the incidence of child labour in terms of a three-sector general equilibrium model. The model considers one formal and two informal sectors, one of the informal sectors produces a non-traded intermediary for the tariff protected import-competing formal manufacturing sector of the economy. Adult labour and child labour are perfect substitutes to each other, and child labour are

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<sup>2</sup> One can also see Ranjan (1999, 2001) in this context.



used in the informal sectors. The effect of trade sanctions is captured by a decrease in the price of the export commodity in which child labour is used. If some of the developed countries impose trade sanctions on the products, world prices of the exportables of the developing countries would fall. The effect of such trade sanctions on the problem of child labour has been studied. Also the outcome of a reduction in import tariff (trade liberalization policy) has been analyzed. The paper finds that under reasonable conditions trade sanctions may produce perverse effect on the incidence of child labour while a liberalized trade policy is effective in reducing the prevalence of the evil in the system.

## **2. The Model:**

I consider a small open economy with three sectors producing an exportable commodity X, an importable Z, and a non-traded intermediary, Y. Commodities X and Y are produced using labour and capital while production of Z requires the non-traded intermediary apart from labour and capital. The per-unit requirement of the intermediate input is assumed to be technologically fixed in sector Z.<sup>3</sup> Z is the formal sector of the economy, which faces a unionized labour market while X and Y are the two informal sectors facing competitive labour market. There are two types of labour in the model: adult labour and child labour. Sector Z uses only adult labour. On the other hand, sectors X and Y being the two informal sectors of the economy use both adult and child labour. Following Basu and Van (1998), it is assumed that adult labour is a perfect substitute for child labour in the informal sectors. It is assumed that an adult worker is equivalent to  $\beta$  number of child workers, where  $\beta > 1$ . Each adult worker employed in the informal sectors earns a wage of W. The child wage rate,  $W_c$ , must be  $(W / \beta)$  when the adult wage rate is W. Capital is perfectly mobile among all the three sectors. But adult labour is completely mobile between the two informal sectors and so is child labour.

It is assumed that the formal sector is the import-competing sector of the economy which is protected by a tariff. Production functions show CRS with diminishing marginal productivity to each factor and all inputs are fully employed. All markets except that of formal sector labour market are perfectly competitive. Prices of the traded goods, X and Z, are given internationally, due to the small open economy assumption. Since Y is non-traded its price is endogenously determined by the demand-supply mechanism. In the domestic market, thus, the supply of Y must equal the demand for Y. Sectors Y and Z, as a whole is more capital intensive than sector X, as considered in Chandra and Khan (1993) and Gupta (1997).

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<sup>3</sup> It rules out the possibility of substitution between the non-traded intermediate input and other factors of production in sector 3.



It is assumed that there exists a critical level of family income from adult wage,  $\bar{W}$ , in the absence of income from child labour. Formal sector workers earn a higher wage  $W^* > \bar{W}$ . As workers engaged in other two sectors earn relatively low wage they are compelled to send their children to job market to compensate low family income. The supply of child labour from each poor working family is endogenously determined.

There are  $L$  numbers of working families, which are classified into two groups with respect to the earnings of their adult members. The adult workers who work in the higher paid formal manufacturing sector comprise the richer section of the working population. On the contrary, the labourers who are engaged in informal sector jobs constitute the poorer section. There is homogeneity of labour within each of these two sections so that each family in the same group has exactly one adult worker and a certain number of children.

The following symbols will be used in the formal presentation of the model.

$a_{Li}$ : labour-output ratio in the  $i$ -th sector,  $i = X, Y, Z$ ;

$a_{Ki}$ : capital-output ratio in the  $i$ -th sector,  $i = X, Y, Z$ ;

$a_{YZ}$ : amount of  $Y$  commodity required to produce one unit of commodity  $Z$ ;

$\theta_{ji}$ : distributive share of the  $j^{\text{th}}$  input in the  $i^{\text{th}}$  industry,  $j = L, K$ ; and,  $i = X, Y, Z$ ;

$\theta_{YZ}$ : distributive share of  $Y$  in the  $Z$  sector;

$\lambda_{ji}$ : proportion of the  $j^{\text{th}}$  input employed in the  $i^{\text{th}}$  sector,  $j = L, K$  and  $i = X, Z$ ;

$P_i$ : world price of the  $i$ -th good,  $i = X, Z$ ;

$P_Y$ : endogenously determined price of the non-traded intermediary;

$t$ : ad-valorem tariff rate on the import of  $Z$ ;

$W^*$ : unionized adult wage rate in sector  $Z$ ;

$W$ : informal sector adult wage rate;

$W_C (= W/\beta)$ : child wage rate;

$r$ : rate of return to capital;

$C_X$ : consumption of commodity  $X$  by each working family;

$C_Z$ : consumption of commodity  $Z$  by each family;

$L$ : adult labour endowment;

$l_C$ : supply of child labour by each informal sector working family;

$L_C$ : aggregate supply of child labour;

$K$ : capital stock of the economy;

$\wedge$ : proportional change.



## 2.1 Supply Function of Child Labour

In this section it is wanted to derive the supply function of child labour from the utility maximizing behaviour<sup>4</sup> of the representative adult worker working in either of the two informal sectors who sends his children to work.

The utility function of the representative adult worker is given by

$$U = W(C_X, C_Z) - V(L_C)$$

The worker derives utility from the consumption of the final goods and disutility from child labour. For analytical simplicity let us consider the following specific algebraic form of the utility function.<sup>5</sup>

$$U = (C_X)^\alpha + (C_Z)^\alpha - (L_C)^\alpha \text{ with } 0 < \alpha < 1 \quad (1)$$

It satisfies all the standard properties. Also it is additive and symmetric. It is homogeneous of degree  $\alpha$  and has the constant elasticity of substitution between any two arguments.

