

IMPLEMENTATION OF INTEGRAL MATHEMATIC APPLICATIONS IN COMMUNITY ECONOMIC LIFE

David

Faculty Engineering, Universitas Harapan Medan. North Sumatera Indonesia
Gedung arca Street, Medan North Sumatera, Indonesia

ABSTRACT

The characteristics of a function are usually investigated by looking at the continuity of the function. But what happens if a function does not have continuous properties? To what extent can the characteristics of continuous function be maintained for discontinuous cases? The stochastic function that is widely involved in solving problems in the field of average financial mathematics is a discontinuous function. This is reflected by the acquisition of a smooth curve from the modeling drawing obtained. Today, the nature of continuous functions in $[a, b]$ has been widely studied and developed. Some properties of the continuous function can be extended to the appropriate discontinuous function. In this paper described some integral reviews for discontinuous functions which are closely related to stochastic functions.

Keywords: *Henstock integral, Lebesgue integral, Riemann integral, stochastic function.*

1. INTRODUCTION

Mathematics for most people was initially limited to simple counting about objects around it and mostly only in dealing with money problems (Solovyeva, 2013). The first scientist to put applied mathematics to the forefront of economics and finance was Loius Bachelier in his dissertation on the theory of speculation who studied the continuous-time stochastic process of Brownian motion and its application to option pricing. Furthermore, Harry Markowitz in the mid-1950s studied portfolio selection to maximize returns by holding several stocks by studying stock moments and finding the mean-variance optimization problem (Dey et al., 2008).

This is the starting point for the growth of modern portfolio theory with a statistical approach (Canny et al., 2014). William Sharpe at the same time used mathematics in determining the correlation between each stock and the market. Later, Robert Merton and Paul Samuelson replaced the one-period model with continuous time, the Brownian model of motion, and the quadratic utility function implied in the mean-variance optimization is replaced by a more general increase in the utility function. Subsequent major leaps in the development of option pricing were undertaken by Fisher Black and Myron Scholes who applied stochastic calculus for options involving various stochastic differential equations (PDS) which subsequently became a new era of stochastic theory (Capovilla et al., 2016).

Along with the development of applied mathematics in terms of economics and finance, and finding solutions for them will impact the development of other sciences (Aryza et al., 2018). Integral theory as an alternative to solving differential equations, which initially in economics and finance is only used to find the original function of the marginal function, the total cost function, the total revenue function of the marginal revenue function, the consumption function

of the marginal consumption function, the saving function of a marginal saving function, and the capital function of the investment function, are now widely researched and used to solve problems related to stochastic differential equations. Integral theory is a branch of analytical mathematics which is deductive-axiomatic and theoretically still growing and developing today (Anggris et al., 2018). However, its application to real problems has not been widely studied and developed. One of the problems that have recently received a lot of attention is the integral theory approach related to the field of financial mathematics (financial) (Nurillah, 2014). In the financial sector, the characteristics of the value of stock prices that change with time with unexpected patterns, cause common stock price movements to be modeled as a stochastic process, one of which is presented in a form of PDS. So far, the PDS is often only solved using stochastic integrals. Stochastic integrals develop from Stieltjes type integrals which can solve the problem \int provided that the function varies limited to this paper will be studied about several theoretical review integrals for discontinuous functions that are applied to models that are close to stochastic functions (Nurillah, 2014).

II. METHOD OF RESEARCH.

This research is basic and is very theoretical, so stages such as experiments to obtain data will certainly not be carried out explicitly. The research begins with a study of literature both presented in the form of books, journals and research reports relevant to the topic to be discussed, namely examining integral theory, in this case the integral application in economics and finance. The systematic literature review that is carried out is by studying the results of previous research related to the topics discussed, whether in books, journals or those that have been standardized in a monograph. In theoretical mathematical research, the research methodology is included in the research process itself through critical and logical thinking, for example in proving theorems or theories. From the results of this analysis, new statements are constructed in the form of propositions. The process of forming propositions is carried out in two ways. First, propositions are compiled based on relevant results that have been cited from literature reviews.

