International Journal in Physical and Applied Sciences
Volume 07 Issue 11, November 2020 ISSN: 2394-5710 Impact Factor: 5.578
Journal Homepage: http://ijmr.net.in, Email: irjmss@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal

JACOBIAN INTEGRAL OF A CABLE-CONNECTED SATELLITES SYSTEM IN ELLIPTICAL ORBIT

Sangam Kumar
P. G. Department of Physics, L. S. College,
B. R. A. Bihar University, Muzaffarpur- 842001, India.
Ashish Kumar
P.G. Department of Physics, M.S. College, Motihari

Abstract

Aim of the present research work is to obtain the Jacobian integral of the motion of a system of two artificial satellites connected by a light, flexible, inextensible and non-conducting cable under the influence of solar radiation pressure, shadow of the earth, earth's magnetic field, earth's oblateness and air resistance. We discuss the case of elliptical orbit of centre of mass of the system. We do not consider neutation and wobbling of the orbit.

Key-words: *Satellites system, Jacobian integral, Elliptical orbit.*

1. Introduction:

We obtain Jacobian integral of the motion of a system of two cable-connected artificial satellites under the influence of solar radiation pressure, earth's magnetic field, shadow of the earth, earth's oblateness and air resistance. Influence of the above mentioned perturbations on the system has been studied singly and by a combination of any two or three or four of them by various workers, but never conjointly all at a time. Therefore, these could not give a real picture of motion of the system. This fact has initiated the present research work. The case of elliptical orbit of the centre of mass of the system is discussed. Shadow of the earth is taken to be cylindrical and the system is allowed to pass through the shadow beam. The satellites are connected by a light, flexible, inextensible and non-conducting cable. The satellites are taken as charged material particles. Since masses of the satellites are small and distances between the satellites and other celestial bodies are very large, the gravitational forces of attraction between the satellites and other celestial bodies including the sun have been neglected.

Beletsky and Novikova (1969) and Beletsky and Novoorebelskii (1969) are the pioneer workers in the area of researches related to a cable – connected satellites system. They studied the motion of a system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of the earth relative to its centre of mass. This study assumed that the two satellites are moving in the plane of the centre of mass of the system. Singh and Demin (1972) and Singh (1973) investigated the problem in two and three dimensional cases. Das et. al. (1976) studied the effect of magnetic force on the motion of a system of two cable-connected satellites in orbit. Kumar and Bhattacharya (1995) studied the stability of equilibrium positions of two cable-connected satellites under the influence of solar radiation pressure, earth's oblateness and earth's magnetic field. Kumar et. al. (2005) obtained the equations of motion of a system of two cable-connected artificial satellites under the influence of solar radiation pressure, earth's oblateness and shadow of the earth.

2. Treatment of the problem:

Equations of motion of one of the satellites when the centre of mass moves along Keplerian elliptical orbit in Nechvill's co-ordinate system (1926) are written as Kumar (2017)

$$X'' - 2Y' - 3X\rho = \lambda_a X - \frac{A}{\rho}\cos i - \gamma\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right).\cos\in\cos(\nu-\alpha) - f\rho\rho' + \frac{12\mu K_2}{R^5}\rho X$$

and

$$Y'' + 2X' = \lambda_a Y - \frac{A\rho'}{\rho^2}\cos i + \gamma\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right).\cos\in\sin(\nu-\alpha) - f\rho^2 - \frac{3\mu K_2}{R^5}\rho Y$$

(1)

With the condition of constraint

$$X^2 + Y^2 \leq \frac{1}{\rho^2} \tag{2}$$

Also,

$$\rho = \frac{1}{(1+e\cos\nu)},$$

$$\lambda_a = \frac{p^3\rho^4}{\mu}\left(\frac{m_1+m_2}{m_1 m_2}\right)\lambda = \rho^4\beta,$$

$$\beta = \frac{p^3}{\mu}\left(\frac{m_1+m_2}{m_1 m_2}\right)\lambda, A = \left(\frac{m_1}{m_1+m_2}\right)\left(\frac{Q_1}{m_1} - \frac{Q_2}{m_2}\right)\frac{\mu_E}{\sqrt{\mu\rho}}$$

$$f = \frac{a_1 p^3}{\sqrt{\mu p}}, a_1 = \rho_a R'(c_2 - c_1)\left(\frac{m_1}{m_1+m_2}\right), K_2 = \frac{\overline{\in} {R_e}^2}{3}, \overline{\in} = \alpha_R - \frac{m}{2} = \frac{\Omega^2 R_e}{g_e} \tag{3}$$