He maximizes this utility function subject to the budget constraint

$$P_X C_X + P_Z (1+t) C_Z = [(W / \beta) l_C + W] \quad (2)$$

where  $W$  is the own income of the adult worker.

The following first-order conditions are satisfied in equilibrium.

$$(C_X / C_Z)^{(\alpha-1)} = (P_X / P_Z (1+t)) \quad (3)$$

$$(C_X / l_C)^{(\alpha-1)} = (P_X / (W / \beta)) \quad (4)$$

and,

$$(C_Z / l_C)^{(\alpha-1)} = \{(P_Z (1+t) / (W / \beta))\} \quad (5)$$

Using equations (4) and (5) one can obtain,

<sup>4</sup> The workers employed in the formal sector of the economy do not send their offspring to the job market. This is because the income of the adult member of each formal sector family,  $W^*$ , is sufficiently above the critical level of income,  $\bar{W}$ . The adult member of each of these families maximizes the family welfare given by  $U = (C_X)^\alpha + (C_Z)^\alpha$  with respect to  $C_X$  and  $C_Z$  and subject to the budget constraint,  $P_X C_X + P_Z (1+t) C_Z = W^*$

<sup>5</sup> If one alternatively considers the utility function to be of the following Cobb-Douglas type:  $U = A.(C_X)^\alpha .(C_Z)^\beta / (l_C)^\rho$  with  $A > 0$  and  $1 > \alpha, \beta, \rho > 0$ , the qualitative results of the model more or less remain unaltered. See footnote 13 in this context.



$$P_X C_X + P_Z C_Z (1+t) = [P_X^{(\alpha/\alpha-1)} + (P_Z (1+t))^{(\alpha/\alpha-1)}](\beta/W)^{(1/\alpha-1)} l_C \quad (6)$$

This means that in equilibrium, total expenditure on final goods of each poor working family is proportional to the supply of child labour given the product prices and the informal sector wage rate.

Using equations (2), (4) and (5) I have

$$l_C = f(W, P_X, t) = [\beta / \{ (P_X)^{\alpha/\alpha-1} + (P_Z (1+t))^{\alpha/\alpha-1} \} \{ (W/\beta)^{\alpha/1-\alpha} \} - 1] \quad (7)$$

(-)(+)(+)

This is the supply function of child labour by each poor family. Its properties are now analyzed. First,  $l_C$  varies negatively with the wage rate,  $W$ . Note that in the informal sectors adult labour and child labour are perfect substitutes. So  $(W/\beta)$  is the child wage rate. Hence a rise in  $W$  produces a negative price effect. Secondly,  $l_C$  varies positively with the tariff rate,  $t$  and the price of commodity  $X$ ,  $P_X$ . An increase in  $t$  or in  $P_X$  lowers  $((P_Z (1+t))^{\alpha/\alpha-1})$  or  $((P_X)^{\alpha/\alpha-1})$  because  $(\alpha/\alpha-1) < 0$ . So  $l_C$  rises given the other parameters. An increase in  $P_X$  (or  $t$ ) raises the price of  $X$  (or  $Z$ ) commodity. So consumption of  $X$  (or  $Z$ ) commodity is substituted by the consumption of  $Z$  (or  $X$ ) commodity. Since the relative consumption of  $Z$  (or  $X$ ) with respect to child labour,  $(C_Z / l_C)$  (or  $C_X / l_C$ ), remains unchanged,  $l_C$  also rises.

Now the aggregate supply function of child labour is given by

$$L_C = [\beta / \{ (P_X)^{\alpha/\alpha-1} + (P_Z (1+t))^{\alpha/\alpha-1} \} \{ (W/\beta)^{\alpha/1-\alpha} \} - 1] (L - a_{LZ} \cdot Z) \quad (8)$$

where  $(L - a_{LZ} \cdot Z)$  is the number of poor or child labour supplying families.<sup>6</sup>

In the subsequent sections of the paper the following general form of the aggregate child labour function will be used for analytical purpose. However, all the properties satisfied by the specific functional form given by equation (8) will be retained.

$$L_C = f(W, P_X, t) (L - a_{LZ} \cdot Z) \quad (8.1)$$

(-)(+)(+)

<sup>6</sup> Note that  $(L - a_{LZ} \cdot Z)$  is the number of adult workers engaged in the two informal or low wage earning sectors of the economy and the supply of child labour comes from these poor families only.



## 2.2 The General Equilibrium Analysis

Given the assumption of perfectly competitive markets the usual price-unit cost equality conditions relating to the three sectors of the economy are given by the following three equations.

$$a_{LX}W + a_{KX}r = P_X \quad (9)$$

$$a_{LY}W + a_{KY}r = P_Y \quad (10)$$

$$a_{LZ}W + a_{KZ}r + P_Y a_{YZ} = P_Z(1+t) \quad (11)$$

Since the intermediary, Y, is used only in the production of Z its full-employment condition is as follows.

$$a_{YZ}Z = Y \quad (12)$$

The capital endowment equation, which shows capital market equilibrium, is given by

$$a_{KX}X + a_{KY}Y + a_{KZ}Z = K \quad (13)$$

There are L numbers of working families,<sup>7</sup> each consisting of one adult member and certain number of children. The number of poor working families is  $(L - a_{LZ}Z)$ . The number of children going to the job market from each poor family is decided by the adult member of the family. Actually, it is determined from the utility maximizing behaviour of the family. The aggregate supply function of child labour in general form is the following.

$$L_c = f(W, P_X, t)(L - a_{LZ}Z) \quad (8.1)$$

(-)(+)(+)

The effective labour endowment of the economy consists of both adult and child labour; and the labour market equilibrium is given by the following equation.

$$a_{LX}X + a_{LY}Y + a_{LZ}Z = L + (L_c / \beta) \quad (14)$$

Using (8.1) equation (14) can be rewritten as

$$a_{LX}X + a_{LY}Y + a_{LZ}Z = L + \{(1/\beta)f(W, P_X, t)(L - a_{LZ}Z)\} \quad (14.1)$$