III. ANALYZED AND RESULT

The discussion in this paper will begin with first examining the integral theory which is then followed by the description of stochastic models and stochastic differential equations related to the financial sector including the Brownian Motion stochastic model and the Black Scholes Model.

3.1. Teori Integral

Integral theory emerges through defining descriptively and constructively. The descriptive definition was carried out by Leibniz and Newton by using an anti-derivative of a function. Then Bernhard Riemann constructively put forward the theory of integrals using partition, which became known as the Riemann integral. (Lee, 1989). The existence of a non-integrated Riemann function for functions valued at 0 and 1 inspired Henri Lebesgue to develop the Lebesgue integral which is carried out using the size theory

approach. Lebesgue integrals can be applied to a finite function defined in a finite set (Burk, 2007). Jaroslav Kurzweil (1957) developed fine-partitioning on straight lines to ensure the convergence of the sequences of the obtained differential equation solutions. On the other hand, because there are still many functions which are quite good in formulation but are not integrated Riemann and Lebesgue integrals, Ralph Henstock (1960) developed the Riemann integral by changing the role of positive constants in Riemann integrals into positive functions. Since the definitions of the integral compiled by Henstock and Kurzweil are both equivalent, the integral has become known as the Henstock-Kurzweil integral. Henstock's greatest motivation in developing this integral is his desire to formulate an integral theory that can be used to solve all existing integral problems (Lee and Vyborny, 2000).

Furthermore, Thomas Joannes Stieltjes, who developed the Riemann integral by adding a certain real value function as an integrator function, inspired the emergence of the definition of the Stieltjes type integral. Riemann integral expanded by Stieltjes, becoming an integral Riemann-Stieltjes. This integral consists of two real-valued functions at a closed and finite interval, namely as an integrated function (integrand) and as an integrator function, written as \int entitled "The Pricing of Options and Corporate Liabilities". In Brown's motion, it can be defined as a motion that changes quite briefly. Some of the important properties of Brownian motion are: finite nature, continuous nature, Markov property, properties. In this case the function is required to be Martingale, has quadratic variance, and normality. Meanwhile, the so-called Brownian motion is a monotonous ascending function, or function and is assumed to have no shared discontinuous points (Rudin, 1976). Furthermore, the discovery of the concept of a finite variable set of functions and the proof that each function that varies finite can be expressed as the difference of two monotonous ascending functions (Lee and Vyborny, 2000) has an impact on the expansion of the theorem from the Riemann-Stieltjes integral with the integrator of the monotonically ascending function to the Riemann-Stieltjes integral with finite variable function integrator, where by the difference, the finite variable function integrator always plays an important role in giving more symmetrical results. (Burkill, 1970). Apostol (1975) guarantees that the Riemann-Stieltjes integral is a more general concept than the Riemann integral, considering that if the integrator function is an identity function then the Riemann-Stieltjes integral becomes a Riemann integral. This then inspired the formation of the definition of the Lebesgue-Stieltjes integral and the Henstock-Stieltjes integral. The Lebesgue-Stieltjes integral is an extension of Riemann-Stieltjes, so that the prevailing properties and theorems are still maintained (Carter and Brunt, 2000). This condition is also guaranteed by Horst (1984) who states that the Riemann-Stieltjes integrated function collection is a subset of the Lebesgue-Stieltjes integral collection. With the same generalization concept, it is also obtained that the Lebesgue-Stieltjes integrated set is a subset of the Henstock-Stieltjes integrated set.

3.2. Model Stokastik Gerak Brown

The Wiener process (or Brownian motion) is the starting point in the development of stochastic integrals (Hassler, 2016). In 1900, Louis Bachelier modeled stock price movements following Brownian motion with constant drift. In 1973, Fischer Black and Myron Scholes published a paper on the theory and calculation of options (Brown Motion) on stochastic processes when the following conditions apply; $\{ \}$ has an increment of independent stability and is normally distributed with zero mean and variance

3.3. Model Black Scholes

The Black - Scholes model was developed by Myron Scholes and Fischer Black in 1973. The assumptions affecting the Black – Scholes model include that the option referred to is the European option, valid when time runs out; volatility (price variance) is constant (fixed) as long as the option's life is known for sure, the shares used are not dividends and taxes and transaction costs are ignored.