m_1 and m_2 are masses of the two satellites. B_1 and B_2 are the absolute values of the forces due to the direct solar pressure on m_1 and m_2 respectively and are small. Q_1 and Q_2 are the charges of the two satellites. μ_E is the magnitude of magnetic moment of the earth's dipole. p is the focal parameter. μ is the product of mass of the earth and gravitational constant. λ is undermined Lagrange's multiplier. g_e is the force of gravity. e is eccentricity of the orbit of the centre of mass. v is the true anomaly of the centre of mass of the system. ∈ is inclination of the oscillatory plane of the masses m_1 and m_2 with the orbital plane of the centre of mass of the system. α is the inclination of the ray. γ is a shadow function which depends on the illumination of the system of satellites by the sun rays. If γ is equal to zero, then the system is affected by the shadow of the earth. If γ is equal to one, then the system is not within the said shadow. R is the modulus of position vector of the centre of mass of the system. c_1 and c_2 are the Ballistic coefficients. ρ_a is the average density of the atmosphere. i is inclination of the orbit with the equatorial plane. θ_2 is the angle between the axis of the cylindrical shadow beam and the line joining the centre of the earth and the end point of the orbit of the centre of mass within the earth's shadow, considering the positive direction towards the motion of the system. Prime denotes differentiation with respect to v.

If motion of one of the satellites m_1 be determined with the help of equations (1), motion of the other satellite of mass m_2 can be determined by Kumar[6]

$$m_1\overline{\rho_1} + m_2\overline{\rho_2} = 0 \tag{4}$$

Where $\overline{\rho_j}$ (j = 1, 2) is the radius vector in the centre of mass system.

For the analysis of the small secular and long periodic effects of the solar pressure and the effects of the earth's shadow on the system, the periodic terms in the equations of motion (1) may be averaged with respect to ν as

(i) From θ_2 to $(2\pi - \theta_2)$ for a period, when the system is under the influence of sun rays directly i.e. $\gamma = 1$.

and

(ii) From $-\theta_2$ to $+\theta_2$ for a period, when the system is under the influence of shadow of the earth i.e. $\gamma = 0$.

The averaged values of the secular terms due to the periodic forces in the equations of motion (1) can be deduced as given below

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\rho}dv = 1, \frac{1}{2\pi}\int_0^{2\pi}\rho dv = \frac{1}{(1-e^2)^{1/2}}, \frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\rho^2}dv = \left(1+\frac{1}{2}e^2\right)$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^2 dv = \frac{1}{(1-e^2)^{3/2}}, \frac{1}{2\pi}\int_0^{2\pi}\rho\rho' dv = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^3 dv = \frac{(2+e^2)}{2(1-e^2)^{5/2}}, \frac{1}{2\pi}\int_0^{2\pi}\rho'\rho^{-2} dv = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\rho^4 dv = \frac{(2+3e^2)}{2(1-e^2)^{7/2}}, \frac{1}{2\pi}\int_0^{2\pi}\rho'\rho^{-1} dv = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi}\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\cos(\nu-\alpha)\,dv = -\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\cos\alpha\sin\theta_2}{\pi}$$

and

$$\frac{1}{2\pi}\int_0^{2\pi}\gamma\left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\cos\in\sin(\nu-\alpha)\,dv = \left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\in\sin\alpha\sin\theta_2}{\pi}$$

(5)

Using (5), equations (1) may be written as

$$X'' - 2Y' - \frac{3X}{(1-e^2)^{\frac{1}{2}}} = \beta\frac{(2+3e^2)}{2(1-e^2)^{\frac{7}{2}}}X - A\cos i + \left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right)\frac{\cos\alpha\cos\in\sin\theta_2}{\pi} + \frac{12\mu k_2}{R^5}.\frac{1}{(1-e^2)^{1/2}}X$$

and

$$Y'' + 2X' = \beta\frac{(2+3e^2)}{2(1-e^2)^{\frac{7}{2}}}Y + \left(\frac{B_1}{m_1}-\frac{B_2}{m_2}\right).\frac{\sin\alpha\cos\in\sin\theta_2}{\pi} - \frac{f}{(1-e^2)^{\frac{3}{2}}} - \frac{3\mu k_2}{R^5}.\frac{1}{(1-e^2)^{1/2}}Y$$

(6)

The condition of constraint given by (2) takes the form

$$X^2 + Y^2 \leq \left(1+\frac{1}{2}e^2\right)$$

(7)

Equations (6) do not contain the time explicitly. Therefore, Jacobian integral of the problem exists over hare.