There are seven endogenous variables (namely, W, r, P<sub>Y</sub>, X, Y, Z and L<sub>C</sub>) and seven independent equations (namely (8.1) and (9–13) and (14.1)) in this system. The parameters in this model are: P<sub>X</sub>, P<sub>Z</sub>, β, L, K, and t. Equations (9) – (11) constitute the price system and the rest of the equations form the output system. It should be noted that the system possesses the decomposition

<sup>7</sup> See footnote 4 in this context.





property since the three unknown input prices,  $W$ ,  $r$  and  $P_Y$ , can be determined from the price system alone, independent of the output system. Once the factor prices are known the factor coefficients,  $a_{ij}$ s, are also known. Now  $X$ ,  $Y$  and  $Z$  are obtained from equations (12), (13) and (14.1). Finally,  $L_C$  is determined from (8.1).

### 3. Comparative Static Exercises:

In this section of the paper I shall study the efficacy of trade sanctions and a liberalized trade policy on the supply of child labour in the economy. Now totally differentiating equations (9) – (11) and solving by Cramer’s rule following expressions are obtained.

$$\hat{W} = (1/|\theta|)[(\theta_{KY}\theta_{YZ} + \theta_{KZ})\hat{P}_x - \theta_{KX}T\hat{t}] \tag{15.1}$$

$$\hat{r} = (1/|\theta|)[\theta_{LX}T\hat{t} - \theta_{LY}\theta_{YZ}\hat{P}_x] \tag{15.2}$$

$$\text{and, } (\hat{W} - \hat{r}) = (1/|\theta|)[(\theta_{YZ} + \theta_{KZ})\hat{P}_x - T\hat{t}] \tag{15.3}$$

where  $|\theta| = [\theta_{LX}(\theta_{KY}\theta_{YZ} + \theta_{KZ}) - \theta_{KX}\theta_{LY}\theta_{YZ}] > 0$  as the vertically integrated import-competing sector is more capital-intensive than the export sector; and,  $T = (t/(1+t)) > 0$ .

Now from (12) one can get

$$\hat{Y} = \hat{Z} \tag{16}$$

Differentiating (13) and (14.1) and using (16) one can get<sup>8</sup>

$$\lambda_{LX}\hat{X} + A\hat{Z} = B\hat{P}_x - C\hat{t} \tag{17}$$

$$\lambda_{KX}\hat{X} + (\lambda_{KY} + \lambda_{LZ})\hat{Z} = D\hat{t} - E\hat{P}_x \tag{18}$$

Where  $A = \{\lambda_{LY} + \lambda_{LZ}(1 + f(\cdot)/\beta)\} > 0$ ;

$$B = (1/|\theta|)[(\theta_{YZ} + \theta_{KZ})(\lambda_{LX}\theta_{KX}\sigma_X + \lambda_{LY}\theta_{KY}\sigma_Y) + \lambda_{LZ}\theta_{KZ}\sigma_Z\theta_{LY}\theta_{YZ}(1 + f(\cdot)/\beta) + (E_W L_C / \beta L^*)(\theta_{KY}\theta_{YZ} + \theta_{KZ}) + (L_C E_{PX} |\theta| / \beta L^*)];$$

$$C = (T/|\theta|)[(\lambda_{LX}\theta_{KX}\sigma_X + \lambda_{LY}\theta_{KY}\sigma_Y) + \lambda_{LZ}\theta_{KZ}\sigma_Z\theta_{LX}(1 + f(\cdot)/\beta) + (E_W L_C \theta_{KX} / \beta L^*) - (E_t L_C |\theta| / \beta L^* T)];$$

$$D = (T/|\theta|)[\lambda_{KX}\theta_{LX}\sigma_X + \lambda_{KY}\theta_{LY}\sigma_Y + \lambda_{KZ}\theta_{LZ}\sigma_Z\theta_{LX}] > 0;$$

$$E = (1/|\theta|)[(\theta_{YZ} + \theta_{KZ})(\lambda_{KX}\theta_{LX}\sigma_X + \lambda_{KY}\theta_{LY}\sigma_Y) + \lambda_{KZ}\theta_{LZ}\sigma_Z\theta_{LY}\theta_{YZ}] > 0;$$

$$E_W = \{(\partial f(\cdot)/\partial W).(W/f(\cdot))\}; E_{PX} = \{(\partial f(\cdot)/\partial P_X).(P_X/f(\cdot))\}; E_t = \{(\partial f(\cdot)/\partial t).(t/f(\cdot))\}.$$

<sup>8</sup> See appendix for detailed derivation.



$$\begin{aligned} & \text{Now, } [(L_C \theta_{KX} / \beta L^*) \{E_W (\theta_{KY} \theta_{YZ} + \theta_{KZ}) + E_{PX} |\theta|\}] \\ & = -(L_C / f(\cdot)) L^* G^2 |\theta| (\alpha / 1 - \alpha) (W / \beta)^{(\alpha / 1 - \alpha)} [(P_X)^{(\alpha / \alpha - 1)} (\theta_{YZ} + \theta_{KZ}) \theta_{KX} \\ & \quad + (\theta_{KY} \theta_{YZ} + \theta_{KZ}) (P_Z (1 + t))^{(\alpha / \alpha - 1)}] < 0; \text{ and,} \end{aligned}$$

$$[(E_W L_C \theta_{KX} / \beta L^*) - (E_t L_C |\theta| / \beta L^* T)] < 0. \text{ (as } E_W < 0 \text{)}$$

$$\text{Note that } G = [\beta / \{ (P_X)^{(\alpha / \alpha - 1)} + (P_Z (1 + t))^{(\alpha / \alpha - 1)} \} \{ (W / \beta)^{(\alpha / 1 - \alpha)} \} - 1] > 0.$$

Using the above expressions it is easy to check that the signs of B and C are uncertain.

