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# EXPLAINING NUMERICAL METHOD FOR SOLVING FUZZY DIFFERENTIAL EQUATION

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## ABSTRACT

*Fuzzy differential equations (FDEs) play important roles in modeling dynamic systems in science, economics and engineering. The modeling roles are important because most problems in nature are indistinct and uncertain. Numerical methods are needed to solve FDEs since it is difficult to obtain exact solutions. Many approaches have been studied and explored by previous researchers to solve FDEs numerically. Most FDEs are solved by adapting numerical solutions of ordinary differential equations. In this study, we propose the extended Trapezoidal method to solve first order initial value problems of FDEs. The computed results are compared to that of Euler and Trapezoidal methods in terms of errors in order to test the accuracy and validity of the proposed method. The results shown that the extended Trapezoidal method is more accurate in terms of absolute error. Since the extended Trapezoidal method has shown to be an efficient method to solve FDEs, this brings an idea for future researchers to explore and improve the existing numerical methods for solving more general FDEs.*

**Keyword: -** Fuzzy, Equations, Fuzzy Differential Equations, Application, Linear

## I. INTRODUCTION

The concept of a crisp set is fundamental in mathematics. According to, a crisp set is a set that consists of elements with either full or no membership in the set. The principle notion in set theory can be defined as element assigned with value of either 1 or 0 to represent whether it belongs or does not belong to the set respectively. For example, the questions, ‘is the woman tall?’, and ‘is the man handsome?’ in crisp set can be answered with yes (1) or no (0). LotfiZadeh in, a mathematician by profession, first introduced the idea of fuzzy mathematical framework in 1965 which defined it as a class of objects that involves partial set membership. It is called as fuzzy theory, which is a reasonable tool for modeling imprecision and ambiguity in mathematical models. The theory appears in many disciplines including medicine, science and engineering. It is also very useful in decision making process that involves human perception. According to [2], a fuzzy set is a set that allows the membership function to take any value in the interval of Uncertainty level of a fuzzy set can be measured by using the level of fuzziness. The measure of fuzziness is represented by the function such that where the set gathering is all subsets of the universal set and is the real number domain.

The study of fuzzy differential equations (FDEs) is considered as a new branch of fuzzy mathematics. Modeling with FDEs is more relevant and appropriate in representing uncertain systems. The researches in FDEs have been rapidly growing over the last few decades. Chang and Zadeh in first introduced the concept of fuzzy derivative. It has been used as conditions to solve fuzzy problems. Meanwhile, introduced concepts that are related to differential equations. Based on the extension principle of Zadeh, the researchers defined the concepts of fuzzy derivative where the differentiation of ordinary functions at a fuzzy point and fuzzy-valued functions at a non-fuzzy point. This approach satisfies the generalized Lipschitz condition in which the fuzzy initial-value problems (IVPs) have specific solution.

Finding solutions for FDEs is very fundamental. Hukuhara differentiability is the most popular approach in solving fuzzy valued functions. Kaleva in has studied further on the concepts of fuzzy-set-valued mappings of a real variable, which the values are normal, convex, upper semi continuous, and support fuzzy sets. Most FDEs can be solved using several approaches, either numerically or analytically. However, for some FDEs, it is difficult to find analytical solutions. Numerical method is an effective way to solve FDEs. According to, numerical solution of FDEs is obtained by extending the current classical methods to the fuzzy version of differential equations. Finding actual solutions for fuzzy IVPs can be quite difficult and at times are almost impossible to obtain. In order to overcome these problems, most researchers employ numerical methods for solving the IVPs which can be made as accurately as possible. Conventionally, most of the numerical methods for solving fuzzy IVPS are adapted from that of the numerical methods for solving ODEs. Researchers such as have worked on various one-step methods. Meanwhile, authors in have worked on family of Adam-Bashforth_Moulton methods.

Since FDEs are applicable in many real life problems, researchers still need to improve and develop numerical methods in order to find better solutions for FDEs. More researchers in have also proposed various methods to solve FDEs numerically. This study aims to improve the accuracy of the numerical solution of FDEs. Trapezoidal method has been seen to be able to solve FDEs but current practice has less accuracy with error in approximating the solution for large step-size. We propose extended Trapezoidal method to solve first-order IVPs of linear FDEs numerically. The results are expected to be more accurate as compared to the existing methods.

## II. FUZZY DIFFERENTIAL EQUATIONS

This section presents various approaches dealing with a definition of a derivative of type-1 or type-2 fuzzy functions. The most important part of history of FDEs is formed by different definitions of fuzzy derivatives. As a matter of fact, since the concept of derivative is the fundamental element of a differential equation, the evolution of fuzzy derivatives plays a key role in the evolution of FDEs. The fuzzy derivatives may be classified as: integer order and fractional order fuzzy derivatives.