Multiplying the first and second equations of (6) by X' and Y' respectively and adding them we obtain,

$$X'X" + Y'Y" - \frac{3}{(1-e^2)^{\frac{1}{2}}} XX' = \beta \frac{(2+3e^2)}{2(1-e^2)^{\frac{7}{2}}} (XX' + YY') - AX' \cos i + \left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right) . \frac{\cos\in \sin\theta_2}{\pi} (X' \cos\alpha + Y' \sin\alpha) - \frac{fY'}{(1-e^2)^{\frac{3}{2}}} + \frac{3\mu k_2}{R^5} . \frac{1}{(1-e^2)^{1/2}} (4XX' - YY') \tag{8}$$

Integrating (8), the expression comes to

$$X'^2 + Y'^2 - \frac{3}{(1-e^2)^{\frac{1}{2}}} X^2 = \beta \frac{(2+3e^2)}{2(1-e^2)^{\frac{7}{2}}} (X^2 + Y^2) - 2AX \cos i + \frac{2}{\pi}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)$$

$$\cos \in \sin\theta_2 (X \cos\alpha + Y \sin\alpha) - \frac{2fY}{(1-e^2)^{\frac{3}{2}}} + \frac{3\mu k_2}{R^5} . \frac{1}{(1-e^2)^{1/2}} (4X^2 - Y^2) + h$$

(9)

h is the constant of integration. It is known as Jacobian constant.
The surface of zero velocity can be obtained in the form

$$\frac{3}{(1-e^2)^{\frac{1}{2}}} X^2 + \beta \frac{(2+3e^2)}{2(1-e^2)^{\frac{7}{2}}} (X^2 + Y^2) - 2AX \cos i + \frac{2}{\pi}\left(\frac{B_1}{m_1} - \frac{B_2}{m_2}\right)$$

$$\cos \in \sin\theta_2 (X \cos\alpha + Y \sin\alpha) - \frac{2fY}{(1-e^2)^{\frac{3}{2}}} + \frac{3\mu k_2}{R^5} . \frac{1}{(1-e^2)^{1/2}} (4X^2 - Y^2) + h = 0$$

(10)

Thus it is concluded that the satellite m_1 moves inside the boundary of different curves of zero velocity represented by (10) for different values of the Jacobian constant.

3. Result and Discussion:

Purpose of the present research work is to obtain Jacobean integral of motion of a system of two cable – connected satellites under the influence of several perturbative forces like shadow of the earth, solar radiation pressure, oblateness of the earth, earth's magnetic field and air resistance in elliptical orbit. The cable connecting the two satellites is light, flexible, non – conducting and inelastic in nature. The satellites are considered as charged material particles. As the body of the satellites is made up of metal, the satellites cut magnetic lines of force of the earth during the motion. As a result of which charges get developed on the two satellites. But magnitudes of the charges is very small. Thus, electrostatic interaction between the satellites is not taken into account.

4. Conclusion:

Equation (9) is the required Jacobean integral of motion of the system, when centre of mass of the system moves in elliptical orbit. It is also known as the first integral of motion of the system. It has wide applications in further studies of the problems related to two cable connected artificial satellites moving in orbit either at high altitude or at low altitude.

5. Acknowledgement:

We are thankful to Prof. R. K. Sharma from Thiruvanathapuram for his encouragement.

References

[1] Beletsky, V. V. and Novikova, E. T.- Kosmicheskie Issledovania **7(6)**, pp. 377-384 (1969).

[2] Beletsky, V. V. and Novoorebelskii, A. B.- Inst. of App. Maths., Acad. of Sci. of the U. S. S. R. **17**, pp. 213-219 (1969).

[3] Das, S. K., Bhattacharya, P. K. and Singh, R. B.- Proc. Nat. Acad. Sci., India **46**, pp. 287-299 (1976).

[4] Kumar, S. and Bhattacharya, P. K.- Proc. Workshop on Spa. Dyn. and Cel. Mech., Muz., India **Eds. K. B. Bhatnagar and B. Ishwar**, pp. 71-74 (1995).

[5] Kumar, S., Srivastava, U. K. and Bhattacharya, P. K.- Proc. Math. Soc., B. H. U., Varanasi **21**, pp. 51-61 (2005).

[6] Kumar, S.- International Jour. in Phy. & Appl. Sci., **04 (Issue-07)**, pp. 01-11 (2017).

[7] Singh, R. B. and Demin, V. G.- Cel. Mech. **06**, pp. 268-277 (1972).

[8] Singh, R. B.- Astronau. Acta **18**, pp. 301-308 (1973).

[9] Nechvile, V.- Acad. Paris Compt. Rend. **182**, pp. 310-322 (1926).