Solving (17) and (18) by Cramer's rule I get<sup>9</sup>

$$\hat{Z} = (|\lambda|) [(\lambda_{LX} D + \lambda_{KX} C) \hat{t} - (\lambda_{LY} E + \lambda_{KX} B) \hat{P}_X] \tag{19}$$

where

$$\begin{aligned} |\lambda| = & \lambda_{KX} (\lambda_{KY} + \lambda_{KZ}) [(\lambda_{LX} / \lambda_{KX}) - (\lambda_{LY} + \lambda_{LZ}) / (\lambda_{KY} + \lambda_{KZ})] \\ & - \lambda_{KX} \lambda_{LZ} (f(\cdot) / \beta) \end{aligned} \tag{20}$$

$$\text{or, } |\lambda| = \lambda_{KX} (\lambda_{KY} + \lambda_{KZ}) [(\lambda_{LX} / \lambda_{KX}) - \{ \lambda_{LY} + \lambda_{LZ} (1 + (f(\cdot) / \beta)) \} / (\lambda_{KY} + \lambda_{KZ})] \tag{20.1}$$

I shall now try to interpret  $|\lambda|$ . It should be remembered that each adult worker employed in the two informal sectors sends  $f(\cdot)$  number of his offspring to the job market to supplement low family income. On the other hand, labourers engaged in the formal sector of the economy are the richer section of the working class and do not send their children to the labour market. In the two informal sectors adult and child labour are perfect substitutes. So, the effective adult labour endowment of the economy must include child labour too, and is given by  $L^* (= L + L_C / \beta)$ . The labour-capital ratio in sector X is given by  $(\lambda_{LX} / \lambda_{KX})$ . Sector Z uses capital and labour both directly as well as indirectly through use of Y as production of one unit of Z requires  $a_{YZ}$  units of Y and sector Y also requires capital and labour in its production. Thus,  $(\lambda_{KY} + \lambda_{KZ})$  and  $(\lambda_{LY} + \lambda_{LZ})$  are the proportions of capital and the effective labour force used directly and indirectly in the production of Z. Thus the effective labour-capital ratio for sector Z is given by  $\{(\lambda_{LY} + \lambda_{LZ}) / (\lambda_{KY} + \lambda_{KZ})\}$ . The expression,  $[(\lambda_{LX} / \lambda_{KX}) - (\lambda_{LY} + \lambda_{LZ}) / (\lambda_{KY} + \lambda_{KZ})]$ , in the right-hand side of (20) gives the difference between the actual labour-capital ratio of sector X and the effective labour-capital ratio of the Z sector. This difference is positive because it is sensible to assume that the export sector (sector X) is more labour intensive vis-à-vis the vertically

<sup>9</sup> This has been shown in appendix.



integrated industrial sector (i.e. sectors Z and Y taken together).<sup>10</sup> Now turning back to interpreting the right-hand side of (20), it is noted that  $\lambda_{LZ}$  proportion of adult workers of the economy are engaged in the Z sector and do not send their offspring to the job market. However, if they were employed in either of the two informal sectors each of them would have sent  $f(\cdot)$  number of children to the labour market. This implies that the economy is deprived of having  $(f(\cdot)a_{LZ}Z)$  number of potential child workers, which is equivalent to  $(f(\cdot)a_{LZ}Z/\beta)$  units of adult worker. Thus,  $[\{\lambda_{LY} + \lambda_{LZ}(1 + (f(\cdot)/\beta))\} / (\lambda_{KY} + \lambda_{KZ})]$  gives the labour-capital ratio of sector Z taking into account the opportunity labour endowment which the economy is denied of because of the employment of  $(f(\cdot)a_{LZ}Z)$  number of adult labourers in sector Z. I call  $[\{\lambda_{LY} + \lambda_{LZ}(1 + (f(\cdot)/\beta))\} / (\lambda_{KY} + \lambda_{KZ})]$  the labour-capital ratio of the Z sector from the social point of view and this may not necessarily be less than the actual labour-capital ratio of the export sector i.e.  $(\lambda_{LX} / \lambda_{KX})$ . Thus,  $|\lambda|$  is negative (positive) under the necessary and sufficient condition that the actual labour-capital ratio of sector X is less (greater) than the labour-capital ratio of sector Z calculated from the social point of view. It should be noted that the  $a_{jis}$  depend on the unknown factor prices, which in turn depend on the parameters in the price system like,  $P_X, P_Z, t$  and  $\beta$ . The magnitude of  $f(\cdot)$  also depends on the values of  $\beta, P_X, P_Z,$  and  $t$ . So, depending on the parameter values  $|\lambda|$  would be negative or positive.

To find out the policy effects on the aggregate supply of child labour after totally differentiating equation (8.1) one can get the following expression.<sup>11</sup>

$$\begin{aligned} \hat{L}_C &= (E_{PX} \hat{P}_X + E_t \hat{t} + E_W \hat{W}) \\ &- \{a_{LZ}Z / (L - a_{LZ}Z)\} [\{(\theta_{LZ} \sigma_Z \theta_{LX} T / |\theta|) + ((\lambda_{LX} D + \lambda_{KX} C) / |\lambda|)\} \hat{t} \\ &- \{(\theta_{LZ} \sigma_Z \theta_{LY} \theta_{YZ} / |\theta|) + ((\lambda_{LX} E + \lambda_{KX} B) / |\lambda|)\} \hat{P}_X] \end{aligned} \quad (21)$$

From (7) it can be verified<sup>12</sup> that

$$\begin{aligned} \hat{l}_C &= (E_{PX} \hat{P}_X + E_t \hat{t} + E_W \hat{W}) \\ &= (\beta / f(\cdot) G^2 |\theta|) (\alpha / 1 - \alpha) (W / \beta)^{(\alpha/1-\alpha)} [-\hat{P}_X \{ (P_X)^{(\alpha/\alpha-1)} \theta_{KX} (\theta_{YZ} + \theta_{KZ}) \} \end{aligned}$$

<sup>10</sup> Chandra and Khan (1993) and Gupta (1997) have also made this assumption.