Integer order fuzzy derivatives are sub-classified as integer order fuzzy derivatives of type-1 fuzzy functions (or type-1 fuzzy derivatives), and integer order fuzzy derivatives of type-2 fuzzy functions (or type-2 fuzzy derivatives). Similarly, there are type-1 and type-2 fuzzy fractional derivatives. It should be noted that corresponding to each class or sub-class of fuzzy derivatives, FDEs may be classified.

For example, what may be referred to as type-1 fuzzy fractional differential equations is associated with FDEs in which the derivative is of the kind of type-1 fuzzy fractional derivatives.

## III. INTEGER ORDER FUZZY DIFFERENTIAL EQUATIONS

Although the term fuzzy differential equations for the first time emerged in the literature in 1978, FDEs, as they are known nowadays, was initiated in 1982 based on a definition of a fuzzy derivative which may be called DuboisPrade derivative. Thereafter, different definitions of fuzzy derivatives were proposed among which were Hukuhara derivative (or Puri–Ralescu derivative) presented in 1983, Goetschel-Voxman derivative in 1986, Seikkala derivative in 1987, and Friedman-Ming-Kandel derivative introduced in 1996, respectively.

In spite of the fact that all these fuzzy derivatives have been presented in different forms, it has been proved that they are equivalent provided that the subjected fuzzy function lower and upper α-level cuts are continuous functions, for more details see.

Among the mentioned fuzzy derivatives, Hukuhara and Seikkala derivatives are more widely known. The difference between the definitions of Hukuhara and Seikkala derivatives is that Hukuhara derivative (H-derivative) is, in essence, defined based on what is called Hukuhara difference (Hdifference), but Seikkala derivative is defined based on derivatives of the lower and upper α-level cuts of the fuzzy function in question. The existence and uniqueness of the solution for FDEs under H-derivative and Seikkala derivative have been investigated in.

A large number of studies conducted on FDEs, for instance see, and demonstrate that Hukuhara and Seikkala derivatives, despite being equivalent, are more palatable definitions. However, the research results have revealed that these derivatives suffer from a number of major limitations among which the most serious is that the diameter of the fuzzy function under study needs to be necessarily non-decreasing.

Such a limitation causes the obtained solution of an FDE, in a great number of cases, to differ from what is realized intuitively from the nature of the system or phenomenon modeled by the FDE.

As an illustration, the diameter of the obtained solution of an FDE in the form of $\dot{\tilde{x}}(t) = -\tilde{x}(t)$ whose initial condition is a fuzzy number, increases as time goes by. This is while we intuitively expect that the natural behavior of such a differential equation show that $\tilde{x}$ decreases as time passes. As a conclusion, considering such definitions in an FDE necessitates that the fuzziness of the solution be non-decreasing which imposes a great restriction on their real case applications.

## IV. FRACTIONAL ORDER FUZZY DIFFERENTIAL EQUATIONS

The idea for studying fuzzy differential equations of fractional order was first presented in 2010. There are different definitions of classical fractional derivatives, namely in the sense of Riemann-Liouville, Caputo, Modified Riemann-Liouville, conformable fractional derivative, Caputo-Fabrizio fractional derivative, to name but a few.

Possible combinations of such derivatives with the concepts of fuzzy derivatives and/or fuzzy differences have resulted in the introduction of different definitions of fuzzy fractional derivatives based on which fuzzy fractional differential equations (FFDEs) have been examined.

The Riemann-Liouville fuzzy fractional derivative in the sense of H-derivative; and the existence and uniqueness of the solution for a class of FFDEs with infinite delay were presented in 2010. In 2011, the Riemann-Liouville fuzzy fractional derivative in the sense of Seikkala derivative was proposed in; and the existence and uniqueness of the solution for FFDEs with fuzzy initial conditions under such a derivative have been shown in.

## V. CONCLUSION

A technique for approximating the solution of a time dependent system of fuzzy linear differential equations is presented. The necessary and sufficient condition for the existence of a fuzzy solution is proposed. The original fuzzy system of differential equations with matrix coefficient A is transformed by a $2n \times 2n$ crisp linear system of differential equations with matrix coefficient S.

This paper has presented a brief survey on the evolution of fuzzy differential equations. The particular attention has been given to FDEs in which a definition of fuzzy derivative of fuzzy number-valued functions has been considered. Through a selective list of papers, the historical motivations and current research progress of FDEs have been outlined.

Great advances on both fundamental aspects and applications of FDEs have been made with a multitude of available publications on the topic. Nonetheless, some issues still remain challenging which provide opportunities for further research on FDEs in the future. What follows presents our personal perspectives on the issues and FDEs.

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