3.4. Stochastic-Type Integral Approach to Stochastic Functions

Defining stochastic integrals in basically develops from a type integral Stieltjes. This type of integral is possible an integrator in the form of a function defined at certain intervals, during the function it still has real value. So existence tribe $\int_0^T g(X(t))dW(t)$ in the PDS still lets solve with integrals of type Stieltjes. In this integration, the characteristics of a function either function as integers and functions as integrators is an important thing that must be considered in order to meet the requirements that are allowed in the Stieltjes type integral. The following will given examples of integral studies in function that approximates the geometry model of stochastic function. An example will be given take the problem of sawtooth function (sawtooth function).

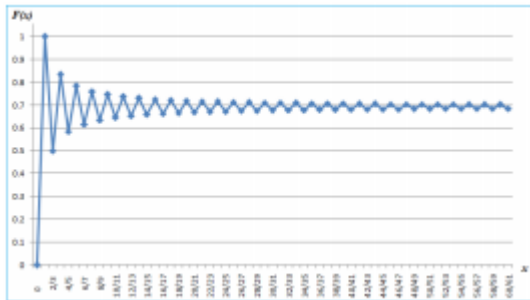
This function is taken given the existence of its function resembles conditions of stochastic functions with order more organized.

$$F(x) = \begin{cases} 0, & \text{untuk } x = 0 \\ \sum_{k=1}^n (-1)^{k+1} \binom{1}{k}, & \text{untuk } x = \left(\frac{n}{n+1}\right), n = 1, 2, 3, \dots \\ \ln 2, & \text{untuk } x = 1 \\ \left(\frac{F\left(\frac{n}{n+1}\right) - F\left(\frac{n-1}{n}\right)}{\frac{n}{n+1} - \frac{n-1}{n}} \right) \left(x - \frac{n-1}{n} \right) + F\left(\frac{n-1}{n}\right), & \text{untuk } x \in \left(\frac{n-1}{n}, \frac{n}{n+1}\right) \end{cases}$$

Figure 1. Function Of differential

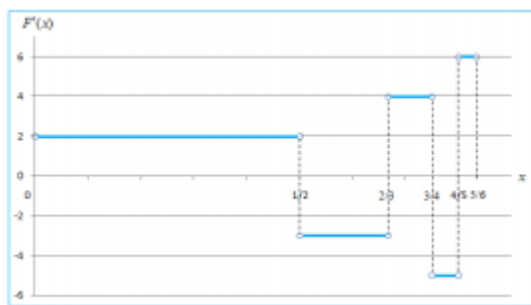
We will investigate whether the function is derived from terintegral Riemann, Lebesgue or Henstock? Furthermore, it will also be investigated whether its a function integrator the function is uniformly distributed still terintegral and integral type Stieltjes still be maintained? To answer that point first will draw the geometric shape of the function $f(x)$ the. An illustration of the

function $f(x)$ can be seen in Figure 2 below:



Figures 2. Function Graph

Illustration function $F(x/)$ we can see figure in below



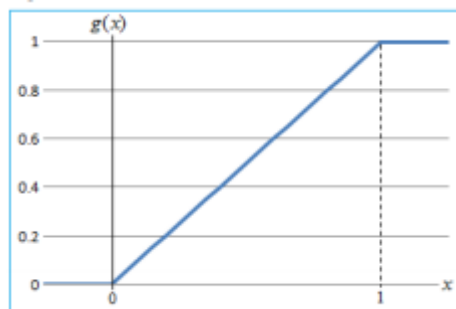
Figures 3. Function Of Graph

In this case the function $f(x)$ is discontinuous at finite point if n is finite, will be but will be a discontinuous function at infinity point if n is headed infinity. It will be investigated whether the derivative function of $f(x)$ is Riemann's integrated $f(x)$, Lebesgue's integrated or Henstock's integral.