<sup>11</sup> See appendix for detailed derivation.

<sup>12</sup> This has been shown in appendix.



$$\begin{aligned}
 &+ (\theta_{KY}\theta_{YZ} + \theta_{KZ})(P_Z(1+t))^{(\alpha/\alpha-1)} + \hat{t} \left\{ t(P_Z(1+t))^{(1/\alpha-1)} |\theta| + \{(P_X)^{(\alpha/\alpha-1)} \right. \\
 &\left. + (P_Z(1+t))^{(\alpha/\alpha-1)} \} (\theta_{KX} \cdot T) \right\} \quad (22)
 \end{aligned}$$

Using (22) from equation (21) it is easy to derive the following results.<sup>13</sup>

(i)  $\hat{L}_C > 0$  when  $\hat{P}_X < 0$  if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} \leq 0$ .

(ii)  $\hat{L}_C < 0$  when  $\hat{t} < 0$  if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} \leq 0$ .

So I have the following proposition.

**PROPOSITION:** Trade sanctions, which result in a reduction in the price of the export commodity accentuate the incidence of child labour in the society if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} \leq 0$ . On the contrary, trade liberalization in the form of a reduction in the tariff rate reduces the problem of child labour if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} \leq 0$ .

The above proposition can be intuitively explained as follows. The aggregate supply of child labour in the economy is a product of the supply of child labour by each poor working family,  $f(\cdot)$ , and the number poor families,  $(L - a_{LZ}Z)$ . Trade sanctions in the form of a reduction in the price of the export commodity,  $P_X$ , in which child labour is used as input lower the competitive wage rate and raise the rental on capital following a Stolper-Samuelson effect as the export sector is more labour intensive than the vertically integrated import-competing sector. First, the effect of the policy on the supply of child labour from each poor working family is considered. As  $P_X$  decreases consumption of Z commodity is substituted by the consumption of X commodity. Since the relative consumption of Z with respect to child labour,  $(C_Z / l_C)$  remains unchanged,  $l_C$  also falls. But as W falls  $l_C$  rises. It is easy check<sup>14</sup> that the net effect would be an increase in  $l_C$ . Next I consider the effect on the number of child labour supplying families,  $(L - a_{LZ}Z)$ . As r rises and W falls producers in all the three sectors of the economy would adopt more labour-intensive techniques of production. As a consequence  $a_{Li}$ s increase and  $a_{Ki}$ s decrease for  $i = X, Y, Z$ . Given the product mix there will be an excess supply of capital, which produces

<sup>13</sup> If the utility function is of the following Cobb-Douglas type:  $U = (A(C_X)^\alpha (C_Z)^\beta / (l_C)^\rho)$  with  $A > 0$  and  $1 > \alpha, \beta, \rho > 0$ , one can check that  $\hat{L}_C = 0$ . So,  $\hat{L}_C > 0$  when  $\hat{P}_X < 0$  if and only if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} < 0$ ; and,  $\hat{L}_C < 0$  when  $\hat{t} < 0$  if and only if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T/|\theta|) + ((\lambda_{LX}D + \lambda_{KX}C)/|\lambda|)\} < 0$ .

<sup>14</sup> This has been shown in appendix.



a Rybczynski type effect. If  $\{(\lambda_{LX}E + \lambda_{KX}B) / |\lambda|\} < 0$ ,  $Z$  takes a lower value. But as  $a_{LZ}$  has increased the effect on  $a_{LZ}Z$  is uncertain. However, under the condition that  $\{(\theta_{LZ}\sigma_Z\theta_{LY}\theta_{YZ} / |\theta|) + ((\lambda_{LX}E + \lambda_{KX}B) / |\lambda|)\} < (=) 0$ ,  $a_{LZ}Z$  decreases (remains unchanged), which in turn implies that the number of poor families supplying child labour,  $(L - a_{LZ}Z)$ , increases (remains unaltered). It has already shown that a decrease in  $P_X$  leads to an increase in the number of child labour supplied by each poor working family. Thus, under the sufficient condition that  $\{(\theta_{LZ}\sigma_Z\theta_{LY}\theta_{YZ} / |\theta|) + ((\lambda_{LX}E + \lambda_{KX}B) / |\lambda|)\} \leq 0$ , the incidence of child labour rises. This is, however, only a sufficient condition because even if  $\{(\theta_{LZ}\sigma_Z\theta_{LY}\theta_{YZ} / |\theta|) + ((\lambda_{LX}E + \lambda_{KX}B) / |\lambda|)\} > 0$ , and  $(L - a_{LZ}Z)$  falls, the proportionate increase in  $l_c$  may outweigh the proportionate decrease in  $(L - a_{LZ}Z)$ . So  $L_C$ , which is a product of  $l_c$  and  $(L - a_{LZ}Z)$ , may increase even in this case. On the other hand, if  $\{(\lambda_{LX}E + \lambda_{KX}B) / |\lambda|\} > 0$ ,  $Z$  increases. As  $a_{LZ}$  has risen,  $(L - a_{LZ}Z)$  definitely falls. But the proportionate increase in  $l_c$  may dominate over the proportionate fall in  $(L - a_{LZ}Z)$ . So  $L_C$  may increase even in this case.

On the contrary, a reduction in the import tariff,  $t$ , lowers  $l_c$ , the supply of child labour by each poor working family in two ways. First, it lowers  $l_c$  directly. As  $t$  falls,  $P_Z(1+t)$ , falls too. It results in a substitution of commodity  $X$  by commodity  $Z$ . But as the ratio between  $l_c$  and  $C_X$  remains unchanged,  $l_c$  falls as  $C_X$  falls. Secondly, a decline in  $t$  lowers the rental rate on capital,  $r$ , but raises the informal sector wage rate,  $W$ . As  $W$  rises  $l_c$  falls due to a negative price effect. As the two effects on  $l_c$  work in the same direction,  $l_c$  falls as  $t$  falls. Let us now find out the effect of this policy on  $(L - a_{LZ}Z)$ . As  $r$  falls and  $W$  rises following a reduction in  $t$ , producers in the different sectors of the economy would adopt more capital-intensive techniques of production than before. So  $a_{Ki}$ s increase and  $a_{Li}$ s decrease for  $i = X, Y, Z$ . Given the composition of output, there would occur a shortage of capital and this produces a Rybczynski type effect. Now if  $\{(\lambda_{LX}D + \lambda_{KX}C) / |\lambda|\} < 0$ ,  $Z$  increases. So the effect on  $a_{LZ}Z$  remains ambiguous.

However, if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T / |\theta|) + ((\lambda_{LX}D + \lambda_{KX}C) / |\lambda|)\} < (=) 0$ ,  $a_{LZ}Z$  increases (remains unchanged) and as a consequence  $(L - a_{LZ}Z)$  decreases (remains unchanged). Thus under the sufficient condition that  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T / |\theta|) + ((\lambda_{LX}D + \lambda_{KX}C) / |\lambda|)\} \leq 0$ ,  $L_C$  falls. It is to be noted that this is only a sufficient condition for  $L_C$  to fall because even if  $\{(\theta_{LZ}\sigma_Z\theta_{LX}T / |\theta|) + ((\lambda_{LX}D + \lambda_{KX}C) / |\lambda|)\} > 0$  and  $a_{LZ}Z$  falls (and hence  $(L - a_{LZ}Z)$  increases)  $L_C$  may fall if the proportionate fall in  $l_c$  is greater than the proportionate increase in  $(L - a_{LZ}Z)$ .



If, on the other hand,  $\{(\lambda_{LX}D + \lambda_{KX}C) / |\lambda|\} > (=) 0$ ,  $Z$  decreases (remains constant). So  $a_{LZ}Z$  decreases and hence  $(L - a_{LZ}Z)$  increases. Still then  $L_C$  may fall if  $l_c$  falls at a greater rate than the rate at which  $(L - a_{LZ}Z)$  rises.

#### 4. Concluding Remarks:

Child labour is a slur on the fair face of the globalized world. Different policies are advocated in reducing the prevalence of the evil in the system. Imposition of trade sanctions on the exportable commodities of the developing countries that are produced using child labour is the most controversial and hotly debated policy suggested by representatives of the developed countries in this context at the WTO meeting. The paper tries to examine whether trade sanctions can become an important tool of reducing child labour in a developing economy.

The paper builds up a three-sector general equilibrium model with informal sectors and a non-traded intermediary for the purpose of analysis. One of the informal sectors produces a non-traded intermediary for the formal manufacturing tariff protected import-competing sector of the economy. Child labour is used in the informal sectors. Adult labour and child labour are perfect substitutes to each other. If some of the developed countries impose trade sanctions on the exportable commodities from the developing countries, which are produced using child labour, the consequence would be a fall in the world prices of the exportable commodities of the developing countries. So, the effect of trade sanctions in this paper is captured by a decrease in the price of the export commodity in which child labour is used. The effect of such trade sanctions on the incidence of child labour has been investigated. The paper finds that the effectiveness of trade sanctions as a means of reducing the problem of child labour can be called into question. This policy may in fact produce a perverse effect on the supply of child labour. The paper then goes onto study the implication of a policy of trade liberalization as an alternative policy to combat child labour. It has been found that under reasonable conditions a liberalized trade policy may be an effective way to fight against the child labour problem.



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**APPENDIX:**

Differentiating equation (12) I get

$$\hat{Y} = \hat{Z} \text{ (Note that } a_{YZ} \text{ is technologically given.)} \tag{16}$$

Again differentiating (13) and using (16) it can be written

$$\begin{aligned} \lambda_{KX} \hat{X} + (\lambda_{KY} + \lambda_{KZ}) \hat{Z} &= -\lambda_{KX} \hat{a}_{KX} - \lambda_{KY} \hat{a}_{KY} - \lambda_{KZ} \hat{a}_{KZ} \\ &= \lambda_{KX} \theta_{LX} \sigma_X (\hat{W} - \hat{r}) - \lambda_{KY} \theta_{LY} \sigma_Y (\hat{W} - \hat{r}) + \lambda_{KZ} \theta_{LZ} \sigma_Z \hat{r} \\ &= -\{(\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y) / |\theta|\} \{(\theta_{YZ} + \theta_{KZ}) \hat{P}_X - T\hat{t}\} \\ &\quad + \{(\lambda_{KZ} \theta_{LZ} \sigma_Z / |\theta|\} \{\theta_{LX} T\hat{t} - \theta_{LY} \theta_{YZ} \hat{P}_X\} \\ &= (T\hat{t} / |\theta|) [\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y + \lambda_{KZ} \theta_{LZ} \sigma_Z \theta_{LX}] \\ &\quad - (\hat{P}_X / |\theta|) [(\theta_{YZ} + \theta_{KZ}) (\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y) + \lambda_{KZ} \theta_{LZ} \sigma_Z \theta_{LY} \theta_{YZ}] \end{aligned} \tag{A.1}$$