$$\begin{aligned}
 (L)\int_0^1 |f| d\mu &= (L)\int_0^1 |F'| d\mu = \ln 2 > (L)\int_0^1 |F^n| d\mu \\
 &= 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{12}\right) + \dots + (n+1)\left(\frac{1}{n(n+1)}\right) \\
 &= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} > \ln 2
 \end{aligned}$$

There is a contradiction, so that presumption must be denied. Are you in or not terintegral Lebesgue on

The illustration of the function $g(x)$ can be seen in Figure 4 follows:



Figures 4. Function Graph $g(x)$

In this case it can be seen that the function $g(x)$ continuous at $[0,1]$, so that function is uniform distribution can be used as an integrator of type integrals Stieltjes. Thus the value is obtained calculate from the Henstock-Stieltjes integral is the expected value of the function $f(x)$ is :

$$E(f) = \int_0^1 f(x) dg(x) = F(1) \cdot 1 - F(0) \cdot 0 = \ln 2 - 0 = \ln 2 = 0,69315$$

If you want it is known which type of integral Stieltjes will be used it requires a study or more research especially related to integrant function to be integrated which is contained in the PDS.

IV. CONCLUSION.

Stochastic integral in essence developed from the integral type Stieltjes so PDS can be solved by approach integral of Stieltjes. The existence of a stochastic function as either an integer or an integrator on each PDS tribe must be investigated one by one making it possible for the PDS solved by one of the integrals of type Stieltjes. Further studies are mainly related on the integrant function to be integrated must be done if you want to know the type integral Stieltjes to be used. With know the characteristics of the integrant function and integrator function on the PDS is expected make it easy and allow it in looking for an exact solution to a PDS.

REFERENCES.

- Anggris, M. F., Ananta, M. T., & Az-zahra, H. M. (2018). *Rancang Bangun Aplikasi Augmented Reality Pengelolaan Rambu-Rambu Lalu Lintas Menggunakan Global Positioning System (GPS) pada Android*. 2(8), 2892–2901.
- Aryza, S., Irwanto, M., Khairunizam, W., Lubis, Z., Putri, M., Ramadhan, A., Hulu, F. N., Wibowo, P., Novalianda, S., & Rahim, R. (2018). An effect sensitivity harmonics of rotor induction motors based on fuzzy logic. *International Journal of Engineering and Technology(UAE)*, 7(2.13 Special Issue 13), 418–420. <https://doi.org/10.14419/ijet.v7i2.13.16936>
- Canny, U., Detector, E., & Filter, R. M. (2014). *The international journal of science & technoledge*. 2(6), 317–325. <http://theijst.com/june2014/51.ST1406-137.pdf>
- Capovilla, D., Hubwieser, P., & Shah, P. (2016). DiCS-Index: Predicting student performance in



- computer science by analyzing learning behaviors. *Proceedings - 2016 International Conference on Learning and Teaching in Computing and Engineering, LaTiCE 2016*, 136–140. <https://doi.org/10.1109/LaTiCE.2016.12>
- Dey, A., Tripathi, A., Singh, B., Dwivedi, B., & Chandra, D. (2008). An Improved Model of a Three phase Induction Motor Incorporating the Parameter Variations. *Electrical Power Quality and Utilisation, XIV(1)*, 73–78.
- Nurillah, A. S. (2014). Pengaruh Kompetensi Sumber Daya Manusia, Penerapan Sistem Akuntansi Keuangan Daerah (SKPD), Pemanfaatan Teknologi Informasi, dan Sistem Pengendalian Intern Terhadap Kualitas Laporan Keuangan Pemerintah Daerah (Studi Empiris Pada SKPD Kota Depok). *Skripsi Universitas Diponegoro*, 3, 1–13. <https://doi.org/2337-3806>
- Solovyeva, E. (2013). *Mathematical Models and Stability Analysis of Induction Motors under Sudden Changes of Load*. https://jyx.jyu.fi/dspace/bitstream/handle/123456789/42614/978-951-39-5521-2_vaitos17122013.pdf