Differentiating (14.1) and using equation (16) one obtains

$$\begin{aligned} \lambda_{LX} \hat{X} + (\lambda_{LY} + \lambda_{LZ}) \hat{Z} &= -\lambda_{LX} \hat{a}_{LX} - \lambda_{LY} \hat{a}_{LY} - \lambda_{LZ} \hat{a}_{LZ} - (f(\cdot) / \beta) \lambda_{LZ} \hat{Z} \\ &\quad - (f(\cdot) / \beta) \lambda_{LZ} \hat{a}_{LZ} + (L - a_{LZ} \cdot Z) [(\partial f(\cdot) / \partial P_X) \cdot (dP_X / \beta \cdot L^*)] \\ &\quad + (\partial f(\cdot) / \partial t) \cdot (dt / \beta \cdot L^*) + (\partial f(\cdot) / \partial W) \cdot (dW / \beta \cdot L^*) \\ &= \lambda_{LX} \theta_{KX} \sigma_X (\hat{W} - \hat{r}) + \lambda_{LY} \theta_{KY} \sigma_Y (\hat{W} - \hat{r}) - \lambda_{LZ} \theta_{KZ} \sigma_Z \hat{r} \\ &\quad + (L_C / \beta L^*) \cdot [E_{PX} \hat{P}_X + E_t \hat{t} + E_W \hat{W}] - (f(\cdot) / \beta) \lambda_{LZ} \hat{Z} - (f(\cdot) / \beta) \lambda_{LZ} \hat{r} \end{aligned} \tag{A.2}$$

where  $E_{PX} = ((\partial f(\cdot) / \partial P_X) \cdot (dP_X / f(\cdot)))$  ;  $E_t = ((\partial f(\cdot) / \partial t) \cdot (dt / f(\cdot)))$  ;

$E_W = ((\partial f(\cdot) / \partial W) \cdot (dW / f(\cdot)))$  are the elasticities of the  $f(\cdot)$  function with respect to  $P_X$ ,  $t$  and

$W$ , respectively. Using the expressions for  $E_{PX}$ ,  $E_t$  and  $E_W$  and after simplification (A.2) becomes

$$\begin{aligned} \lambda_{LX} \hat{X} + \{\lambda_{LY} + \lambda_{LZ} (1 + f(\cdot) / \beta)\} \hat{Z} \\ &= \{(\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y) / |\theta|\} \{\hat{P}_X (\theta_{YZ} + \theta_{KZ}) - T\hat{t}\} \\ &\quad - (\lambda_{LZ} \theta_{KZ} \sigma_Z / |\theta|) \cdot [\theta_{LX} T\hat{t} - \theta_{LY} \theta_{YZ} \hat{P}_X] + (L_C / \beta L^*) [E_{PX} \hat{P}_X + E_t \hat{t} + E_W \hat{W}] \\ &\quad - (f(\cdot) / \beta) \lambda_{LZ} \hat{Z} - \{(f(\cdot) / \beta) \lambda_{LZ} \theta_{KZ} \sigma_Z / |\theta|\} \cdot [\theta_{LX} T\hat{t} - \theta_{LY} \theta_{YZ} \hat{P}_X] \end{aligned}$$





Further simplification gives

$$\begin{aligned} & \lambda_{LX} \hat{X} + \{\lambda_{LY} + \lambda_{LZ}(1 + f(\cdot) / \beta)\} \hat{Z} \\ &= \hat{P}_X \{[(\theta_{YZ} + \theta_{KZ})(\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y) + \lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LY} \theta_{YZ}] |\theta| \\ &+ \{(E_W L_C / \beta L^*) \cdot (\theta_{KY} \theta_{YZ} + \theta_{KZ}) / |\theta|\} + \{(f(\cdot) / \beta)(\lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LY} \theta_{YZ}) / |\theta|\} + (L_C \cdot E_{PX} / \beta L^*)] \\ &- \hat{t}(T / |\theta|)[(\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y) + \lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LX} \\ &+ \{(f(\cdot) / \beta) \lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LX}\} + (E_W L_C / \beta L^*) \theta_{KX} - \{(E_i L_C / \beta L^*) \cdot (|\theta| / T)\}] \end{aligned} \tag{A.3}$$

I rewrite (A.3) and (A.1) as follows.

$$\lambda_{LX} \hat{X} + A \hat{Z} = B \hat{P}_X - C \hat{t} \tag{17}$$

$$\lambda_{KX} \hat{X} + (\lambda_{KY} + \lambda_{KZ}) \hat{Z} = D \hat{t} - E \hat{P}_X \tag{18}$$

where  $A = \{\lambda_{LY} + \lambda_{LZ}(1 + f(\cdot) / \beta)\} > 0$ ;

$$\begin{aligned} B &= (1 / |\theta|) \{[(\theta_{YZ} + \theta_{KZ})(\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y) + \lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LY} \theta_{YZ}(1 + f(\cdot) / \beta)] \\ &+ \{(E_W L_C / \beta L^*) \cdot (\theta_{KY} \theta_{YZ} + \theta_{KZ})\} + (L_C \cdot E_{PX} / \beta \cdot L^*) \cdot (|\theta|)\}; \end{aligned}$$

$$\begin{aligned} C &= (T / |\theta|) \{[(\lambda_{LX} \theta_{KX} \sigma_X + \lambda_{LY} \theta_{KY} \sigma_Y) + \lambda_{LZ} \theta_{KZ} \sigma_Z \theta_{LX} \cdot (1 + f(\cdot) / \beta)] \\ &+ (E_W L_C \theta_{KX} / \beta L^*) - \{(E_i L_C / \beta L^*) \cdot (|\theta| / T)\}\}; \end{aligned}$$

$$D = (T / |\theta|) [\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y + \lambda_{KZ} \theta_{LZ} \sigma_Z \theta_{LX}] > 0; \text{ and,}$$

$$E = (1 / |\theta|) [(\theta_{YZ} + \theta_{KZ})(\lambda_{KX} \theta_{LX} \sigma_X + \lambda_{KY} \theta_{LY} \sigma_Y) + \lambda_{KZ} \theta_{LZ} \sigma_Z \theta_{LY} \theta_{YZ}] > 0.$$

Arranging equations (17) and (18) in a matrix notation it can be written as

$$\begin{pmatrix} \lambda_{LX} & A \\ \lambda_{KX} & (\lambda_{KY} + \lambda_{KZ}) \end{pmatrix} \begin{pmatrix} \hat{X} \\ \hat{Z} \end{pmatrix} = \begin{pmatrix} B \hat{P}_X - C \hat{t} \\ D \hat{t} - E \hat{P}_X \end{pmatrix} \tag{A.4}$$

Solving (A.4) by Cramer's rule I get

$$\hat{Z} = (1 / |\lambda|) [(\lambda_{LX} D + \lambda_{KX} C) \hat{t} - (\lambda_{LX} \cdot E + \lambda_{KX} B) \hat{P}_X] \tag{19}$$

$$\text{where } |\lambda| = [\lambda_{LX} (\lambda_{KY} + \lambda_{KZ}) - \lambda_{KX} (\lambda_{LY} + \lambda_{LZ}) - \lambda_{KX} \lambda_{LZ} \cdot (f(\cdot) / \beta)] \tag{20}$$

Totally differentiating equation (8.1) one can get

$$\hat{L}_C L_C = (L - a_{LZ} Z) [(\partial f(\cdot) / \partial P_X) dP_X + (\partial f(\cdot) / \partial t) dt + (\partial f(\cdot) / \partial W) dW] - f(\cdot) a_{LZ} Z (\hat{a}_{LZ} + \hat{Z})$$

$$\hat{L}_C = (E_{PX} \hat{P}_X + E_i \hat{t} + E_W \hat{W}) - \{[a_{LZ} Z / (L - a_{LZ} Z)] \cdot (\theta_{LZ} \cdot \sigma_Z \hat{t} + \hat{Z})\}$$

$$= (E_{PX} \hat{P}_X + E_i \hat{t} + E_W \hat{W})$$

$$- \{[a_{LZ} Z / (L - a_{LZ} Z)] \cdot [(\theta_{LZ} \cdot \sigma_Z / |\theta|) \{ \theta_{LX} \hat{t} - \theta_{LY} \theta_{YZ} \hat{P}_X \}]\}$$



$$\begin{aligned}
 &+ \{(\lambda_{LX}D + \lambda_{KX}C).(\hat{t} / |\lambda|)\} - \{(\lambda_{LX}E + \lambda_{KX}B).(\hat{P}_X / |\lambda|)\} \\
 &= (E_{PX}\hat{P}_X + E_t\hat{t} + E_W\hat{W}) \\
 &- \{a_{LZ}Z / (L - a_{LZ}Z)\} [\{(\theta_{LZ}\sigma_Z\theta_{LX}.T / |\theta|) + ((\lambda_{LX}.D + \lambda_{KX}.C) / |\lambda|)\} \hat{t} \\
 &- \{(\theta_{LZ}\sigma_Z\theta_{LY}\theta_{YZ} / |\theta|) + ((\lambda_{LX}E + \lambda_{KX}B) / |\lambda|)\} \hat{P}_X] \quad (21)
 \end{aligned}$$

Now totally differentiating equation (7) one gets

$$dl_c = (\partial f(\cdot) / \partial P_X)dP_X + (\partial f(\cdot) / \partial t)dt + (\partial f(\cdot) / \partial W)dW$$

After a simple manipulation this becomes

$$\hat{l}_c = (E_{PX}\hat{P}_X + E_t\hat{t} + E_W\hat{W})$$

Differentiating equation (7) partially with respect to  $P_X$ ,  $t$  and  $W$  one can find out the expressions for the elasticities of  $l_c$  with respect to  $P_X$ ,  $t$  and  $W$  i.e. the expressions for  $E_{PX}$ ,  $E_t$  and  $E_W$ .

Inserting those expressions into the above equation one gets

$$\begin{aligned}
 \hat{l}_c &= (\beta / l_c G^2)(\alpha / 1 - \alpha)(P_X)^{(\alpha/\alpha-1)} .(W / \beta)^{(\alpha/1-\alpha)} \hat{P}_X \\
 &+ (\beta / l_c G^2)(\alpha / 1 - \alpha) t (P_Z(1+t))^{(1/\alpha-1)} (W / \beta)^{(\alpha/1-\alpha)} \hat{t} \\
 &- (\beta / l_c G^2)(\alpha / 1 - \alpha)(W / \beta)^{(\alpha/1-\alpha)} \{(P_X)^{(\alpha/\alpha-1)} \\
 &+ (P_Z(1+t))^{(\alpha/\alpha-1)}\} \{[\hat{P}_X(\theta_{KY}\theta_{YZ} + \theta_{KZ}) - \theta_{KX}\hat{t}] / |\theta|\}
 \end{aligned}$$

where  $G = [\beta / \{ \{(P_X)^{(\alpha/\alpha-1)} + (P_Z(1+t))^{(\alpha/\alpha-1)} \} \{(W / \beta)^{(\alpha/1-\alpha)}\} - 1\}] > 0$ .

After simplification this may be reduced to the following expression.

$$\begin{aligned}
 \hat{l}_c &= (\beta / f(\cdot).G^2|\theta|).(\alpha / 1 - \alpha).(W / \beta)^{(\alpha/1-\alpha)} [-\hat{P}_X \{(P_X)^{(\alpha/\alpha-1)} \theta_{KX} (\theta_{YZ} + \theta_{KZ}) \\
 &+ (\theta_{KY}.\theta_{YZ} + \theta_{KZ})(P_Z(1+t))^{(\alpha/\alpha-1)}\} + \hat{t} \{t(P_Z(1+t))^{(1/\alpha-1)} |\theta| + \{(P_X)^{(\alpha/\alpha-1)} \\
 &+ (P_Z(1+t))^{(\alpha/\alpha-1)}\} \theta_{KX} T\}] \quad (22)
 \end{aligned}$$